NAME: KEY

# MATH 126 TEST 1 SAMPLE KEY

**NOTE: The actual exam will only have 13 questions. The different parts of each question (part A, B, etc.) are variations. Know how to do all the variations on this exam.**

1A.) (6 pts) Given: A = (–4, 3) and B = (8, –2), find the following:

i.) The distance from A to B. i. 13

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$d=\sqrt{(8--4)^2+(-2-3)^2}$$
$$d=\sqrt{(12)^2+(-5)^2}$$
$$d=\sqrt{169}$$
$$d=13$$

ii.) The midpoint of a line segment containing A and B. ii. $M=\left(2,\frac{1}{2}\right)$

$$M=\left(\frac{x_2+x_1}{2},\frac{y_2+y_1}{2}\right)$$
$$M=\left(\frac{-4+8}{2},\frac{3+-2}{2}\right)$$
$$M=\left(\frac{4}{2},\frac{1}{2}\right)$$
$$M=\left(2,\frac{1}{2}\right)$$

iii.) The slope of a line passing through A and B. iii. $-\frac{5}{12}$

$$m=\frac{y_2-y_1}{x_2-x_1}$$
$$m=\frac{-2-3}{8--4}=-\frac{5}{12}$$

1B.) (6 pts) Given: A = (4, –3) and B = (6, 4), find the following:

i.) The distance from A to B. i. $\sqrt{53}$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(6-4)^2 + (4--3)^2}$$
$$d = \sqrt{(2)^2 + (7)^2}$$
$$d = \sqrt{53}$$

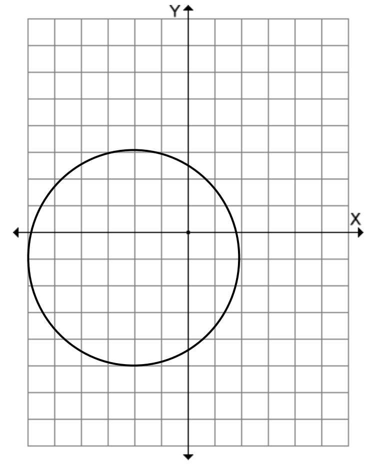
ii.) The midpoint of a line segment containing A and B. ii. $M = \left(5, \frac{1}{2}\right)$

$$M = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$$
$$M = \left(\frac{4+6}{2}, \frac{-3+4}{2}\right)$$
$$M = \left(\frac{10}{2}, \frac{1}{2}\right)$$
$$M = \left(5, \frac{1}{2}\right)$$

iii.) The slope of a line passing through A and B. iii. $\frac{7}{2}$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{4--3}{6-4} = \frac{7}{2}$$

2A.) (5 pts) Identify the center and radius: $x^2 + y^2 + 4x + 2y - 11 = 0$
Then graph.




Center: $(-2, -1)$

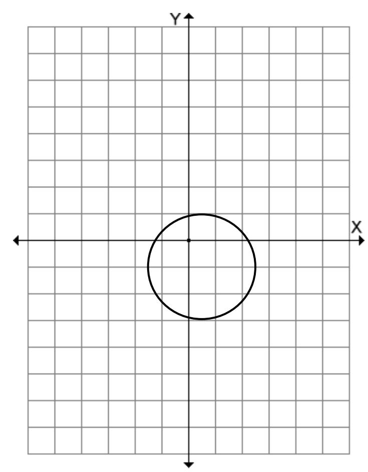
Radius: 4

$$x^2 + 4x + y^2 + 2y = 11$$
$$x^2 + 4x + 4 + y^2 + 2y + 1 = 11 + 4 + 1$$
$$(x+2)^2 + (y+1)^2 = 16$$

Center: $(-2, -1)$
Radius: 4

2B.) (5 pts) Identify the center and radius: $x^2 + y^2 - x + 2y - \frac{11}{4} = 0$

Then graph.




Center: $\left(\frac{1}{2}, -1\right)$

Radius: 2

$$x^2 - x + y^2 + 2y = \frac{11}{4}$$
$$x^2 - x + \frac{1}{4} + y^2 + 2y + 1 = \frac{11}{4} + \frac{1}{4} + 1$$
$$\left(x - \frac{1}{2}\right)^2 + (y+1)^2 = 4$$

Center: $\left(\frac{1}{2}, -1\right)$
Radius: 2

3A.) (3 pts) Find the domain: $f(x)=\frac{|x-4|}{\sqrt{7x-3}}$ 3A. $\left(\frac{3}{7},\infty\right)$

Write your answer in interval notation.

Domain is all x values that make the function defined. Therefore we are not allowed to divide by zero or take the square root of a negative number.

$7x-3>0$
$7x>3$
$x>3/7$
$\left(\frac{3}{7},\infty\right)$

3B.) (3 pts) Find the domain: $f(x)=\frac{\sqrt{3-4x}}{30}$ 3B. $\left(-\infty,\frac{3}{4}\right]$

Write your answer in interval notation.

In this case there is no variable in the denominator. Therefore, the denominator will never be zero, so we only need to consider the numerator. We cannot take the square root of a negative number. This time zero is allowed since 0 divided by anything is zero.

$3-4x\geq 0$
$-4x\geq -3$
$x\leq 3/4$ Flip the inequality sign whenever you divide or multiply by a negative number.
$\left(-\infty,\frac{3}{4}\right]$

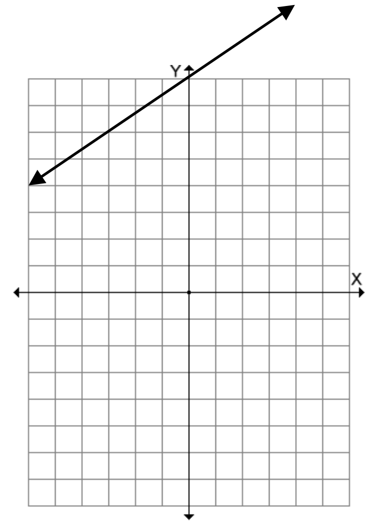
3C.) (3 pts) Find the domain: $f(x)=\frac{5x}{x^2-5x+6}$ 3C. $(-\infty,\ 2)\cup(2,\ 3)\cup(3,\ \infty)$

Write your answer in interval notation.

There are no square roots this time, so we only need to consider division by zero. It is okay if the numerator is zero, so we can ignore the top since nothing makes that undefined. Whatever is in the denominator cannot be equal to zero.

$x^2-5x+6\neq 0$
$(x-2)(x-3)\neq 0$
$x\neq 2,\ 3$ These are the numbers not allowed in the domain.
$(-\infty,\ 2)\cup(2,\ 3)\cup(3,\ \infty)$

4A.) (4 pts) Find the equation, in slope-intercept form, of a line that is perpendicular to the line $5x + 4y + 8 = 0$ and passes through (–5, 4). Graph your equation.




$$5x+4y=-8 \qquad y-y_1=m(x-x_1)$$

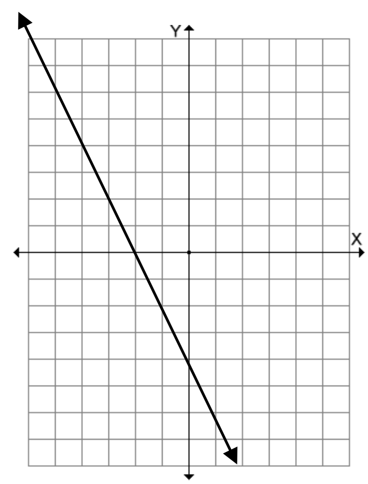
$$4y=-5x-8 \qquad y-4=\frac{4}{5}(x+5)$$

$$y=-\frac{5}{4}x-2 \qquad y-4=\frac{4}{5}x+4$$

Slope perpendicular is $\frac{4}{5}$. $\qquad y=\frac{4}{5}x+8$

Equation: $y=\frac{4}{5}x+8$

4B.) (4 pts) Find the equation, in slope-intercept form, of a line that is parallel to the line $2x + y + 4 = 0$ and passes through (–1, –2). Graph your equation.




$$2x+y=-4 \qquad y-y_1=m(x-x_1)$$

$$y=-2x-4 \qquad y+2=-2(x+1)$$

$$y+2=-2x-2$$

Slope parallel is –2. $\qquad y=-2x-4$

Equation: $y=-2x-4$

5A.) (4 pts) A company plans to manufacture a certain product with fixed costs of \$50000. It will cost \$140 to manufacture each unit, and each will be sold for \$300. Write a function that describes the profit, P, in terms of units sold $x$.

5A. $P(x)=160x-50000$

First set up equations:

$C(x)=50000+140x$

$R(x)=300x$ (Revenue = Price*Quantity)

$P(x)=R(x)-C(x)$ (Profit = Revenue – Costs)

$P(x)=300x-(50000+140x)$

$P(x)=160x-50000$

5B.) (4 pts) Suppose that a company purchases a new car for \$28000. After 3 years, the car is worth \$16000. Write a linear function that expresses the value, V, of the car as a function of its age, t.

5B. $V(t) = -4000t + 28000$

First set up two coordinates. In this case age is in the $x$ position, and V is in the $y$ position.
$(0,\ 28000),\ (3,\ 16000)$ Now find the slope.

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{16000 - 28000}{3 - 0} = -4000$ Now use the point-slope formula to find the equation.

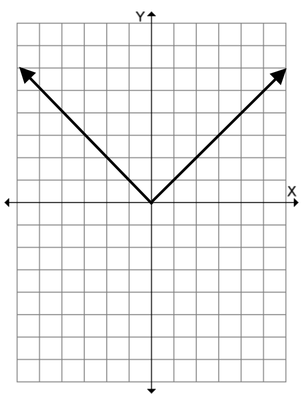
$y - y_1 = m(x - x_1)$

$y - 28000 = -4000(x - 0)$

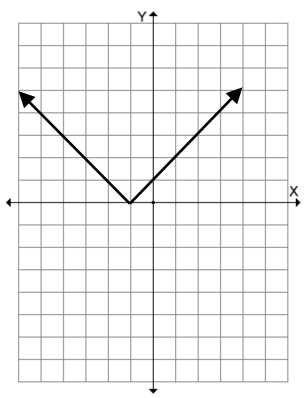
$y = -4000x + 28000$

$V(t) = -4000t + 28000$ Be sure to use the correct variables and use function notation.

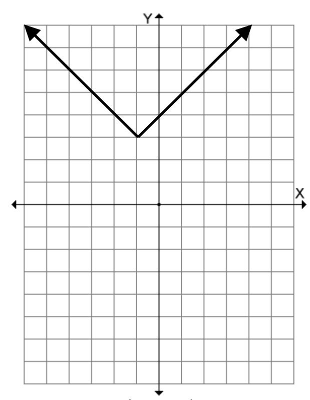
6A.) (5 pts) Graph using transformations: $y = -|x+1| + 3$. Start with the base graph (library function) and then graph each successive transformation. The final graph will be your graph of $y = -|x+1| + 3$.



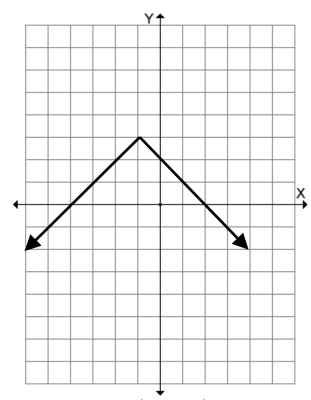

$y = |x|$




$y = |x+1|$



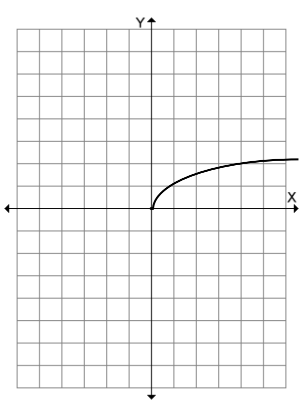

$y = |x+1| + 3$



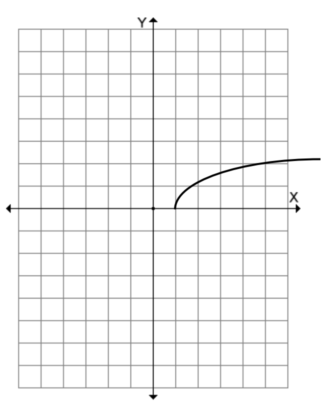

$y = -|x+1| + 3$

6B.) (5 pts) Graph using transformations: $y = \sqrt{1-x} - 2$. Start with the base graph (library function) and then graph each successive transformation. The final graph will be your graph of $y = \sqrt{1-x} - 2$.

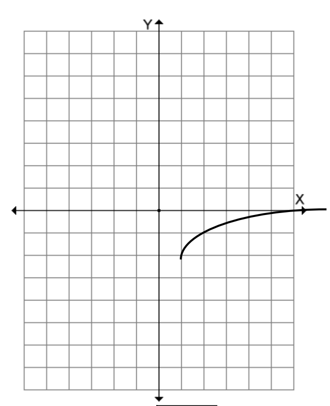
First we need to rewrite our original equation: $y = \sqrt{-x+1} - 2$. Now factor out a negative: $y = \sqrt{-(x-1)} - 2$



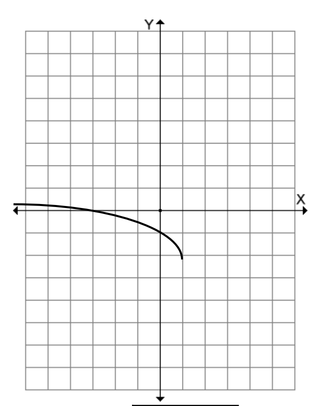

$y = \sqrt{x}$




$y = \sqrt{x-1}$




$y = \sqrt{x-1} - 2$




$y = \sqrt{-(x-1)} - 2$

7A.) (5 pts) Indicate what kind of symmetry ($x$-axis, $y$-axis, origin) this graph has: $y = \frac{9-x^2}{3x}$. Also find its $x$ and $y$ intercepts.

Sym: origin symmetry

x-int: $x = \pm 3$

y-int: none

x-int: y = 0

$$y = \frac{9-x^2}{3x}$$
$$0 = \frac{9-x^2}{3x}$$
$$0 = 9 - x^2$$
$$x = \pm 3$$

y-int: x = 0

$$y = \frac{9-0^2}{3(0)}$$
$$y = \frac{9}{0}$$
$$y = \text{undefined}$$

x-axis sym: -y for y

$$-y = \frac{9-x^2}{3x}$$

Not equal to original, so no x-axis sym.

y-axis sym: -x for x

$$y = \frac{9-(-x)^2}{3(-x)} = \frac{9-x^2}{-3x}$$

Not equal to original, so no y-axis sym.

origin-sym: -x for x, -y for y

$$-y = \frac{9-(-x)^2}{3(-x)},\ -y = \frac{9-x^2}{-3x},\ y = \frac{9-x^2}{3x}$$

The negatives cancel on both sides, so this is the same as the original. Origin sym.

7B.) (5 pts) Indicate what kind of symmetry ($x$-axis, $y$-axis, origin) this graph has: $x^2 - y = 4$. Also find its $x$ and $y$ intercepts.

Sym: y-axis

x-int: $x = \pm 2$

y-int: $y = -4$

x-int: y = 0

$$x^2 - y = 4$$
$$x^2 - 0 = 4$$
$$x^2 = 4$$
$$x = \pm 2$$

y-int: x = 0

$$x^2 - y = 4$$
$$0^2 - y = 4$$
$$y = -4$$

x-axis sym: -y for y

$$x^2 - (-y) = 4$$
$$x^2 + y = 4$$

Not equal to original, so no x-axis sym.

y-axis sym: -x for x

$$(-x)^2 - y = 4$$
$$x^2 - y = 4$$

Same as original, so it has y-axis sym.

origin-sym: -x for x, -y for y

$$(-x)^2 - (-y) = 4$$
$$x^2 + y = 4$$

No equal to original, so no origin sym.

8A. (4 pts) Determine algebraically whether $f(x)=\dfrac{-x^3}{3x^2-9}$ is even, odd, or neither.

8A. Odd

$$f(x)=\frac{-x^3}{3x^2-9}$$
$$f(-x)=\frac{-(-x)^3}{3(-x)^2-9}$$
$$f(-x)=\frac{x^3}{3x^2-9}, \quad \text{so } f(-x)=-f(x)$$

Since $f(-x)=-f(x)$, then this function is odd.

8B. (4 pts) Determine algebraically whether $f(x)=\sqrt[3]{2x^2-3}$ is even, odd, or neither.

8A. Even

$$f(x)=\sqrt[3]{2x^2-3}$$
$$f(-x)=\sqrt[3]{2(-x)^2-3}$$
$$f(-x)=\sqrt[3]{2x^2-3}, \quad \text{so } f(-x)=f(x)$$

Since $f(-x)=f(x)$, then this function is even.

9A.) (4 pts) Find the difference quotient for $f(x)=3x-2x^2$ by using $\dfrac{f(x+h)-f(x)}{h}$.

9A. $3-4x-2h$

$$\frac{f(x+h)-f(x)}{h}$$
$$\frac{3(x+h)-2(x+h)^2-(3x-2x^2)}{h}$$
$$\frac{3x+3h-2(x^2+2xh+h^2)-3x+2x^2}{h}$$
$$\frac{3x+3h-2x^2-4xh-2h^2-3x+2x^2}{h}$$
$$\frac{3h-4xh-2h^2}{h}$$
$$\frac{h(3-4x-2h)}{h}$$
$$3-4x-2h$$

9B.) (4 pts) Find the difference quotient for $f(x)=\frac{x}{3}-4$ by using $\frac{f(x+h)-f(x)}{h}$.

9B. $\frac{1}{3}$

$$\frac{f(x+h)-f(x)}{h}$$

$$\frac{\frac{(x+h)}{3}-4-\left(\frac{x}{3}-4\right)}{h}$$

$$\frac{\frac{x+h}{3}-4-\frac{x}{3}+4}{h}$$

$$\frac{\frac{x+h-x}{3}}{h} = \frac{\frac{h}{3}}{h}=\frac{h}{3}\cdot\frac{1}{h}=\frac{1}{3}$$

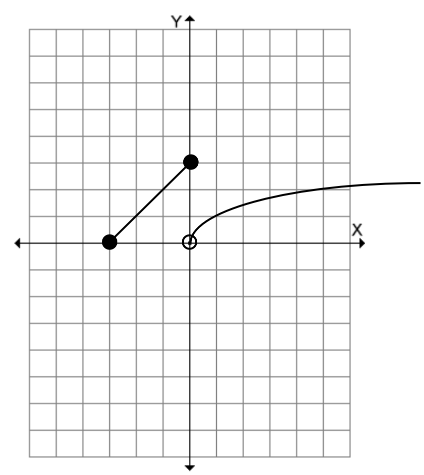
10A.) (6 pts) Given $f(x)=\begin{cases} 3+x & if \quad -3\le x<0 \\ 3 & if \quad x=0 \\ \sqrt{x} & if \quad x>0 \end{cases}$ find the following and graph.

a.) $f(0)$ = 3

b.) $f\left(-\frac{3}{2}\right) = 3+\left(-\frac{3}{2}\right)=\frac{3}{2}$

c.) $f(9)$ = $\sqrt{9}=3$

d.) $f(-4)$: undef



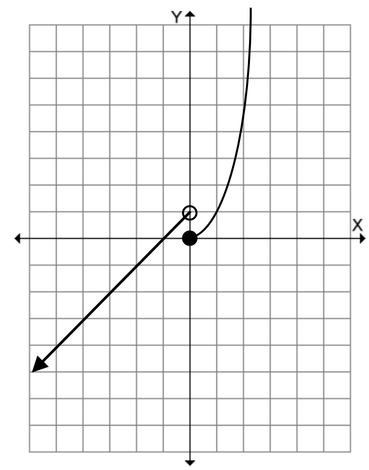
10B.) (6 pts) Given $f(x)=\begin{cases}1+x & if \quad x<0\\ x^2 & if \quad x\geq 0\end{cases}$ find the following and graph.

a.) $f(0) \;=\; (0)^2=0$

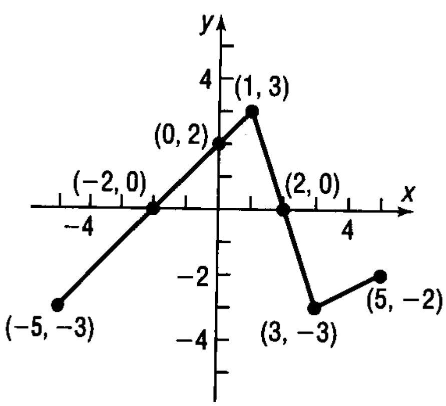
b.) $f(-3) \;=\; 1+(-3)=-2$

c.) $f\left(\frac{2}{3}\right) \;=\; \left(\frac{2}{3}\right)^2=\frac{4}{9}$

d.) $f\left(-\frac{1}{2}\right) \;=\; 1+\left(-\frac{1}{2}\right)=\frac{1}{2}$

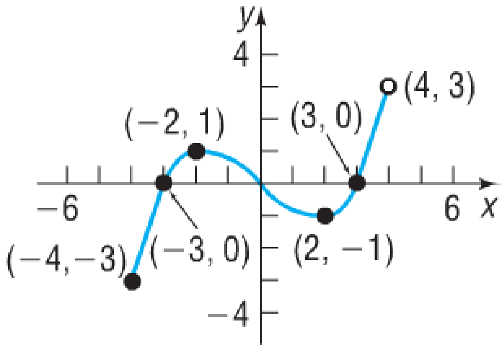



11A.) (4 pts) Use the graph of $f(x)$ below to answer the questions.




| | | |
|---|---|---|
| a.) Find $f(5)$ | 11a. | –2 |
| b.) Find all values of x such that $f(x)=-3$. | 11b. | –5, 3 |
| c.) Find the domain. | 11c. | [–5, 5] |
| d.) Find the range. | 11d. | [–3, 3] |
| e.) Interval(s) of decreasing | 11e. | (1, 3) |
| f.) List the number at which the graph has relative max. | 11f. | 1 |
| g.) What is the value of the relative minimum? | 11g. | –3 |
| h.) How many times does the line y = –5/2 intersect *f*? | 11h. | 3 |

11B.) (4 pts) Use the graph of $f(x)$ below to answer the questions.




| | | |
|---|---|---|
| a.) Find $f(-4)$ | 11a. | –3 |
| b.) Find all values of x such that $f(x)=0$. | 11b. | –3, 0, 3 |
| c.) Find the domain. | 11c. | [–4, 4) |
| d.) Find the range. | 11d. | [–3, 3) |
| e.) Interval(s) of increasing | 11e. | $(-4,\ -2)\cup(2,\ 4)$ |
| f.) List the number at which the graph has relative min. | 11f. | 2 |
| g.) What is the value of the relative maximum? | 11g. | 1 |
| h.) How many times does the line y = –1/2 intersect *f*? | 11h. | 3 |

12A.) (6 pts) Let $f(x)=\dfrac{1}{2x-3}$ and $g(x)=\dfrac{x+3}{2}$

i.) Find $(f\circ g)(0)$ if possible. i. undefined

This means $f(g(0))$. First find $g(0)$. So $g(0)=\dfrac{0+3}{2}=\dfrac{3}{2}$.

Now we will find $f\left(\dfrac{3}{2}\right)=\dfrac{1}{2\left(\dfrac{3}{2}\right)-3}=\dfrac{1}{3-3}=\dfrac{1}{0}=\textit{undefined}$.

Therefore $(f\circ g)(0)$ is undefined.

ii.) Find $(g\circ f)(x)$. Write as a single fraction. ii. $g(f(x))=\dfrac{3x-4}{2x-3}$

$$g(f(x))=g\left(\frac{1}{2x-3}\right)=\frac{\left(\frac{1}{2x-3}\right)+3}{2}=\frac{\frac{1}{2x-3}+\frac{3}{1}\cdot\left(\frac{2x-3}{2x-3}\right)}{2}=\frac{\frac{1+6x-9}{2x-3}}{2}=\frac{6x-8}{2x-3}\cdot\frac{1}{2}=\frac{2(3x-4)}{2x-3}\cdot\frac{1}{2}$$

$$g(f(x))=\frac{3x-4}{2x-3}$$

iii.) Find $(f \circ g)(x)$. Simplify.  iii. $f(g(x)) = \frac{1}{x}$

$$f(g(x)) = f\left(\frac{x+3}{2}\right) = \frac{1}{2\left(\frac{x+3}{2}\right) - 3} = \frac{1}{x+3-3} = \frac{1}{x}$$

$$f(g(x)) = \frac{1}{x}$$

12B.) (6 pts) Let $f(x) = x + 3$ and $g(x) = x^2 - 4$

i.) Find $(f \circ g)(1)$ if possible.  i. 0

This means $f(g(1))$. First find $g(1)$. So $g(1) = 1^2 - 4 = -3$.
Now we will find $f(-3) = -3 + 3 = 0$.
Therefore $(f \circ g)(0)$ is 0.

ii.) Find $(g \circ f)(x)$. Factor your answer.  ii. $g(f(x)) = (x+1)(x+5)$

$$g(f(x)) = g(x+3)$$
$$g(f(x)) = (x+3)^2 - 4$$
$$g(f(x)) = x^2 + 6x + 9 - 4$$
$$g(f(x)) = x^2 + 6x + 5$$
$$g(f(x)) = (x+1)(x+5)$$

iii.) Find $(f \circ g)(x)$. Factor your answer.  iii. $f(g(x)) = (x+1)(x-1)$

$$f(g(x)) = f(x^2 - 4)$$
$$f(g(x)) = x^2 - 4 + 3$$
$$f(g(x)) = x^2 - 1$$
$$f(g(x)) = (x+1)(x-1)$$

13A.) (4 pts) Find $f^{-1}(x)$ if $f(x)=\sqrt[5]{3x+5}$. You do NOT need to do a check to verify your answer.

13A. $f^{-1}(x)=\dfrac{x^5-5}{3}$

$$f(x)=\sqrt[5]{3x+5}$$
$$y=\sqrt[5]{3x+5}$$
$$x=\sqrt[5]{3y+5}$$
$$x^5=\left(\sqrt[5]{3y+5}\right)^5$$
$$x^5=3y+5$$
$$x^5-5=3y$$
$$\frac{x^5-5}{3}=y, \quad \text{so } f^{-1}(x)=\frac{x^5-5}{3}$$

13B.) (4 pts) Find $f^{-1}(x)$ if $f(x)=\dfrac{x-2}{2x+3}$. You do NOT need to do a check to verify your answer.

13B. $f^{-1}(x)=\dfrac{-3x-2}{2x-1}$

$$f(x)=\frac{x-2}{2x+3}$$
$$y=\frac{x-2}{2x+3}$$

$$x=\frac{y-2}{2y+3}$$

$x(2y+3)=y-2$ Cross multiply

$$2xy+3x=y-2$$

$$2xy-y=-3x-2$$

$$y(2x-1)=-3x-2$$

$$y=\frac{-3x-2}{2x-1}, \quad \text{so } f^{-1}(x)=\frac{-3x-2}{2x-1}$$