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MATH 126/126E FINAL EXAM SAMPLE

NOTE: The final exam will have 19 questions. The different parts of each question (part A, B, etc.) are variations. Know how to do all the variations on this exam.

1A.) (3 pts) Solve $\log_3(x-5)-\log_3(2x+3)=0$ 1A. No solution

$$\log_3\left(\frac{x-5}{2x+3}\right)=0$$
$$3^0=\frac{x-5}{2x+3}$$
$$1=\frac{x-5}{2x+3}$$

$2x+3=x-5 \quad \Rightarrow \quad x=-8$ However, this is not in the domain, so no solution.

1B.) (3 pts) Solve $\log_2(x+5)+\log_2(2x-5)=3$ 1B. $x=3$

$$\log_2(x+5)(2x-5)=3$$
$$2^3=(x+5)(2x-5)$$
$$8=2x^2+5x-25$$
$$2x^2+5x-33=0$$
$$(2x+11)(x-3)=0$$

$x=-11/2$ and $x=3$
However, if you put -11/2 into the original equation, you will get a negative number inside the log. Therefore 3 is the only answer.

2A.) (2 pts) Solve $2^{4x-3}-27=5$ 2A. $x=2$

$$2^{4x-3}=32$$
$$2^{4x-3}=2^5$$
$$4x-3=5$$
$$4x=8 \;\Rightarrow\; x=2$$

2B.) (2 pts) Solve $3^{1-2x}-21=6$ 2B. $x=-1$

$$3^{1-2x}=27$$
$$3^{1-2x}=3^3$$
$$1-2x=3$$
$$-2x=2$$
$$x=-1$$

3A.) (2 pts) Solve $10^{-x} = 1.6$ Round your answer to four decimal places. 3A. $x \approx -0.2041$

$$\log 10^{-x} = \log 1.6$$
$$-x = \log 1.6$$
$$x = -\log 1.6 \approx -0.2041$$

3B.) (2 pts) Solve $e^{x} = 4.5$ Round your answer to four decimal places. 3B. $x \approx 1.5041$

$$\ln e^{x} = \ln 4.5$$
$$x = \ln 4.5 \approx 1.5041$$

4A. (4 pts) Use the polynomial inequality $x^2 + 4x - 5 \geq 0$ to write the open intervals determined by the boundary points as they appear from left to right on a number line by filling in the table below. Then solve the inequality by writing the solution set in interval notation.

$(x-1)(x+5) = 0 \quad \Rightarrow \quad x = 1,\ x = -5$

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Test –6 Test 0 Test 2

Open Interval	$(-\infty, -5)$	$(-5, 1)$	$(1, \infty)$
Sign (+ or –)	+	–	+

The solution set (in interval notation) is: $(-\infty, -5] \cup [1, \infty)$

4B. (4 pts) Use the polynomial inequality $4x - x^2 < 0$ to write the open intervals determined by the boundary points as they appear from left to right on a number line by filling in the table below. Then solve the inequality by writing the solution set in interval notation.

$x(4-x) = 0 \quad \Rightarrow \quad x = 0,\ x = 4$

--------------- 0 ++++++++++ 4 --------------

Test –1 Test 1 Test 5

Open Interval	$(-\infty, 0)$	$(0, 4)$	$(4, \infty)$
Sign (+ or –)	–	+	–

The solution set (in interval notation) is: $(-\infty, 0) \cup (4, \infty)$

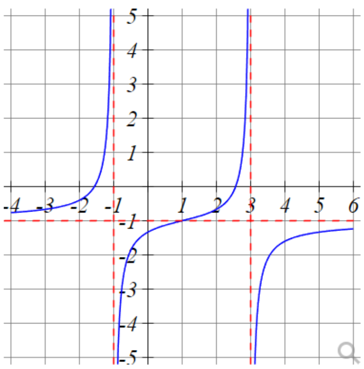
5A.) (3 pts) The graph of $f(x)$ has the below characteristics. Use this information to choose the correct graph of f below.

x-intercept: $(1, 0)$

y-intercept: $\left(0, \frac{1}{3}\right)$

vertical asymptotes: $x = -1,\ x = 3$

horizontal asymptote: $y = -1$



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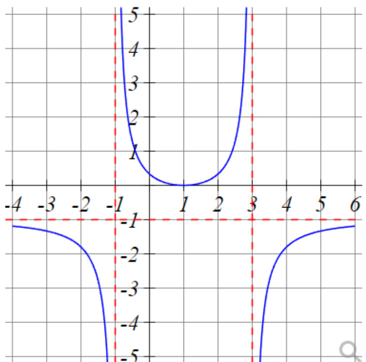

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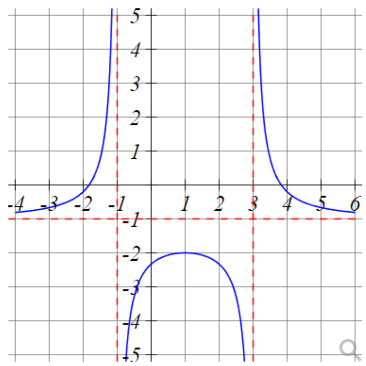

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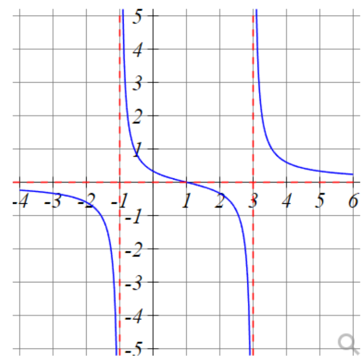


(d)

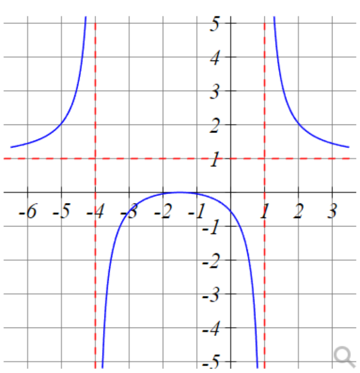
5B.) (3 pts) The graph of $f(x)$ has the below characteristics. Use this information to choose the correct graph of f below.

x-intercept: $\left(-\frac{3}{2}, 0\right)$

y-intercept: $\left(0, -\frac{3}{8}\right)$

vertical asymptotes: $x = -4,\ x = 1$

horizontal asymptote: $y = 0$



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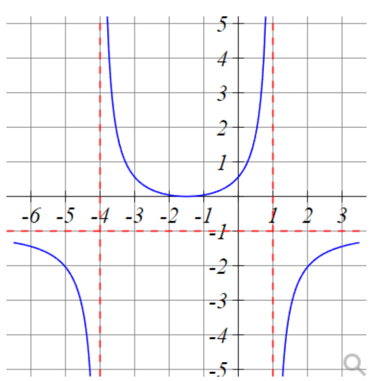

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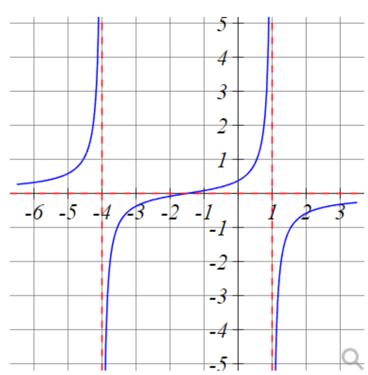

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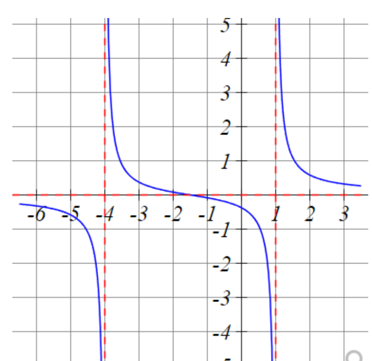

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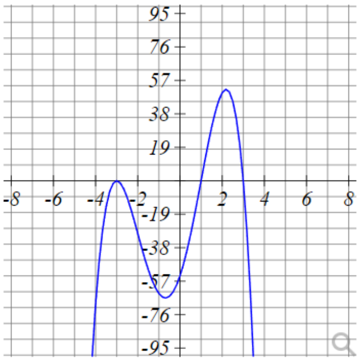
6A.) (3 pts) The graph of $f(x)$ has the below characteristics. Use this information to choose the correct graph of f below.

zeros: -3 (multiplicity 1), 1 (multiplicity 2), 3 (multiplicity 1)
leading term (power function): $-2x^4$



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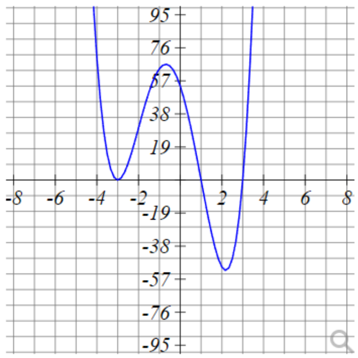

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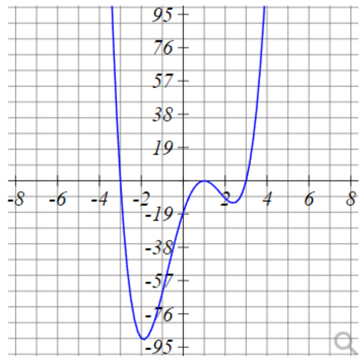

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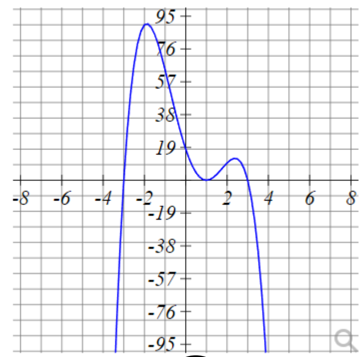

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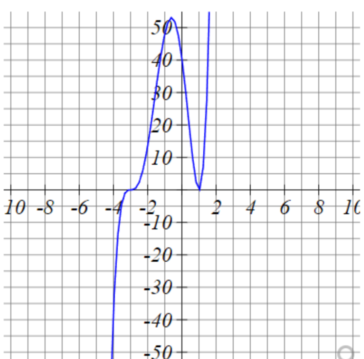
6B.) (3 pts) The graph of $f(x)$ has the below characteristics. Use this information to choose the correct graph of f below.

zeros: -3 (multiplicity 3), 1 (multiplicity 2)
leading term (power function): $\frac{3}{2}x^5$



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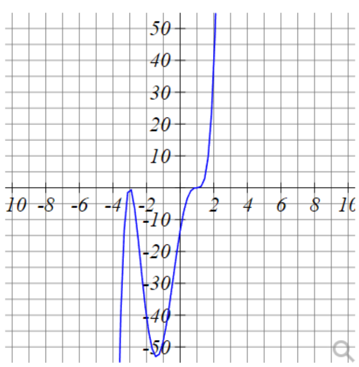

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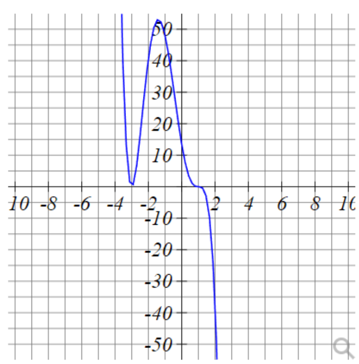

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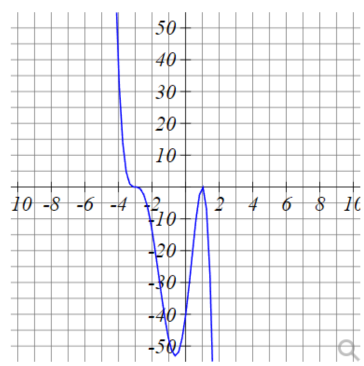

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7A. (3 pts) For the functions $f(x)=\sqrt{6-3x}$ and $g(x)=2x^2-3$, find $(f+g)(-1)$.

7A. $(f+g)(-1)=2$

$$(f+g)(x)=\sqrt{6-3x}+2x^2-3$$

$$(f+g)(-1)=\sqrt{6-3(-1)}+2(-1)^2-3$$

$$(f+g)(-1)=\sqrt{9}+2-3$$

$$(f+g)(-1)=2$$

7B. (3 pts) For the functions $f(x)=\dfrac{30}{2+x^3}$ and $g(x)=5-3x$, find $(f-g)(2)$.

7B. $(f-g)(2)=4$

$$(f-g)(x)=\frac{30}{2+x^3}-(5-3x)$$

$$(f-g)(2)=\frac{30}{2+2^3}-(5-3(2))$$

$$(f-g)(2)=\frac{30}{2+8}-(5-6)$$

$$(f-g)(2)=3-(-1)=4$$

8A.) (4 pts) Let $f(x)=3-2x^2$. Find the difference quotient. Use $\dfrac{f(x+h)-f(x)}{h}$.

8A. $-4x-2h$

$$f(x+h)=3-2(x+h)^2$$
$$f(x+h)=3-2\left(x^2+2xh+h^2\right)$$
$$f(x+h)=3-2x^2-4xh-2h^2$$

$$\frac{3-2x^2-4xh-2h^2-\left(3-2x^2\right)}{h}=\frac{3-2x^2-4xh-2h^2-3+2x^2}{h}=\frac{-4xh-2h^2}{h}=\frac{h(-4x-2h)}{h}=-4x-2h$$

8B.) (4 pts) Let $f(x)=3x^2-\dfrac{x}{5}$. Find the difference quotient. Use $\dfrac{f(x+h)-f(x)}{h}$.

8B. $6x+3h-\dfrac{1}{5}$

$$f(x+h)=3(x+h)^2-\frac{x+h}{5}$$
$$f(x+h)=3\left(x^2+2xh+h^2\right)-\frac{x+h}{5}$$
$$f(x+h)=3x^2+6xh+3h^2-\frac{x}{5}-\frac{h}{5}$$

$$\frac{3x^2+6xh+3h^2-\frac{x}{5}-\frac{h}{5}-\left(3x^2-\frac{x}{5}\right)}{h}=\frac{3x^2+6xh+3h^2-\frac{x}{5}-\frac{h}{5}-3x^2+\frac{x}{5}}{h}=\frac{6xh+3h^2-\frac{h}{5}}{h}=6x+3h-\frac{1}{5}$$

9A.) (3 pts) Given $f(x)=\begin{cases} x^2 & if \quad x\le 0 \\ -\sqrt{x} & if \quad x>0 \end{cases}$ find the following.

i.) $f(0)=0^2=0$
We need to use the first equation since 0 is included here.

ii.) $f(9)=-\sqrt{9}=-3$
We are using the second equation since 9 is greater than 0.

iii.) $f(-4)=(-4)^2=16$
We are using the first equation since 16 is less than 0.

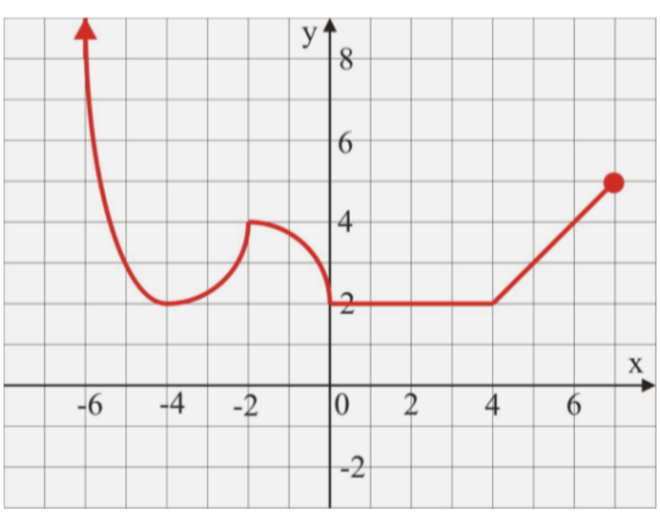
9B.) (3 pts) Given $f(x)=\begin{cases} |x| & if \quad x<0 \\ -4 & if \quad 0\le x<3 \\ -x-3 & if \quad x\ge 3 \end{cases}$ find the following.

i.) $f(5)=-5-3=-8$
We are using the third equation here since 5 is greater than 3.

ii.) $f(0)=-4$
We are using the second equation here since 0 is included.

iii.) $f(-3)=|-3|=3$
We are using the first equation here since -3 is less than 0.

10A.) (3 pts) Use the graph of $f(x)$ below to find the following.

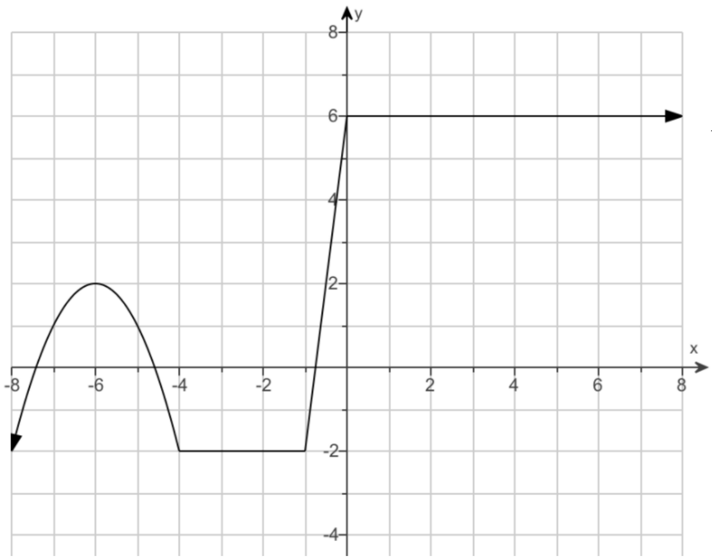


a.) interval(s) of increasing: $(-4,-2)\cup(4,7)$

b.) interval(s) of decreasing: $(-\infty,-4)\cup(-2,0)$

c.) interval(s) f is constant: $(0,4)$

10B.) (3 pts) Use the graph of $f(x)$ below to find the following.



a.) interval(s) of increasing: $(-\infty,-6)\cup(-1,0)$

b.) interval(s) of decreasing: $(-6,-4)$

c.) interval(s) *f* is constant: $(-4,-1)\cup(0,\infty)$

11A.) (3 pts) The function $f(x)=6x^3-5x^2-29x+10$ has a zero at $x=-2$. Use synthetic division and factoring to find the other zeros.

11A. $x=\frac{5}{2},\ \frac{1}{3}$

$$\begin{array}{r|rrrr} -2 & 6 & -5 & -29 & 10 \\ & & -12 & 34 & -10 \\ \hline & 6 & -17 & 5 & 0 \end{array}$$

$6x^2-17x+5=0$
$(2x-5)(3x-1)=0$

$x=\frac{5}{2},\ \frac{1}{3}$

11B.) (3 pts) The function $f(x)=27x^3-54x^2+27x-4$ has a zero at $x=\frac{4}{3}$. Use synthetic division and factoring to find the other zeros.

11B. $x=\frac{1}{3}$

$$\begin{array}{r|rrrr} 4/3 & 27 & -54 & 27 & -4 \\ & & 36 & -24 & 4 \\ \hline & 27 & -18 & 3 & 0 \end{array}$$

$27x^2-18x+3=0$
$3(9x^2-6x+1)=0$
$3(3x-1)(3x-1)=0$
$x=\frac{1}{3}$

12A.) (3 pts) Solve the system: $\begin{matrix} 2x-3y=-8 \\ 4x+5y=6 \end{matrix}$. Write your answer as coordinates. 12A. $(-1,2)$

These kinds of problems can be solved using either the elimination or substitution method. Here, it will be easier to use the elimination method since we do not see a single x or y term. So we can choose to eliminate either the x or y. If we eliminate x, then we only need to multiply one equation, but if we choose to eliminate the y, then we need to multiply both equations. Therefore, we will choose to eliminate the x terms. To do this, we will multiply the first equation by –2:

$$\begin{matrix} -2(2x-3y=-8) \\ 4x+5y=6 \end{matrix} = \begin{matrix} -4x+6y=16 \\ 4x+5y=6 \end{matrix}$$ Now we will add the equations together.

$11y=22$

$y=2$ Now we will plug this into the first equation to solve for x.

$2x-3(2)=-8$

$2x-6=-8$

$2x=-2$

$x=-1$ Our answer is the point $(-1,2)$.

12B.) (3 pts) Solve the system: $\begin{matrix} 4x-3y=5 \\ 6x+2y=1 \end{matrix}$. Write your answer as coordinates. 12B. $\left(\frac{1}{2},-1\right)$

Here, it will be easier to use the elimination method since we do not see a single x or y term. So we can choose to eliminate either the x or y. If we eliminate x or y, then we need to multiply both equations. Therefore, it does not matter which variable we choose to eliminate. Let's eliminate the y. To do this, we will multiply the first equation by 2 and the second equation by 3:

$$\begin{matrix} 2(4x+3y=5) \\ 3(6x-2y=1) \end{matrix} = \begin{matrix} 8x+6y=10 \\ 18x-6y=3 \end{matrix}$$ Now we will add the equations together.

$26x=13$

$x=\frac{1}{2}$ Now we will plug this into the first equation to solve for y.

$8\left(\frac{1}{2}\right)+6y=10$

$4+6y=10$

$6y=-6$

$y=-1$ Our answer is the point $\left(\frac{1}{2},-1\right)$.

13A.) (4 pts) Given $f(x)=\dfrac{2x-7}{x^3+16x}$ and $g(x)=\dfrac{\sqrt{3x-4}}{45}$, find the following:

i.) Domain of $f(x)$ in interval notation   i. $(-\infty,0)\cup(0,\infty)$

$f(x)=\dfrac{2x-7}{x(x^2+16)}$ $x(x^2+16)=0$ => $x=0$ or $x^2+16=0$.

Since x^2+16 can never be 0, the answer is all real numbers except 0.

ii.) Domain of $g(x)$ in interval notation   ii. $\left[\dfrac{4}{3},\infty\right)$

$3x-4\geq 0$

$3x\geq 4$

$x\geq \dfrac{4}{3}$

$\left[\dfrac{4}{3},\infty\right)$ We do include $\dfrac{4}{3}$ since it does not make the bottom zero.

13B.) (4 pts) Given $f(x)=\dfrac{x-4}{x^2-9}$ and $g(x)=\dfrac{8x-3}{\sqrt{11-5x}}$, find the following:

i.) Domain of $f(x)$ in interval notation   i. $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

$x^2-9\neq 0$ => $x\neq 3$ and $x\neq -3$

The domain is all real numbers not including 3 and -3.

ii.) Domain of $g(x)$ in interval notation   ii. $\left(-\infty,\dfrac{11}{5}\right)$

$11-5x>0$

$-5x>-11$

$x<\dfrac{11}{5}$

$\left(-\infty,\dfrac{11}{5}\right)$ We do not include $\dfrac{11}{5}$ since it makes the bottom zero.

14A. (3 pts) Use the function $f(x)=\frac{1-x}{x^2-9}$,

i.) Find the intercepts. x-int: $(1, 0)$

x-int: top=0 y-int: $y=0$

$1-x=0$ $y=\frac{1-0}{0^2-9}=-\frac{1}{9}$ y-int: $\left(0,-\frac{1}{9}\right)$

$x=1$

ii.) Find the asymptotes. Vertical: $x=3, x=-3$

V.A.: Set bottom = 0 H.A. (Rules)

$x^2-9=0$ $n<m$ so $y=0$ Horizontal: $y=0$

$x^2=9$

$x=\pm 3$

iii.) Find the range in interval notation. range: $(-\infty,\infty)$

14B. (3 pts) Use the function $f(x)=\frac{x+1}{4-x}$,

i.) Find the intercepts. x-int: $(-1,0)$

x-int: top=0 y-int: $y=0$

$x+1=0$ $y=\frac{0+1}{4-0}=\frac{1}{4}$ y-int: $\left(0,\frac{1}{4}\right)$

$x=-1$

ii.) Find the asymptotes. Vertical: $x=4$

V.A.: Set bottom = 0 H.A. (Rules)

$4-x=0$ $n=m$ so $y=\frac{a_n}{b_m}=\frac{1}{-1}=-1$ Horizontal: $y=-1$

$x=4$

iii.) Find the range in interval notation. range: $(-\infty,-1)\cup(-1,\infty)$

Since the graph does not cross the horizontal asymptote, it never uses the y-value of -1. Therefore, it is not included in the range.

15A.) (4 pts) For the polynomial $y=-5(x-5)^2(x+2)^3(x+1)$:

i.) Find the leading term (a power function): $y=-5x^6$

ii.) Find the degree: 6

iii.) Choose the correct statement below:

A. As $x\to-\infty$, $y\to\infty$. As $x\to\infty$, $y\to\infty$.
B. As $x\to-\infty$, $y\to-\infty$. As $x\to\infty$, $y\to-\infty$.
C. As $x\to-\infty$, $y\to\infty$. As $x\to\infty$, $y\to-\infty$.
D. As $x\to-\infty$, $y\to-\infty$. As $x\to\infty$, $y\to\infty$.

iv.) Find the y-intercept of the graph of *f* (as an ordered pair): $(0,-1000)$

$$y-\text{int}: \quad x=0$$
$$y=-5(0-5)^2(0+2)^3(0+1)$$
$$y=-5(-5)^2(2)^3(1)$$
$$y=-5(25)(8)(1)$$
$$y=-1000$$

15B.) (4 pts) For the polynomial $y=4x(x-5)^6$:

i.) Find the leading term (a power function): $y=4x^7$

ii.) Find the degree: 7

iii.) Choose the correct statement below:

A. As $x\to-\infty$, $y\to\infty$. As $x\to\infty$, $y\to\infty$.
B. As $x\to-\infty$, $y\to-\infty$. As $x\to\infty$, $y\to-\infty$.
C. As $x\to-\infty$, $y\to\infty$. As $x\to\infty$, $y\to-\infty$.
D. As $x\to-\infty$, $y\to-\infty$. As $x\to\infty$, $y\to\infty$.

iv.) Find the y-intercept of the graph of *f* (as an ordered pair): $(0,0)$

$$y-\text{int}: \quad x=0$$
$$y=4(0)(0-5)^6$$
$$y=0$$

16A.) (3 pts) Find the equation of a line (in slope-intercept form) that passes through $\left(\frac{3}{2},-4\right)$ and (-3, -7). NOTE: $m=\frac{y_2-y_1}{x_2-x_1}$ $\quad y-y_1=m(x-x_1)$

16A. $y=\frac{2}{3}x-5$

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-7-(-4)}{-3-\frac{3}{2}}=\frac{-3}{-\frac{9}{2}}=3\cdot\frac{2}{9}=\frac{2}{3}$$

$$y-y_1=m(x-x_1) \quad \Rightarrow \quad y-(-7)=\frac{2}{3}(x-(-3)) \quad \Rightarrow \quad y+7=\frac{2}{3}(x+3) \quad \Rightarrow \quad y+7=\frac{2}{3}x+2 \quad \Rightarrow \quad y=\frac{2}{3}x-5$$

16B.) (3 pts) Find the equation of a line (in slope-intercept form) that is perpendicular to $2x-3y=4$ and passes through (-2, 5).
NOTE: $y-y_1=m(x-x_1)$

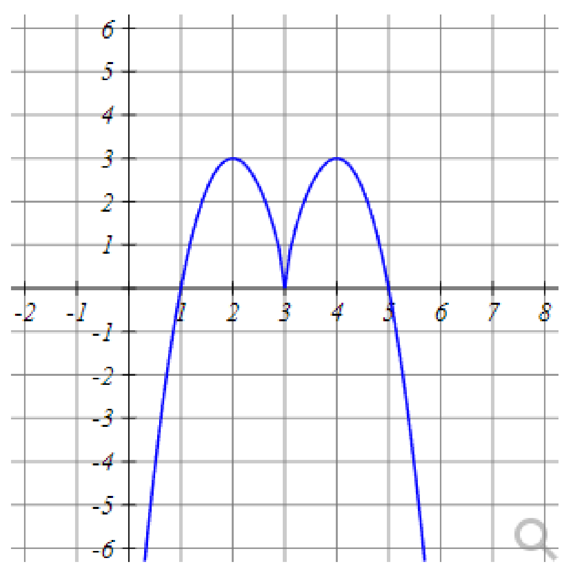
16B. $y=-\frac{3}{2}x+2$

$$2x-3y=4$$
$$-3y=-2x+4$$
$$y=\frac{2}{3}x-\frac{4}{3}$$

Slope perpendicular is $-\frac{3}{2}$

$$y-y_1=m(x-x_1)$$
$$y-5=-\frac{3}{2}(x+2)$$
$$y-5=-\frac{3}{2}x-3$$
$$y=-\frac{3}{2}x+2$$

17A.) (3 pts) Given the graph of f (to the right), find the following:

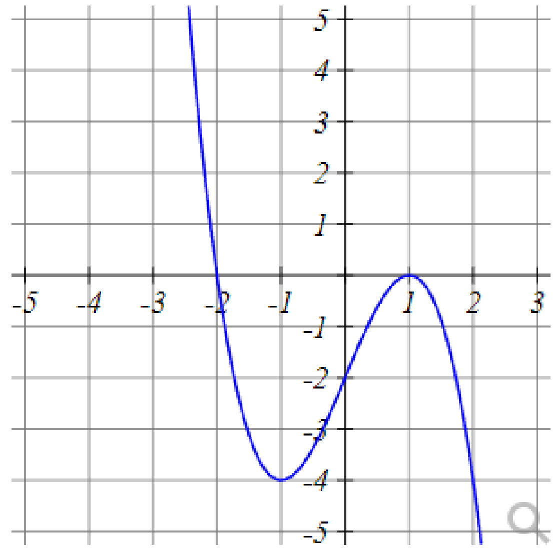


i.) $f(4)=2$

ii.) the solution(s) to $f(x)=0$: $\quad x=1,3,5$

iii.) the coordinate(s) of the relative maximum: $\quad (2,3),\ (4,3)$

17B.) (3 pts) Given the graph of f (to the right), find the following:

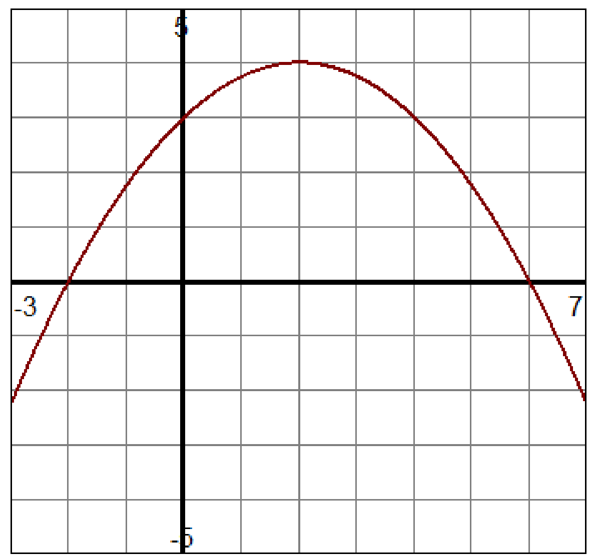


i.) $f(2) = -4$

ii.) the solution(s) to $f(x) = 0$: $x = -2, 1$

iii.) the coordinate(s) of the relative minimum: $(-1, -4)$

18A.) (5 pts) Use the graph to answer the following.



i.) The graph has: (choose) **max**

Find the max (y-value): 4

ii.) vertex: $(2, 4)$

iii.) axis of symmetry: $x = 2$

iv.) domain:_ $(-\infty, \infty)$
(write in interval notation)

v.) range:_ $(-\infty, 4]$
(write in interval notation)

vi.) Write the function's equation in the form $f(x) = a(x-h)^2 + k$

We found the vertex, which means $h = 2$ and $k = 4$. We can plug this into the given formula: $y = a(x-2)^2 + 4$. Now we need to find the a value. We need to plug in a point on the curve and solve for a. We can use the y-intercept, (0, 3). Put in a 0 for x and a 3 for y:

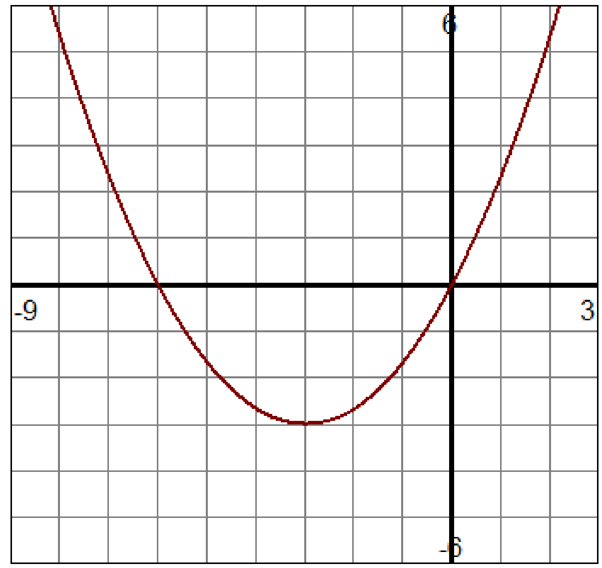
$3 = a(0-2)^2 + 4$

$3 = 4a + 4$

$-1 = 4a$

$a = -\frac{1}{4}$. Therefore, our equation is $f(x) = -\frac{1}{4}(x-2)^2 + 4$

18B.) (5 pts) Use the graph to answer the following.



i.) The graph has: (choose) **min**.

Find the min (y-value): -3

ii.) vertex:_ $(-3,-3)$

iii.) axis of symmetry: $x=-3$

iv.) domain: $(-\infty,\infty)$
(write in interval notation)

v.) range: $[-3,\infty)$
(write in interval notation)

vi.) Write the function's equation in the form $f(x)=a(x-h)^2+k$

We found the vertex, which means *h* = -3 and *k* = -3. We can plug this into the given formula: $y=a(x+3)^2-3$. Now we need to find the *a* value. We need to plug in a point on the curve and solve for a. We can use the y-intercept, (0, 0). Put in a 0 for x and a 0 for y:

$0=a(0+3)^2-3$

$0=9a-3$

$3=9a$

$a=\frac{1}{3}$. Therefore, our equation is $f(x)=\frac{1}{3}(x+3)^2-3$

19A.) (2 pts) Given $h(x)=\frac{2}{(7-9x)^3}$, find two functions $f(x)$ and $g(x)$ such that $h(x)=f(g(x))$. NOTE: $g(x)$ cannot equal just *x*.

$f(x)=\frac{2}{x^3}$

$g(x)=7-9x$

To do this one, first find the inside function, which is always the g. The g is always the expression that is inside something else. It could be inside parenthesis, or inside a root. In this problem, $g(x)=7-9x$. Now take the original equation and take out the $7-9x$ and replace it with *x*. You will get $f(x)=\frac{2}{x^3}$.

19B.) (2 pts) Given $h(x)=9\sqrt{2-3x}$, find two functions $f(x)$ and $g(x)$ such that $h(x)=f(g(x))$. NOTE: $g(x)$ cannot equal just *x*.

$f(x)=9\sqrt{x}$

$g(x)=2-3x$

To do this one, first find the inside function, which is always the g. The g is always the expression that is inside something else. It could be inside parenthesis, or inside a root. In this problem, $g(x)=2-3x$. Now take the original equation and take out the $2-3x$ and replace it with *x*. You will get $f(x)=9\sqrt{x}$.