NAME: ___________KEY_______________ **/ 60 = [%]**

MATH 126 FINAL EXAM SAMPLE

NOTE: The final exam will only have 14 questions. The different parts of each question (part A, B, etc.) are variations. Know how to do all the variations on this exam.

1A.) (3 pts) Find the equation of a line (in slope-intercept form) that passes through $\left(\frac{3}{2},-4\right)$ and (-3, -7).

1A. $y=\frac{2}{3}x-5$

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{-7-(-4)}{-3-\frac{3}{2}}=\frac{-3}{-\frac{9}{2}}=3\cdot\frac{2}{9}=\frac{2}{3}$$

$$y-y_1=m\left(x-x_1\right) \quad \Rightarrow \quad y-(-7)=\frac{2}{3}\left(x-(-3)\right) \quad \Rightarrow \quad y+7=\frac{2}{3}\left(x+3\right) \quad \Rightarrow \quad y+7=\frac{2}{3}x+2 \quad \Rightarrow \quad y=\frac{2}{3}x-5$$

1B.) (3 pts) Find the equation of a line (in slope-intercept form) that is perpendicular to $2x-3y=4$ and passes through (-2, 5).

1B. $y=-\frac{3}{2}x+2$

$2x-3y=4$

$-3y=-2x+4$

$y=\frac{2}{3}x-\frac{4}{3}$

Slope perpendicular is $-\frac{3}{2}$

$y-y_1=m\left(x-x_1\right)$

$y-5=-\frac{3}{2}\left(x+2\right)$

$y-5=-\frac{3}{2}x-3$

$y=-\frac{3}{2}x+2$

2A.) (3 pts) Given $f(x)=\frac{2x-7}{x^3+16x}$ and $g(x)=\frac{\sqrt{3x-4}}{45}$, find the following:

i.) Domain of $f\left(x\right)$ in interval notation

2i. $\left(-\infty,0\right)\cup\left(0,\infty\right)$

$$f(x)=\frac{2x-7}{x\left(x^2+16\right)}$$

$x\left(x^2+16\right)=0 \quad \Rightarrow \quad x=0 \text{ or } x^2+16=0.$

Since x^2+16 can never be 0, the answer is all real numbers except 0.

ii.) Domain of $g(x)$ in interval notation 2ii. $\left[\frac{4}{3},\infty\right)$

$3x-4\geq 0$
$3x\geq 4$
$x\geq\frac{4}{3}$
$\left[\frac{4}{3},\infty\right)$ We do include $\frac{4}{3}$ since it does not make the bottom zero.

iii.) $(f+g)(2)$ 2iii. $-\frac{3}{40}+\frac{\sqrt{2}}{45}$

$$(f+g)(x)=\frac{2x-7}{x^3+16x}+\frac{\sqrt{3x-4}}{45}$$

$$(f+g)(2)=\frac{2(2)-7}{(2)^3+16(2)}+\frac{\sqrt{3(2)-4}}{45}$$

$$(f+g)(2)=-\frac{3}{40}+\frac{\sqrt{2}}{45}$$

2B.) (3 pts) Given $f(x)=\frac{x-4}{x^2-9}$ and $g(x)=\frac{8x-3}{\sqrt{11-5x}}$, find the following:

i.) Domain of $f(x)$ in interval notation 2i. $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

$x^2-9\neq 0$ => $x\neq 3$ and $x\neq -3$
The domain is all real numbers not including 3 and -3.

ii.) Domain of $g(x)$ in interval notation 2ii. $\left(-\infty,\frac{11}{5}\right)$

$11-5x>0$
$-5x>-11$
$x<\frac{11}{5}$
$\left(-\infty,\frac{11}{5}\right)$ We do not include $\frac{11}{5}$ since it makes the bottom zero.

iii.) $(f-g)(1)$ 2iii. $\frac{3}{8}-\frac{5}{\sqrt{6}}$

$$(f-g)(x)=\frac{x-4}{x^2-9}-\frac{8x-3}{\sqrt{11-5x}}$$

$$(f-g)(1)=\frac{1-4}{1^2-9}-\frac{8(1)-3}{\sqrt{11-5(1)}}=\frac{3}{8}-\frac{5}{\sqrt{6}}$$

3A.) (4 pts) Let $f(x) = 3 - 2x^2$. Find the difference quotient. 3A. $-4x - 2h$

Use $\dfrac{f(x+h) - f(x)}{h}$.

$$f(x+h) = 3 - 2(x+h)^2$$
$$f(x+h) = 3 - 2\left(x^2 + 2xh + h^2\right)$$
$$f(x+h) = 3 - 2x^2 - 4xh - 2h^2$$

$$\frac{3 - 2x^2 - 4xh - 2h^2 - \left(3 - 2x^2\right)}{h} = \frac{3 - 2x^2 - 4xh - 2h^2 - 3 + 2x^2}{h} = \frac{-4xh - 2h^2}{h} = \frac{h\left(-4x - 2h\right)}{h} = -4x - 2h$$

3B.) (4 pts) Let $f(x) = 3x^2 - \dfrac{x}{5}$. Find the difference quotient. 3B. $6x + 3h - \dfrac{1}{5}$

Use $\dfrac{f(x+h) - f(x)}{h}$.

$$f(x+h) = 3(x+h)^2 - \frac{x+h}{5}$$
$$f(x+h) = 3\left(x^2 + 2xh + h^2\right) - \frac{x+h}{5}$$
$$f(x+h) = 3x^2 + 6xh + 3h^2 - \frac{x}{5} - \frac{h}{5}$$

$$\frac{3x^2 + 6xh + 3h^2 - \frac{x}{5} - \frac{h}{5} - \left(3x^2 - \frac{x}{5}\right)}{h} = \frac{3x^2 + 6xh + 3h^2 - \frac{x}{5} - \frac{h}{5} - 3x^2 + \frac{x}{5}}{h} = \frac{6xh + 3h^2 - \frac{h}{5}}{h} = 6x + 3h - \frac{1}{5}$$

4A. (8 pts) Find the following and graph: $f(x) = \dfrac{1-x}{x^2 - 9}$

i.) Find the intercepts. x-int: (1, 0)

x-int: top=0 y-int: $y = 0$

$1 - x = 0$ $\quad y = \dfrac{1-0}{0^2 - 9} = -\dfrac{1}{9}$ y-int: $\left(0, -\dfrac{1}{9}\right)$

$x = 1$

ii.) Find the asymptotes. Vertical: $x = 3, x = -3$

V.A.: Set bottom = 0 H.A. (Rules)

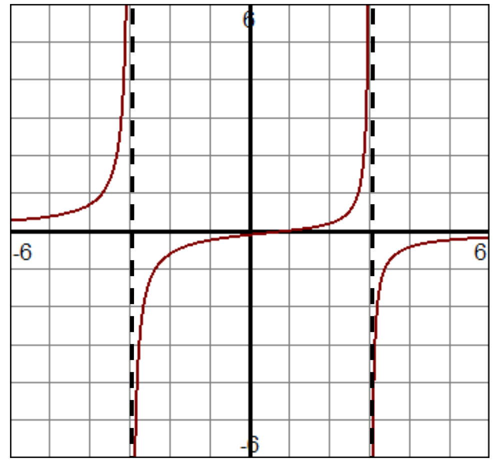
$x^2 - 9 = 0$ $\quad n < m$ so $y = 0$ Horizontal: $y = 0$

$x^2 = 9$

$x = \pm 3$

iii.) Does *f* cross the horizontal asymptote? (Yes or No) iii. Yes

iv.) Graph.



v.) Find the domain: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

vi.) Find the range: $(-\infty,\infty)$

4B. (8 pt bs) Find the following and graph: $f(x)=\dfrac{x+1}{4-x}$

i.) Find the intercepts. x-int: $(-1,0)$

x-int: top=0 y-int: $y=0$

$x+1=0$ $y=\dfrac{0+1}{4-0}=\dfrac{1}{4}$ y-int: $\left(0,\dfrac{1}{4}\right)$

$x=-1$

ii.) Find the asymptotes. Vertical: $x=4$

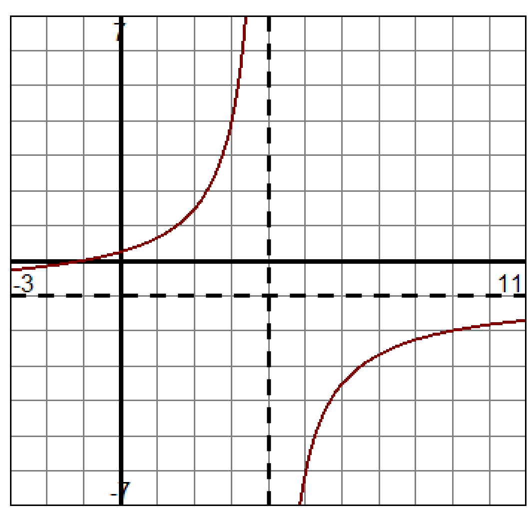
V.A.: Set bottom = 0 H.A. (Rules)

$4-x=0$ $n=m$ so $y=\dfrac{a_n}{b_m}=\dfrac{1}{-1}=-1$ Horizontal: $y=-1$

$x=4$

iii.) Does *f* cross the horizontal asymptote? (Yes or No) iii. No

iv.) Graph.



v.) Find the domain: $(-\infty,4)\cup(4,\infty)$

vi.) Find the range: $(-\infty,-1)\cup(-1,\infty)$

Since the graph does not cross the horizontal asymptote, it never uses the y-value of -1. Therefore, it is not included in the range.

5A.) (3 pts) Given $f(x)=\begin{cases} x^2 & if \quad x\le 0 \\ -\sqrt{x} & if \quad x>0 \end{cases}$ find the following.

i.) $f(0)=0^2=0$
We need to use the first equation since 0 is included here.

ii.) $f(9)=-\sqrt{9}=-3$
We are using the second equation since 9 is greater than 0.

iii.) $f(-4)=(-4)^2=16$
We are using the first equation since 16 is less than 0.

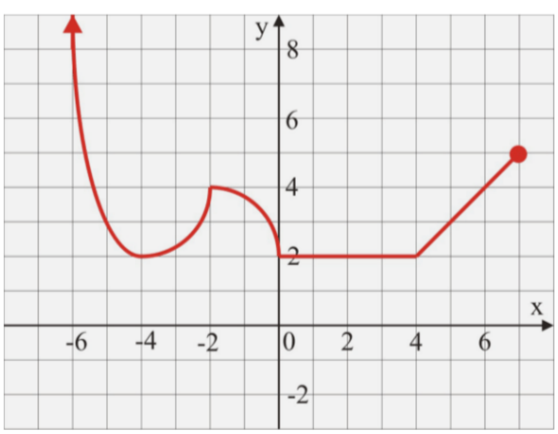
5B.) (3 pts) Given $f(x)=\begin{cases} |x| & if \quad x<0 \\ -4 & if \quad 0\le x<3 \\ -x-3 & if \quad x\ge 3 \end{cases}$ find the following.

i.) $f(5)=-5-3=-8$
We are using the third equation here since 5 is greater than 3.

ii.) $f(0)=-4$
We are using the second equation here since 0 is included.

iii.) $f(-3)=|-3|=3$
We are using the first equation here since -3 is less than 0.

6A.) (3 pts) Use the graph of $f(x)$ below to find the following.

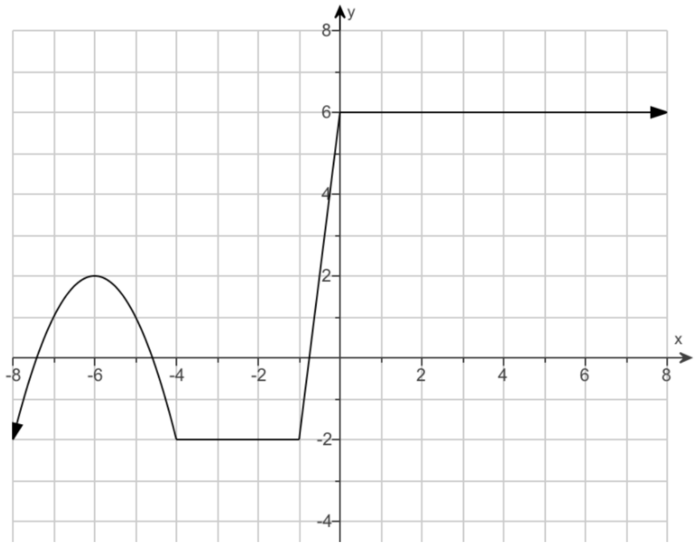


a.) interval(s) of increasing
$(-4,-2)\cup(4,7)$
We are looking for when the graph is going uphill as we move from left to right.

b.) interval(s) of decreasing
$(-\infty,-4)\cup(-2,0)$
We are looking for when the graph is going downhill as we move from left to right.

c) interval(s) f is constant
$(0,4)$
The graph is constant anywhere there is a horizontal line.

6B.) (3 pts) Use the graph of $f(x)$ below to find the following.

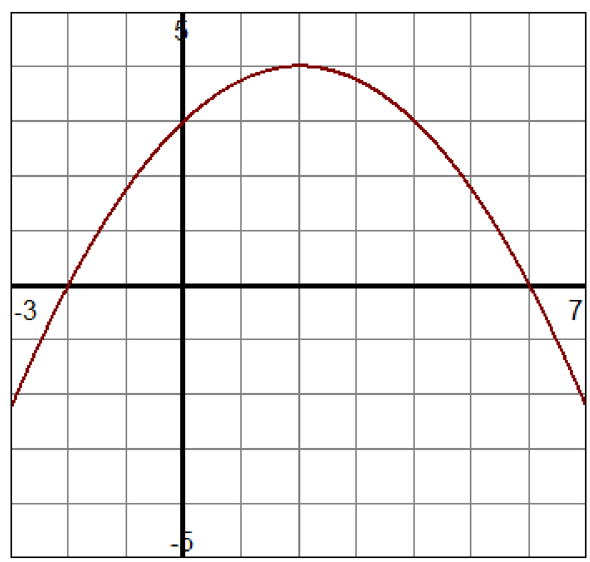


a.) interval(s) of increasing
$(-\infty,-6)\cup(-1,0)$
We are looking for when the graph is going uphill as we move from left to right.

b.) interval(s) of decreasing
$(-6,-4)$
We are looking for when the graph is going downhill as we move from left to right.

c) interval(s) *f* is constant
$(-4,-1)\cup(0,\infty)$
The graph is constant anywhere there is a horizontal line.

7A.) (8 pts) Use the graph to answer the following.



i.) The graph has: (choose) **max**

Find the max (y-value): 4

ii.) x-intercept(s): $(-2,0),(6,0)$

iii.) y-intercept: $(0,3)$

iv.) vertex: $(2,4)$

v.) axis of symmetry: $x=2$

vi.) domain: $(-\infty,\infty)$

vii.) range: $(-\infty,4]$

viii.) Write the function's equation in the form $f(x)=a(x-h)^2+k$

We found the vertex, which means $h=2$ and $k=4$. We can plug this into the given formula: $y=a(x-2)^2+4$. Now we need to find the a value. We need to plug in a point on the curve and solve for a. We can use the y-intercept, (0, 3). Put in a 0 for x and a 3 for y:

$3=a(0-2)^2+4$

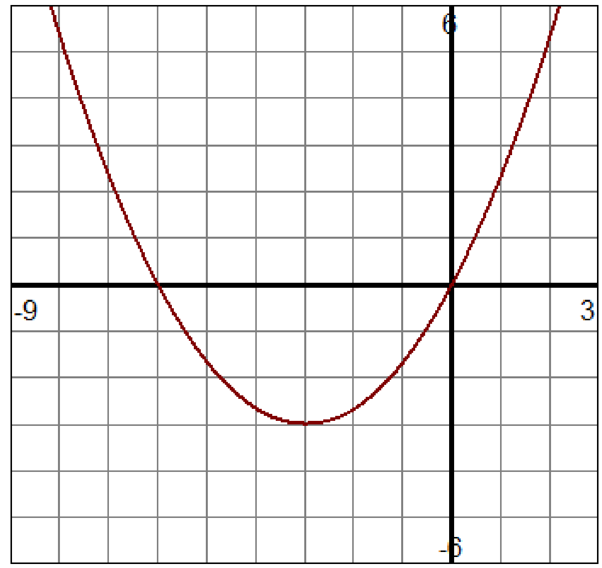
$3=4a+4$

$-1=4a$

$a=-\frac{1}{4}$. Therefore, our equation is $f(x)=-\frac{1}{4}(x-2)^2+4$

7B.) (8 pts) Use the graph to answer the following.

i.) The graph has: (choose) **min**.



Find the min (y-value): -3

ii.) x-intercept(s): $(-6,0),(0,0)$

iii.) y-intercept: $(0,0)$

iv.) vertex: $(-3,-3)$

v.) axis of symmetry: $x=-3$

vi.) domain: $(-\infty,\infty)$

vii.) range: $[-3,\infty)$

viii.) Write the function's equation in the form $f(x)=a(x-h)^2+k$

We found the vertex, which means h = -3 and k = -3. We can plug this into the given formula: $y=a(x+3)^2-3$. Now we need to find the a value. We need to plug in a point on the curve and solve for a. We can use the y-intercept, (0, 0). Put in a 0 for x and a 0 for y:

$0=a(0+3)^2-3$

$0=9a-3$

$3=9a$

$a=\frac{1}{3}$. Therefore, our equation is $f(x)=\frac{1}{3}(x+3)^2-3$

8A.) (4 pts) Given $h(x)=\dfrac{2}{(7-9x)^3}$, find two functions $f(x)$ and $g(x)$ such that $h(x)=f(g(x))$. NOTE: $g(x)$ cannot equal just x.

$f(x)=\dfrac{2}{x^3}$

$g(x)=7-9x$

To do this one, first find the inside function, which is always the g. The g is always the expression that is inside something else. It could be inside parenthesis, or inside a root. In this problem, $g(x)=7-9x$. Now take the original equation and take out the $7-9x$ and replace it with x. You will get $f(x)=\dfrac{2}{x^3}$.

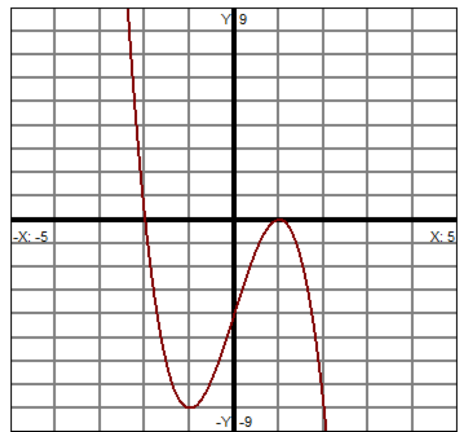
8B.) (4 pts) Given $h(x)=9\sqrt{2-3x}$, find two functions $f(x)$ and $g(x)$ such that $h(x)=f(g(x))$. NOTE: $g(x)$ cannot equal just x.

$f(x)=9\sqrt{x}$

$g(x)=2-3x$

To do this one, first find the inside function, which is always the g. The g is always the expression that is inside something else. It could be inside parenthesis, or inside a root. In this problem, $g(x)=2-3x$. Now take the original equation and take out the $2-3x$ and replace it with x. You will get $f(x)=9\sqrt{x}$.

9A.) (7 pts) Graph $y=-2(x-1)^2(x+2)$ Find the following and graph.

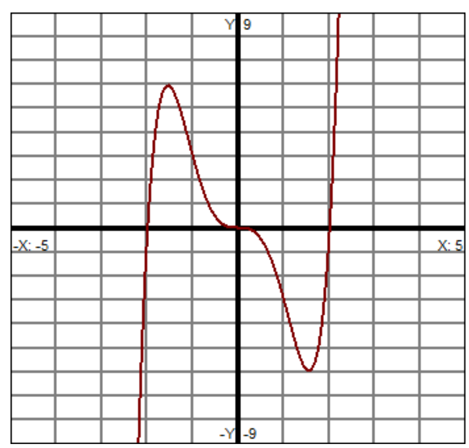


zero: 1 Multiplicity: 2 y-intercept: $y=-2(0-1)^2(0+2)=-4$

zero: -2 Multiplicity: 1 Degree: 3

Leading term: $y=-2\cdot x^2\cdot x=-2x^3$
(Power function)

9B.) (7 pts) Graph $y=x^3(x-2)(x+2)$ Find the following and graph.



zero: 0 Multiplicity: 3 y-intercept: $y=0^3(0-2)(0+2)=0$

zero: 2 Multiplicity: 1 Degree: 5

zero: -2 Multiplicity: 1 Leading term: $y=x^3\cdot x\cdot x=x^5$
(Power function)

10A.) (4 pts) The function $f(x) = 6x^3 - 5x^2 - 29x + 10$ has a zero at $x = -2$. Use synthetic division and factoring to find the other zeros.

10A. $x = \frac{5}{2}, \frac{1}{3}$

$$\begin{array}{r|rrrr} -2 & 6 & -5 & -29 & 10 \\ & & -12 & 34 & -10 \\ \hline & 6 & -17 & 5 & 0 \end{array}$$

$$6x^2 - 17x + 5 = 0$$
$$(2x-5)(3x-1) = 0$$
$$x = \frac{5}{2}, \frac{1}{3}$$

10B.) (4 pts) The function $f(x) = 27x^3 - 54x^2 + 27x - 4$ has a zero at $x = \frac{4}{3}$. Use synthetic division and factoring to find the other zeros.

10B. $x = \frac{1}{3}$

$$\begin{array}{r|rrrr} 4/3 & 27 & -54 & 27 & -4 \\ & & 36 & -24 & 4 \\ \hline & 27 & -18 & 3 & 0 \end{array}$$

$$27x^2 - 18x + 3 = 0$$
$$3(9x^2 - 6x + 1) = 0$$
$$3(3x-1)(3x-1) = 0$$
$$x = \frac{1}{3}$$

11A.) (2 pts) Use a calculator to evaluate each expression. Round your answers to four decimal places where appropriate. (No partial credit on this one.)

i.) $3^{1.2} \approx 3.7372$

ii.) $\ln\left(\frac{10}{3.5}\right) \approx 1.0498$

11B.) (2 pts) Use a calculator to evaluate each expression. Round your answers to four decimal places where appropriate. (No partial credit on this one.)

i.) $e^{0.03} \approx 1.0305$

ii.) $\log\left(\frac{3}{11}\right) \approx -0.5643$

12A.) (4 pts) Convert each to an exponential equation. (Do not solve for any unknowns.)

i.) $\ln(5x-6)=3$ — i. $e^3=5x-6$

This equation can be rewritten as $\log_e(5x-6)=3$.

ii.) $\ln(5x)=4$ — ii. $e^4=5x$

This equation can be rewritten as $\log_e(5x)=4$

12B.) (4 pts) Convert each to an exponential equation. (Do not solve for any unknowns.)

i.) $\log(2-7x)=6$ — i. $10^6=2-7x$

Remember a log with no base written is automatically base 10.

ii.) $\log_{13}(60)=x$ — ii. $13^x=60$

13A.) (2 pts) Convert the equation $2^{-5}=\frac{1}{32}$ into a logarithmic equation. — 13A. $\log_2\left(\frac{1}{32}\right)=-5$

13B.) (2 pts) Convert the equation $\left(\frac{1}{3}\right)^3=\frac{1}{27}$ into a logarithmic equation. — 13B. $\log_{\frac{1}{3}}\left(\frac{1}{27}\right)=3$

14A.) (6 pts) Solve for each equation for *x*.

i.) $\log_3(x-5)-\log_3(2x+3)=0$ — i. No solution

$$\log_3\left(\frac{x-5}{2x+3}\right)=0$$

$$3^0=\frac{x-5}{2x+3}$$

$$1=\frac{x-5}{2x+3}$$

$$2x+3=x-5$$

$x=-8$ Causes a negative number inside the log, so no solution.

ii.) $2^{4x-3} - 27 = 5$ ii. $x = 2$

$$2^{4x-3} = 32$$
$$2^{4x-3} = 2^5$$
$$4x - 3 = 5$$
$$4x = 8 \;\; \Rightarrow \;\; x = 2$$

iii.) $10^{-x} = 1.6$ iii. -0.2041

$$\log 10^{-x} = \log 1.6$$
$$-x = \log 1.6$$
$$x = -\log 1.6 \approx -0.2041$$

14B.) (6 pts) Solve for each equation for *x*.

i.) $\log_2(x+5) + \log_2(2x-5) = 3$ i. 3

$$\log_2(x+5)(2x-5) = 3$$
$$2^3 = (x+5)(2x-5)$$
$$8 = 2x^2 + 5x - 25$$
$$2x^2 + 5x - 33 = 0$$
$$(2x+11)(x-3) = 0$$

$x = -11/2$ and $x = 3$

However, if you put -11/2 into the original equation, you will get a negative number inside the log. Therefore 3 is the only answer.

ii.) $3^{1-2x} - 21 = 6$ ii. $x = -1$

$$3^{1-2x} = 27$$
$$3^{1-2x} = 3^3$$
$$1 - 2x = 3$$
$$-2x = 2$$
$$x = -1$$

iii.) $e^x = 4.5$ iii. 1.5041

$$\ln e^x = \ln 4.5$$
$$x = \ln 4.5 \approx 1.5041$$