NAME:_____________KEY______________

MATH 127 FINAL EXAM SAMPLE

NOTE: The actual exam will only have 13 questions. The different parts of each question (part A, B, etc.) are variations. Know how to do all the variations on this exam.

1A.) (4 pts) Find the exact value: $1+\tan^2\left(\frac{\pi}{6}\right)-\csc^2\left(\frac{\pi}{4}\right)$ 1A. $-\frac{2}{3}$

Section 4.2

$$1+\left(\tan\left(\frac{\pi}{6}\right)\right)^2-\left(\frac{1}{\sin\left(\frac{\pi}{4}\right)}\right)^2 \Rightarrow 1+\left(\frac{\sqrt{3}}{3}\right)^2-\frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} \Rightarrow 1+\frac{3}{9}-\frac{1}{\left(\frac{2}{4}\right)}$$

$$\Rightarrow 1+\frac{1}{3}-2=-\frac{2}{3}$$

1B.) (4 pts) Find the exact value: $3\cot^2\left(\frac{\pi}{3}\right)+2\cos\left(\frac{\pi}{6}\right)$ 1B. $1+\sqrt{3}$

Section 4.2

$$3\left(\frac{1}{\tan\left(\frac{\pi}{3}\right)}\right)^2+2\cos\left(\frac{\pi}{6}\right) \Rightarrow 3\cdot\frac{1}{\left(\sqrt{3}\right)^2}+2\left(\frac{\sqrt{3}}{2}\right) \Rightarrow \frac{3}{3}+\sqrt{3} \Rightarrow 1+\sqrt{3}$$

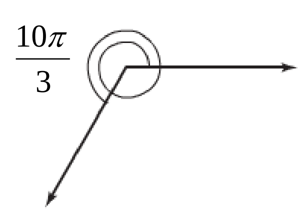
2A.) (5 pts) Find the <u>exact</u> value of $\csc\left(\frac{10\pi}{3}\right)$ using reference angles. 2A. $\frac{-2\sqrt{3}}{3}$

Indicate the ref. angle and draw in standard position. **Section 4.4**

Ref. angle: $\frac{\pi}{3}$

$$\frac{10\pi}{3}\cdot\frac{180}{\pi}=600^\circ \qquad 600^\circ-360^\circ=240^\circ \qquad \csc\left(\frac{10\pi}{3}\right)=\frac{1}{\sin\left(240^\circ\right)}$$

1.) R.A. = $240^\circ-180^\circ=60^\circ$



2.) $\sin(60^\circ)=\frac{\sqrt{3}}{2}$

3.) $\sin(240^\circ)=-\frac{\sqrt{3}}{2} \Rightarrow \frac{1}{\sin(240^\circ)}=-\frac{2}{\sqrt{3}}=\frac{-2\sqrt{3}}{3}$

2B.) (5 pts) Find the exact value of $\sec(480^\circ)$ using reference angles. Indicate the ref. angle and draw in standard position. **Section 4.4**

2B. -2

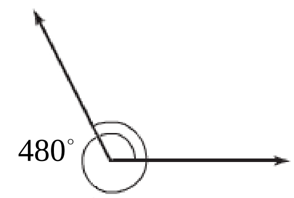
Ref. angle: 60°

$$480^\circ - 360^\circ = 120^\circ \qquad \sec(480^\circ) = \frac{1}{\cos(120^\circ)}$$

1.) R.A. = $180^\circ - 120^\circ = 60^\circ$

2.) $\cos(60^\circ) = \frac{1}{2}$

3.) $\cos(120^\circ) = -\frac{1}{2} \quad \Rightarrow \quad \frac{1}{\cos(120^\circ)} = -2$



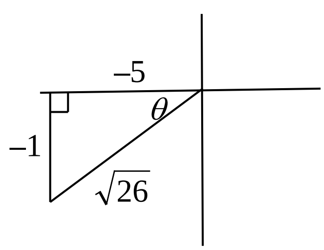
3A.) (5 pts) Find the following EXACT values if you are given $\cot\theta = 5$ and $180^\circ < \theta < 270^\circ$. **Section 4.4**

$\cos\theta = -\frac{5\sqrt{26}}{26} \qquad \sec\theta = -\frac{\sqrt{26}}{5}$

$\sin\theta = -\frac{\sqrt{26}}{26} \qquad \csc\theta = -\sqrt{26}$

$\tan\theta = \frac{1}{5}$

$$a^2 + b^2 = c^2$$
$$(-1)^2 + (-5)^2 = c^2$$
$$c^2 = 26\text{, so } b = \pm\sqrt{26}$$



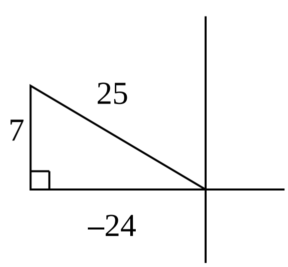
3B.) (5 pts) Find the following given:

$\sin\theta = \frac{7}{25}$ and $\tan\theta < 0$. **Section 4.4**

$\cos\theta: \ -\frac{24}{25} \qquad \sec\theta: \ -\frac{25}{24}$

$\sin\theta: \ \frac{7}{25} \qquad \csc\theta: \ \frac{25}{7}$

$\tan\theta: \ -\frac{7}{24} \qquad \cot\theta: \ -\frac{24}{7}$



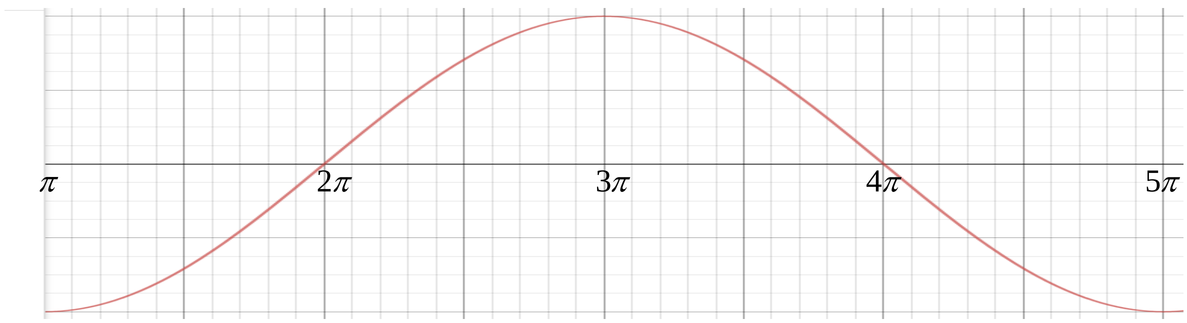
4A.) (5 pts) Use the following equation to answer the questions: $y=-2\cos\left(\frac{1}{2}x-\frac{\pi}{2}\right)$ **Section 4.5**

i.) Find the period. $\frac{2\pi}{B}=\frac{2\pi}{\frac{1}{2}}=4\pi$ 4i. 4π

ii.) Find the amplitude. $|A|=|-2|=2$ 4ii. 2

iii.) Find the phase shift. $\frac{opp\ sign\ of\ C}{B}=\frac{\frac{\pi}{2}}{\frac{1}{2}}=\pi$ 4iii. π

iv.) Graph the function over one period.



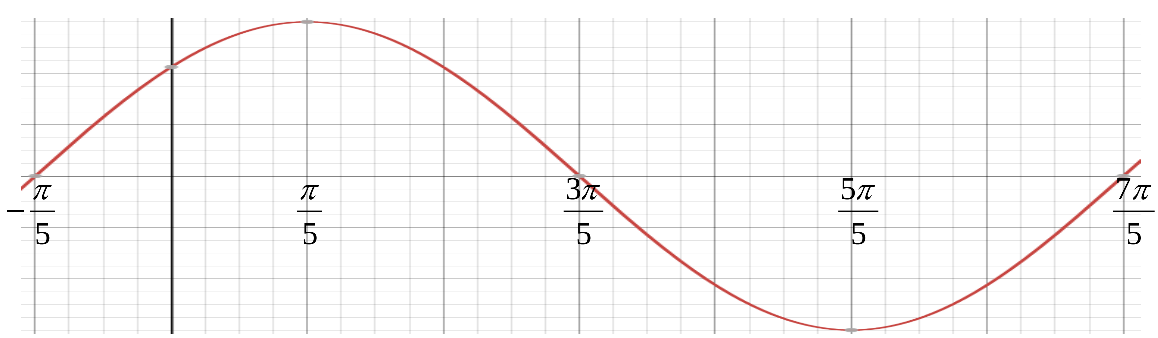
4B.) (5 points) Use the following equation to answer the questions: $y=3\sin\left(\frac{5}{4}x+\frac{\pi}{4}\right)$ **Section 4.5**

i.) Find the period. $\frac{2\pi}{B}=\frac{2\pi}{\frac{5}{4}}=\frac{8\pi}{5}$ 4i. $\frac{8\pi}{5}$

ii.) Find the amplitude. $|A|=|3|=3$ 4ii. 3

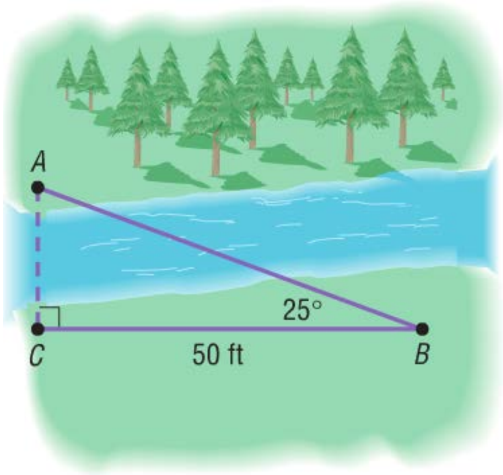
iii.) Find the phase shift. $\frac{opp\ sign\ of\ C}{B}=\frac{-\frac{\pi}{4}}{\frac{5}{4}}=-\frac{\pi}{5}$ 4iii. $-\frac{\pi}{5}$

iv.) Graph the function over one period.



5A.) (4 pts) Find the distance from **A to C** in the figure below: 5A. 23.32 ft

Section 4.3



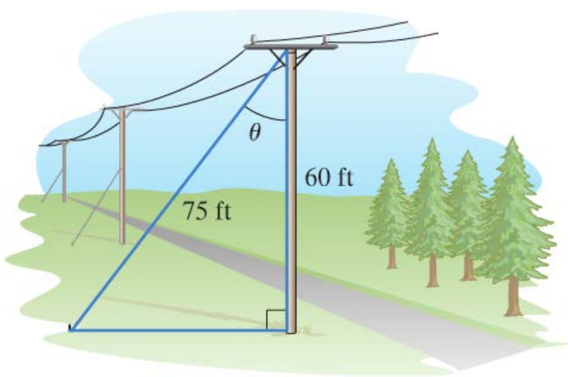
$$\tan 25^\circ = \frac{x}{50}$$

$$x = 50\tan 25^\circ$$

$$x = 23.32 \quad ft$$

5B.) (4 pts) Find the angle between the wire and the pole to the nearest degree. 5B. 37°

Section 4.3, 4.7



$$\cos\theta = \frac{60}{75}$$

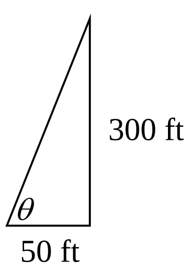
$$\cos\theta = 0.8$$

$$\theta = \cos^{-1}(0.8)$$

$$\theta = 36.87^\circ$$

6A.) (4 pts) At 10:00 AM on April 26, 2005, a building 300 feet high casts a shadow 50 feet long. What is the angle of elevation of the Sun? 6A. 80.54°

Section 4.3, 4.7



$$\tan\theta = \frac{300}{50}$$

$$\tan\theta = 6$$

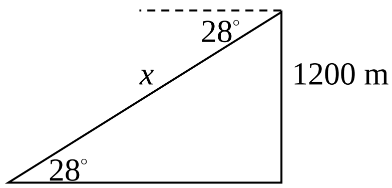
$$\theta = \tan^{-1}(6)$$

$$\theta = 80.54^\circ$$

6B.) (4 pts) From an airplane at an altitude (height) of 1200m, the angle of depression to a rock on the ground measures 28°. Find the distance from the plane to the rock.

6B. 2556.07 m

Section 4.3



$$\sin 28^\circ = \frac{1200}{x}$$

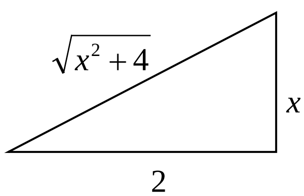
$$x \sin 28^\circ = 1200$$

$$x = \frac{1200}{\sin 28^\circ} \approx 2556.07 \ m$$

7A.) (4 pts) Use a right triangle to write $\sec\left(\sin^{-1}\left(\frac{x}{\sqrt{x^2+4}}\right)\right)$ as an algebraic expression. Assume that x is positive and that the given inverse trigonometric function is defined for the expression in *x*.

7A. $\frac{\sqrt{x^2+4}}{2}$

Section 4.7



$$\sec\left(\sin^{-1}\left(\frac{x}{\sqrt{x^2+4}}\right)\right) = \frac{\sqrt{x^2+4}}{2}$$

$$a^2 + x^2 = \left(\sqrt{x^2+4}\right)^2$$

$$a^2 + x^2 = x^2 + 4$$

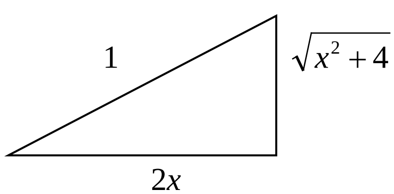
$$a^2 = 4$$

$$a = 2$$

7B.) (4 pts) Use a right triangle to write $\sin\left(\cos^{-1}(2x)\right)$ as an algebraic expression. Assume that x is positive and that the given inverse trigonometric function is defined for the expression in *x*.

7B. $\sqrt{1-4x^2}$

Section 4.7



$$\sin\left(\cos^{-1}(2x)\right) = \frac{\sqrt{1-4x^2}}{1}$$

$$(2x)^2 + b^2 = 1^2$$

$$4x^2 + b^2 = 1$$

$$b^2 = 1 - 4x^2$$

$$b = \sqrt{1-4x^2}$$

8A.) (5 pts) Verify the identity: $\cos x \cot x + \sin x = \csc x$ **Section 5.1**

$$\cos x \cdot \frac{\cos x}{\sin x} + \sin x = \frac{1}{\sin x}$$

$$\frac{\cos^2 x}{\sin x} + \frac{\sin x}{1}\left(\frac{\sin x}{\sin x}\right) = \frac{1}{\sin x}$$

$$\frac{\cos^2 x + \sin^2 x}{\sin x} = \frac{1}{\sin x}$$

$$\frac{1}{\sin x} = \frac{1}{\sin x}$$

8B.) (5 pts) Verify the identity: $\frac{1+\sin\theta}{1-\sin\theta} - \frac{1-\sin\theta}{1+\sin\theta} = 4\tan\theta\sec\theta$ **Section 5.1**

$$\frac{1+\sin\theta}{1-\sin\theta}\left(\frac{1+\sin\theta}{1+\sin\theta}\right) - \frac{1-\sin\theta}{1+\sin\theta}\left(\frac{1-\sin\theta}{1-\sin\theta}\right) = 4 \cdot \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$$

$$\frac{1+2\sin\theta+\sin^2\theta-\left(1-2\sin\theta+\sin^2\theta\right)}{1-\sin^2\theta} = \frac{4\sin\theta}{\cos^2\theta}$$

$$\frac{1+2\sin\theta+\sin^2\theta-1+2\sin\theta-\sin^2\theta}{1-\sin^2\theta} = \frac{4\sin\theta}{\cos^2\theta}$$

$$\frac{4\sin\theta}{\cos^2\theta} = \frac{4\sin\theta}{\cos^2\theta}$$

9A.) (5 pts) Verify the identity: $\cos\left(\theta - \frac{3\pi}{2}\right) = -\sin\theta$ **Section 5.2**

$$\cos\theta\cos\left(\frac{3\pi}{2}\right) + \sin\theta\sin\left(\frac{3\pi}{2}\right) = -\sin\theta$$

$$\cos\theta \cdot (0) + \sin\theta \cdot (-1) = -\sin\theta$$

$$-\sin\theta = -\sin\theta$$

9B.) (5 pts) Verify the identity: $\dfrac{\sin\left(x+\frac{3\pi}{2}\right)}{\cos(\pi-x)}=1$ **Section 5.2**

$$\frac{\sin x\cos\left(\frac{3\pi}{2}\right)+\cos x\sin\left(\frac{3\pi}{2}\right)}{\cos\pi\cos x+\sin\pi\sin x}=1$$

$$\frac{\sin x\cdot(0)+\cos x\cdot(-1)}{(-1)\cdot\cos x+(0)\cdot\sin x}=1$$

$$\frac{-\cos x}{-\cos x}=1$$

$$1=1$$

10A.) (5 pts) Verify the identity: $\sec(2\theta)=\dfrac{\sec^2\theta}{2-\sec^2\theta}$ **Section 5.3**

$$\frac{1}{\cos(2\theta)}=\frac{\frac{1}{\cos^2\theta}}{2-\frac{1}{\cos^2\theta}} \quad\Rightarrow\quad \frac{1}{\cos(2\theta)}=\frac{\frac{1}{\cos^2\theta}}{\frac{2\cos^2\theta}{\cos^2\theta}-\frac{1}{\cos^2\theta}} \quad\Rightarrow\quad \frac{1}{\cos(2\theta)}=\frac{\frac{1}{\cos^2\theta}}{\frac{2\cos^2\theta-1}{\cos^2\theta}}$$

$$\Rightarrow\quad \frac{1}{\cos(2\theta)}=\frac{1}{2\cos^2\theta-1} \quad\Rightarrow\quad \frac{1}{\cos(2\theta)}=\frac{1}{\cos(2\theta)}$$

10B.) (5 pts) Verify the identity: $\cot x=\dfrac{1+\cos 2x}{\sin 2x}$ **Section 5.3**

$$\cot x=\frac{1+2\cos^2 x-1}{2\sin x\cos x}$$

$$\cot x=\frac{2\cos^2 x}{2\sin x\cos x}$$

$$\cot x=\frac{\cos x}{\sin x}$$

$$\cot x=\cot x$$

11A.) (5 pts) Solve for θ on the interval $[0,\ 2\pi]$:

$$\left(2\cos\theta-\sqrt{3}\right)\left(\sin^2\theta-1\right)=0$$

11A. $\frac{\pi}{6},\ \frac{11\pi}{6},\ \frac{\pi}{2},\ \frac{3\pi}{2}$

Section 5.5

$2\cos\theta-\sqrt{3}=0$ $\qquad$ $\sin^2\theta-1=0$

$\cos\theta=\frac{\sqrt{3}}{2}$ $\qquad$ $\sin^2\theta=1 \quad \Rightarrow \quad \sin\theta=\pm1$

$\theta=\frac{\pi}{6},\ \frac{11\pi}{6}$ $\qquad$ $\theta=\frac{\pi}{2},\ \frac{3\pi}{2}$

11B.) (5 pts) Solve for θ on the interval $[0,\ 360^\circ]$:

$$\sin x+2\sin x\cos x=0$$

11B. $\theta=0^\circ,\ 120^\circ,\ 180^\circ,\ 240^\circ$

Section 5.5

$\sin x(1+2\cos x)=0$

$\sin x=0$ $\qquad$ $1+2\cos x=0$

$\theta=0^\circ,\ 180^\circ$ $\qquad$ $\cos\theta=-\frac{1}{2}$

$\theta=120^\circ,\ 240^\circ$

12A.) (4 pts) Convert the rectangular equation $x^2y=4$ into a **polar** equation that expresses r in terms of θ.

$r=\sqrt[3]{\frac{4}{\cos^2\theta\sin\theta}}$

Section 7.1

$(r\cos\theta)^2\cdot r\sin\theta=4$

$r^2\cos^2\theta\cdot r\sin\theta=4$

$r^3\cos^2\theta\sin\theta=4$

$r^3=\frac{4}{\cos^2\theta\sin\theta} \quad \Rightarrow \quad r=\sqrt[3]{\frac{4}{\cos^2\theta\sin\theta}}$ or $r=\sqrt[3]{4\sec^2\theta\csc\theta}$

12B.) (4 pts) Convert the rectangular equation $(x-2)^2 + y^2 = 4$ into a **polar** equation that expresses r in terms of θ. $r = 4\cos\theta$

Section 7.1

$x^2 - 4x + 4 + y^2 = 4$

$x^2 - 4x + y^2 = 0$

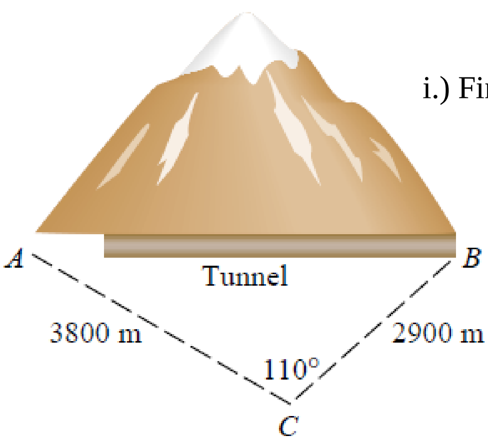
$x^2 + y^2 = 4x$ → $r^2 = 4r\cos\theta$

$r^2 - 4r\cos\theta = 0$

$r(r - 4\cos\theta) = 0$

~~$r = 0$~~ or $r - 4\cos\theta = 0 \Rightarrow r = 4\cos\theta$

13A.) (5 pts) To measure the distance through a mountain for a proposed tunnel, a point C is chosen that can be reached from each end of the tunnel (see figure). Given: AC = 3800 meters, BC = 2900 meters, and angle C is 110 degrees. Round all answers to two decimal places. **Section 6.2, 6.3**



i.) Find the length of the tunnel. i. 5512.54 *miles*

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 2900^2 + 3800^2 - 2(2900)(3800)\cos 110^\circ$

$c^2 = 30388123.96$

$c = 5512.54$

ii.) Find $m\angle B$. ii. 40.37°

$$\frac{\sin 110^\circ}{5512.54} = \frac{\sin B}{3800}$$

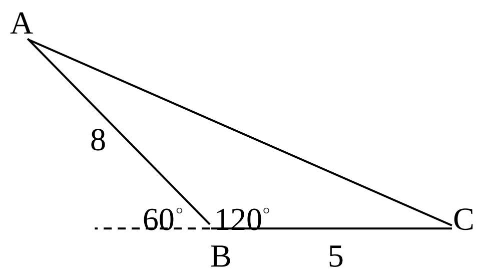
$5512.54 \sin B = 3800 \sin 110^\circ$

$$\sin B = \frac{3800 \sin 110^\circ}{5512.54}$$

$\sin B = 0.647765...$

$B = \sin^{-1}(0.647765...) \approx 40.37^\circ$

13B.) (5 pts) In the figure below, find AC and the measure of angle C. AC: 11.36
Round your answers to the nearest hundredth. **Section 6.2, 6.3**



$m\angle C$: 37.58°

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$b^2 = 5^2 + 8^2 - 2(5)(8)\cos 120^\circ$$

$$b^2 = 129$$

$$b = 11.36$$

$$\frac{\sin 120^\circ}{11.36} = \frac{\sin C}{8}$$

$$11.36\sin C = 8\sin 120^\circ$$

$$\sin C = \frac{8\sin 120^\circ}{11.36}$$

$$\sin C = 0.609877...$$

$$C = \sin^{-1}(0.609877...) \approx 37.58^\circ$$