NAME: <u>KEY</u>

MATH 126 TEST 2 SAMPLE KEY

NOTE: The actual exam will only have 12 questions. The different parts of each question (part A, B, etc.) are variations. Know how to do all the variations on this exam.

1A.) (3 pts) Write a degree 4 polynomial $f(x)$ with a leading coefficient of 1 that has the following zeros: 0, –1, $\pm\sqrt{2}$
Leave your answer in factored form.

1A. $f(x) = x(x+1)(x-\sqrt{2})(x+\sqrt{2})$

To form a polynomial, use the formula: (x – zero1)(x – zero2)(x – xero3)….

So for this problem plug in the zeros:
$f(x) = (x-0)(x-(-1))(x-\sqrt{2})(x-(-\sqrt{2}))$. Now simplify.
$f(x) = x(x+1)(x-\sqrt{2})(x+\sqrt{2})$.

1B.) (3 pts) Write a degree 5 polynomial $f(x)$ with a leading coefficient of 1 that has the following zeros: –5, –2, 0, 1, 6
Leave your answer in factored form.

1B. $f(x) = x(x+5)(x+2)(x-1)(x-6)$

To form a polynomial, use the formula: (x – zero1)(x – zero2)(x – xero3)….

So for this problem plug in the zeros:
$f(x) = (x-(-5))(x-(-2))(x-0)(x-1)(x-6)$. Now simplify.
$f(x) = x(x+5)(x+2)(x-1)(x-6)$.

2.) (4 pts) Find the intercepts for: $y = 6x^2 - 11x - 10$

Use any method.

x-int: $x = -\frac{2}{3},\ x = \frac{5}{2}$

y-int: –10

x-int: y = 0
$0 = 6x^2 - 11x - 10$ (Factor using Bottom's Up)
$0 = x^2 - 11x - 60$
$0 = (x+4)(x-15)$
$0 = \left(x + \frac{4}{6}\right)\left(x - \frac{15}{6}\right)$
$0 = \left(x + \frac{2}{3}\right)\left(x - \frac{5}{2}\right)$, $x = -\frac{2}{3},\ x = \frac{5}{2}$

y-int: $x = 0$
$y = 6(0)^2 - 11(0) - 10$
$y = -10$

2B.) (4 pts) Find the intercepts for: $y=(2-x)^2-5$ x-int: $x=2\pm\sqrt{5}$

Use any method.

x-int: y = 0

$$0=(2-x)^2-5$$
$$5=(2-x)^2$$
$$\pm\sqrt{5}=2-x$$
$$-2\pm\sqrt{5}=-x$$
$$(-1)\left(-2\pm\sqrt{5}\right)=(-x)(-1)$$
$$x=2\pm\sqrt{5}$$

y-int: $x=0$ y-int: $y=-1$

$$y=(2-0)^2-5$$
$$y=4-5$$
$$y=-1$$

3A.) (4 pts) Solve: $0=(2x-3)^2-9$ 3. 0, 3

$$0=(2x-3)^2-9$$
$$(2x-3)^2=9$$
$$\sqrt{(2x-3)^2}=\pm\sqrt{9}$$

$2x-3=3$	$2x-3=-3$
$2x=6$	$2x=0$
$x=3$	$x=$

3B.) (4 pts) Solve: $2x^2-8x-14=0$ 3B. $2\pm\sqrt{11}$

We see that there is a common factor of 2. We can divide both sides of the equation by 2 so that it will make our numbers smaller:

$\frac{2x^2}{2}-\frac{8x}{2}-\frac{14}{2}=\frac{0}{2}$. This simplifies to $x^2-4x-7=0$. This problem cannot be factored, so we will use the quadratic formula to solve it.

In this case, $a=1$, $b=-4$, and $c=-7$.

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-7)}}{2(1)} = \frac{4\pm\sqrt{16+28}}{2} = \frac{4\pm\sqrt{44}}{2} = \frac{4\pm2\sqrt{11}}{2} = 2\pm\sqrt{11}$$

4A.) (3 pts) Simplify: i^{239} 4A. $-i$

$\frac{239}{4}=59$ remainder 3. So $i^{239}=i^3=-i$

4B.) (3 pts) Simplify: i^{108} 4B. 1

$\frac{108}{4} = 27$ remainder 0. So $i^{108} = i^0 = 1$

5A.) (4 pts) Divide and write in standard form: $\frac{4+i}{3+5i}$ 5A. $\frac{1}{2} - \frac{1}{2}i$

$$\frac{4+i}{3+5i} \cdot \frac{(3-5i)}{(3-5i)} = \frac{12-20i+3i-5i^2}{9-15i+15i-25i^2} = \frac{12-17i-5(-1)}{9-25(-1)} = \frac{17-17i}{34} = \frac{1}{2} - \frac{1}{2}i$$

5B.) (4 pts) Divide and write in standard form: $\frac{52i}{2-3i}$ 5B. $-12+8i$

$$\frac{52i}{2-3i} \cdot \frac{(2+3i)}{(2+3i)} = \frac{104i+156i^2}{4+6i-6i-9i^2} = \frac{104i+156(-1)}{4-9(-1)} = \frac{-156+104i}{13} = -12+8i$$

6A.) (3 pts) Write the expression in terms of *i* and simplify: $\sqrt{-32}\sqrt{-6}$ 6A. $-8\sqrt{3}$

$\sqrt{-32}\sqrt{-6}$

$\sqrt{16 \cdot 2 \cdot -1}\sqrt{-6}$

$4i\sqrt{2} \cdot i\sqrt{6}$

$4i^2\sqrt{12}$

$4(-1) \cdot 2\sqrt{3}$

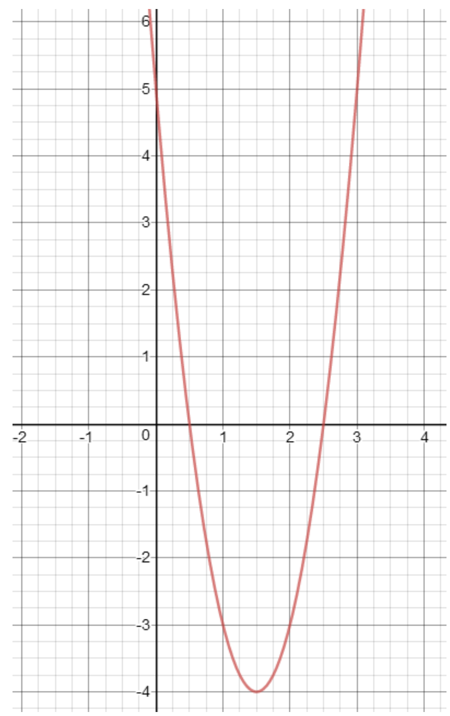
$-8\sqrt{3}$

6B.) (3 pts) Write the expression in terms of i and simplify: $\sqrt{-40}\sqrt{-5}$ 6B. $-10\sqrt{2}$

$$\sqrt{-40}\sqrt{-5}$$
$$2i\sqrt{10}\sqrt{-5}$$
$$2i\sqrt{10}\cdot i\sqrt{5}$$
$$2i^2\sqrt{50}$$
$$2(-1)\cdot 5\sqrt{2}$$
$$-10\sqrt{2}$$

7A.) (8 points) Find the vertex, axis of symmetry, and intercepts

for $y = 4x^2 - 12x + 5$ and graph.



vertex: $x = -b/2a$ $y = 4x^2 - 12x + 5$

$x = -(-12)/2(4)$ $y = 4(3/2)^2 - 12(3/2) + 5$

$x = 12/8 = 3/2$ $y = -4$, So vertex is (3/2, –4)

Axis of symmetry: $x = 3/2$

x-int: $y = 0$

$0 = 4x^2 - 12x + 5$

$0 = (2x-1)(2x-5)$

$x = 1/2$, $x = 5/2$

y-int: $x = 0$

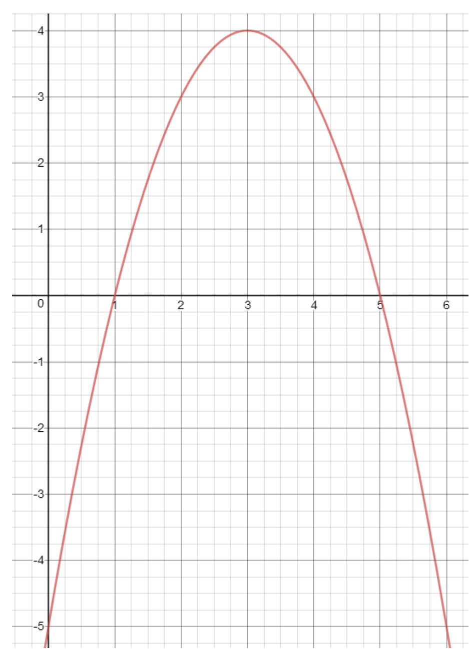
$y = 4(0)^2 - 12(0) + 5$

$y = 5$

Range: (y-values graph uses): $[-4,\infty)$

7B.) (8 points) Find the vertex, axis of symmetry, and intercepts

for $y = -(x-3)^2 + 4$ and graph.



If in the form $y = a(x-h)^2 + k$ then the vertex is (h, k):

vertex: (3, 4) Axis of Symmetry: $x = 3$

x-int: $y = 0$

$0 = -(x-3)^2 + 4$

$(x-3)^2 = 4$

$\sqrt{(x-3)^2} = \pm\sqrt{4}$

$x - 3 = 2$ and $x - 3 = -2$

$x = 5$ and $x = 1$

y-int: $x = 0$

$y = -(0-3)^2 + 4$

$y = -9 + 4$

$y = -5$

Range: (y-values graph uses): $(-\infty,4]$

8A.) (5 pts) Among all pair of numbers whose sum is 19, find a pair whose product is as large as possible. What is the maximum product?

First set up equations

$x + y = 19$

$P = xy$

Pair: 19/2, 19/2

Product: 361/4

Solve for y and substitute into the second equation

$x + y = 19 \rightarrow y = 19 - x$

$P = x(19 - x)$

$P = 19x - x^2$ Now find the vertex: $x = -19/(2(-1))$ so $x = 19/2$. Then $y = 19 - 19/2 = 19/2$

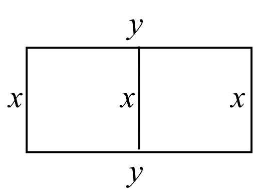
$P = xy$, so $P = (19/2)*(19/2) = 361/4$

8B.) (5 pts) A rancher with 600 feet of fencing wants to enclose a rectangular horse corral and then divide it into two pens with fencing parallel to one side of the rectangle as shown below.

i.) Find a function, $A(x)$, that models the area of corral in terms of the width x of the corral.

i. $A(x) = x\left(300 - \frac{3x}{2}\right)$

Let's label the drawing with x as the width and y as the length. This is because they want a function in terms of width x of the corral. Notice you need three x's, one for each width:



One equation will result from adding all the sides and having it equal 600 (amount of fencing available). The other equation is the area formula, with x as the width and y as the length.

$3x + 2y = 600$

$A = xy$

Solve for y and substitute into the second equation

$2y = 600 - 3x \rightarrow y = 300 - \frac{3x}{2}$

$A = x\left(300 - \frac{3x}{2}\right)$

$A = 300x - \frac{3x^2}{2}$

ii.) What is the maximum area that can be enclosed by the corral? ii. 15000 sq. ft.

We will use our equation $A(x) = x\left(300 - \frac{3x}{2}\right)$ to find the vertex using the vertex formula:

$$x = \frac{-300}{2\left(-\frac{3}{2}\right)} = \frac{-300}{-3} = 100\text{ ft.}$$

Then $y = 300 - \frac{3(100)}{2} = 150\text{ ft.}$

We now have the dimensions, so plug these into the formula $A = xy$.

$A = xy$, so $A = 100 * 150 = 15000$ sq. ft.

9A.) (5 pts) The daily revenue, R, achieved by selling x wristwatches is figured to be $R(x) = 75x - 0.2x^2$. The daily cost, C, of selling x wristwatches is $C(x) = 33x - 1750$.

i.) Write a profit function, P, as a function of x. i. $P(x) = -0.2x^2 + 42x + 1750$

$P(x) = R(x) - C(x)$ (Profit = Revenue – Costs)
$P(x) = 75x - 0.2x^2 - (33x - 1750)$
$P(x) = -0.2x^2 + 42x + 1750$

ii.) How many wristwatches must be sold in order to maximize the profit? ii. 105 watches

$x = -b/2a$
$x = -42/(2(-0.2))$
$x = 105$

iii.) What is the maximum profit? iii. $ 3955

$P(105) = -0.2(105)^2 + 42(105) + 1750$
$P(105) = \$3955$

9B (5 pts) A quarterback throws a football with an initial velocity of 72 ft/sec at an angle of 25°. The height of the ball can be modeled by $h(t) = -16t^2 + 30.4t + 5$, where $h(t)$ is the height (in ft) and t is the time in seconds after release.

i.) Determine the time at which the ball reaches its maximum height. Round to the nearest hundredth. — i. 0.95 sec

$$x = -b/2a$$
$$x = -30.4/(2(-16))$$
$$x = 0.95$$

ii.) Determine the maximum height of the football. Round to the nearest hundredth. — ii. 19.44 ft

$$h(0.95) = -16(0.95)^2 + 30.4(0.95) + 5$$
$$h(0.95) = 19.44$$

10A.) (4 pts) Solve and write in interval notation: $\dfrac{(x+4)^2(x-1)}{(x+2)^3} \le 0$ — 10A. $[-4] \cup (-2, 1]$

First set each factored piece equal to zero to get x = -4, 1, and -2. Then create the table. We will use test values of -5, -3, 0, and 2. Put these into the expressions in the left column and indicate positive or negative. Then multiply all the signs in each column to get the final sign configuration.

	+	+	–	+
$(x+4)^2$	+	+	+	+
$(x-1)$	–	–	–	+
$(x+2)^3$	–	–	+	+

-4 -2 1

If ≤ 0, we look for negatives. Can't include -2 (division by 0): (–2, 1]
We also must include -4 as well since this also make the equation correct.
To include one number, write [-4]. Therefore, we write: $[-4] \cup (-2, 1]$.

10B.) (4 pts) Solve and write in interval notation: $(x+1)(x-3)^2 > 0$ — 10B. $(-1, 3) \cup (3, \infty)$

First set each factored piece equal to zero to get x = –1 and 3. Then create the table. We will use test values of –2, 0, and 4. Put these into the expressions in the left column and indicate positive or negative. Then multiply all the signs in each column to get the final sign configuration.

	–	+	+
$(x+1)$	–	+	+
$(x-3)^2$	+	+	+

–1 3

If >0, we look for positives. Can't include 3 since this would result in $0 > 0$ which is not a true statement. Therefore, the answer is $(-1, 3) \cup (3, \infty)$

11A.) (10 pts) Graph $y = 0.2x(x-2)^2(x+2)^3$ Find the intercepts and multiplicities, and behavior at each (what equation the graph resembles at each zero), max. number of turning pts, degree.

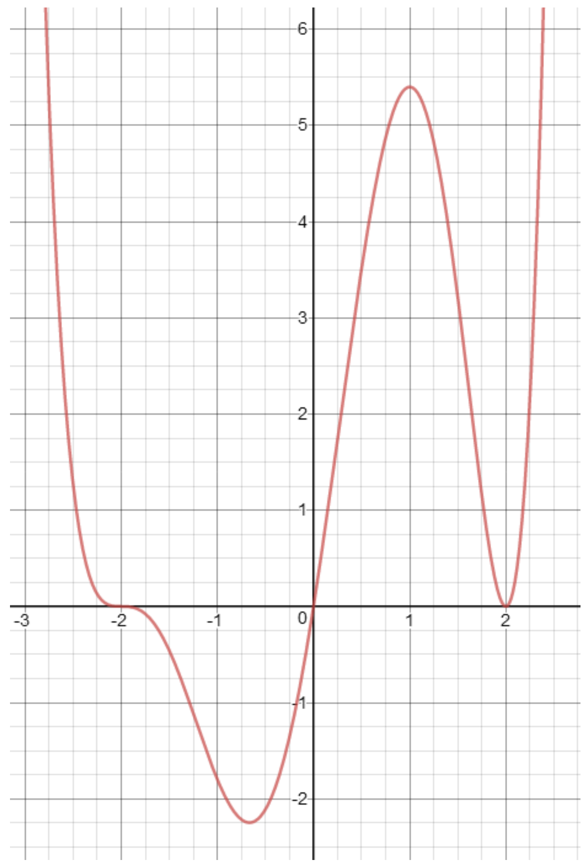
zero: 0 Multiplicity: 1 Crosses

zero: 2 Multiplicity: 2 Touches

zero: -2 Multiplicity: 3 Crosses

y-int: 0 Degree: 6 Max turning pts: 5

To determine the end behavior power function, look at each factor's leading term. The first factor is $0.2x$. If you expand the second factor you will have $y = x^2 + \ldots$ If you expand the third factor you will have $y = x^3 + \ldots$ So if you multiply the leading terms, you will get $y = 0.2x \cdot x^2 \cdot x^3 = 0.2x^6$. Therefore, the graph of *f* behaves like $y = 0.2x^6$ for large values of $|x|$.



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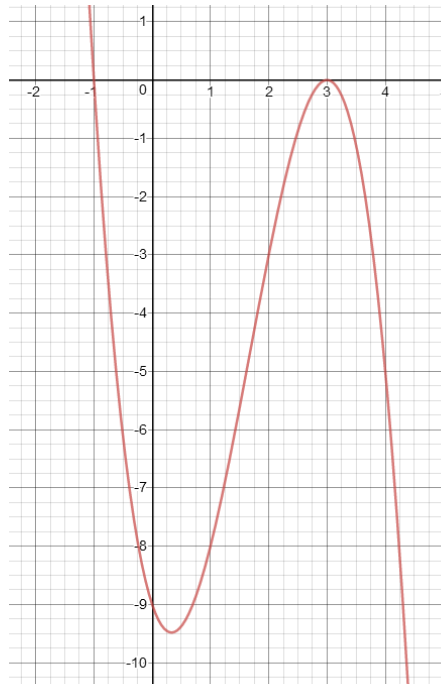
11B.) (10 pts) Graph $y=-(x+1)(x-3)^2$ Find the intercepts and multiplicities, and behavior at each zero (what equation the graph resembles at each zero), max. number of turning pts, degree.



zero: -1 Multiplicity: 1 Crosses

zero: 3 Multiplicity: 2 Touches

y-int: -9 Degree: 3 Max turning pts: 2

y-intercept: $x=0$ $y=-(0+1)(0-3)^2$

$y=-9$

To determine the end behavior power function, look at each factor's leading term. The first factor is $-(x...)$. If you expand the second factor you will have $y=x^2+...$ So if you multiply the leading terms, you will get $y=-x\cdot x^2=-x^3$. Therefore, the graph of f behaves like $y=-x^3$ for large values of $|x|$.

12A.) (7 pts) Given $f(x)=3x^3-23x^2+31x+13$ and $f\left(-\frac{1}{3}\right)=0$, answer the following:

i.) Use the Rational Zero Theorem to find the list of possible zeros. i. $\pm 1, \pm 13, \pm\frac{1}{3}, \pm\frac{13}{3}$

list of possible zeros.

$$\frac{factors\ of\ 13}{factors\ of\ 3}=\frac{\pm 1, \pm 13}{\pm 1, \pm 3}=\pm 1, \pm 13, \pm\frac{1}{3}, \pm\frac{13}{3}$$

ii.) Use SYNTHETIC DIVISION to find the other zeros. ii. $4\pm\sqrt{3}$

-1/3 |	3	-23	31	13
		-1	8	-13
	3	-24	39	0

$3x^2-24x+39=0$
$3(x^2-8x+13)=0$
$x^2-8x+13=0$

$$x=\frac{-(-8)\pm\sqrt{(-8)^2-4(1)(13)}}{2(1)}=\frac{8\pm\sqrt{12}}{2}=\frac{8\pm 2\sqrt{3}}{2}=4\pm\sqrt{3}$$

iii.) Use the zeros to factor $f(x)$. iii. $f(x)=(3x+1)(x^2-8x+13)$

Part of this is already factored from part b: $3(x^2-8x+13)$. We also need to use the zero -1/3. $3(x^2-8x+13)\left(x+\frac{1}{3}\right)$. Now rearrange: $3\left(x+\frac{1}{3}\right)(x^2-8x+13)$. To get rid of the fraction, multiply the first two terms to get the final answer: $f(x)=(3x+1)(x^2-8x+13)$.

12B.) (7 pts) Given $f(x) = 6x^4 - 5x^3 - 15x^2 + 4$ and $f(-1) = 0$ and $f\left(\frac{1}{2}\right) = 0$, answer the following:

i.) Use the Rational Zero Theorem to find the list of possible zeros.

i. $\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{1}{6}$

$$\frac{factors\ of\ 4}{factors\ of\ 6} = \frac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2 \pm 3, \pm 6} = \pm 1, \pm 2, \pm 4, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{1}{6}$$

i.) Use SYNTHETIC DIVISION to find the other zeros.

ii. $x = -\frac{2}{3},\ 2$

When you set this up for synthetic division, notice that in $f(x)$ there is no x term. Remember you need a 0 place keeper for the x term:

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-1 | 6   -5   -15   0    4
           -6    11   4   -4
     6  -11   -4    4    0
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1/2 | 6  -11   -4    4
            3   -4   -4
      6   -8   -8    0
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$$6x^2 - 8x - 8 = 0$$
$$2(3x^2 - 4x - 4) = 0$$
$$2(3x + 2)(x - 2) = 0$$
$$x = -\frac{2}{3},\ 2$$

iii.) Use the zeros to factor $f(x)$.

iii. $f(x) = (2x - 1)(3x + 2)(x - 2)(x + 1)$

Part of this is already factored from part b: $2(3x + 2)(x - 2)$. We also need to use the zeros -1 and 1/2. $2(3x + 2)(x - 2)(x + 1)\left(x - \frac{1}{2}\right)$. Now rearrange: $2\left(x - \frac{1}{2}\right)(3x + 2)(x - 2)(x + 1)$. To get rid of the fraction, multiply the first two terms to get the final answer: $f(x) = (2x - 1)(3x + 2)(x - 2)(x + 1)$.