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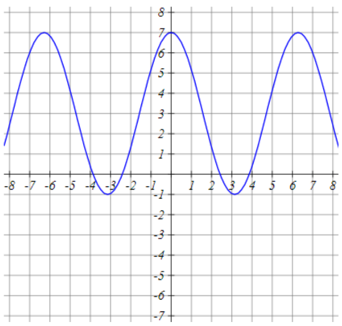
MATH 127 FINAL EXAM SAMPLE

NOTE: The actual exam will only have 14 questions. The different parts of each question (part A, B, etc.) are variations. Know how to do all the variations on this exam.

1A.) (4 points) Choose which function best represents the trigonometric graph below.



If we consider the period starting at x=0, we see that the graph goes down and then up. This means we have a cosine graph.

To find the amplitude, take the highest y-value minus the lowest y-value and divide by 2. (7-(-1))/2 = 8/2 = 4. If we go down 4 units from the highest y-value, we will be at 3, which is the center of the graph. Normally the x-axis is the center of the graph, but now it is at 3. This means the graph is shifted up 3 units. The equation of the graph is $y = 4\cos x + 3$.

(A) $y = 4\sin x + 3$

(B) $y = 4\cos x + 3$

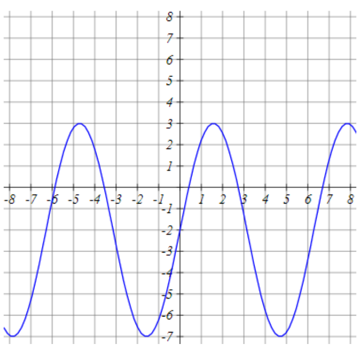
(C) $y = 5\sin x + 3$

(D) $y = 4\cos x$

(E) $y = 5\cos x + 3$

(F) $y = 4\cos x + 2$

1B.) (4 points) Drawn on the coordinate system below is one period of the graph of which equation?



If we consider the period starting at x=0, we see that the graph goes up, down, and then up. This means we have a sine graph.

To find the amplitude, take the highest y-value minus the lowest y-value and divide by 2. (3-(-7))/2 = 10/2 = 5. If we go down 5 units from the highest y-value, we will be at -2, which is the center of the graph. Normally the x-axis is the center of the graph, but now it is at -2. This means the graph is shifted down 2 units. The equation of the graph is $y = 5\sin x - 2$.

(A) $y = 6\sin x - 2$

(B) $y = 5\sin x - 3$

(C) $y = 5\sin x$

(D) $y = 5\sin x - 2$

(E) $y = 6\cos x - 2$

(E) $y = 5\cos x - 2$

2A.) (3 pts) What is the graph of the polar equation $r = 81\tan\theta\sec\theta$?

First use identities for tangent and secant: $r = \frac{81}{1}\cdot\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}$ This becomes $r = \frac{81\sin\theta}{\cos^2\theta}$. Now multiply both sides by $\cos^2\theta$ to get $r\cos^2\theta = 81\sin\theta$. Next, we need to multiply both sides by *r*: $r\left(r\cos^2\theta = 81\sin\theta\right) \Rightarrow r^2\cos^2\theta = 81r\sin\theta$. We can also write this as $\left(r\cos\theta\right)^2 = 81r\sin\theta$. Now we will use the substitutions $x = r\cos\theta$ and $y = r\sin\theta$. We will get $x^2 = 81y$. This is a parabola.

(A) a line

(B) an ellipse

(C) a parabola

(D) a circle

(E) a hyperbola

2B.) (3 pts) What is the graph of the polar equation $r = 3.4$?

First, we want to square both sides. $r^2 = 11.56$. We will replace r^2 with $x^2 + y^2$. We will get $x^2 + y^2 = 11.56$. This will be a graph of a circle.

(A) a parabola

(B) a circle

(C) a line

(D) a hyperbola

(E) an ellipse

3A.) (5 pts) Establish the identity: $\cos x\cot x + \sin x = \csc x$

$$\cos x\cdot\frac{\cos x}{\sin x} + \sin x = \frac{1}{\sin x}$$

$$\frac{\cos^2 x}{\sin x} + \frac{\sin x}{1}\left(\frac{\sin x}{\sin x}\right) = \frac{1}{\sin x}$$

$$\frac{\cos^2 x + \sin^2 x}{\sin x} = \frac{1}{\sin x}$$

$$\frac{1}{\sin x} = \frac{1}{\sin x}$$

3B.) (5 pts) Verify the identity: $\dfrac{1+\sin\theta}{1-\sin\theta}-\dfrac{1-\sin\theta}{1+\sin\theta}=4\tan\theta\sec\theta$

$$\frac{1+\sin\theta}{1-\sin\theta}\left(\frac{1+\sin\theta}{1+\sin\theta}\right)-\frac{1-\sin\theta}{1+\sin\theta}\left(\frac{1-\sin\theta}{1-\sin\theta}\right)=4\cdot\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}$$

$$\frac{1+2\sin\theta+\sin^2\theta-\left(1-2\sin\theta+\sin^2\theta\right)}{1-\sin^2\theta}=\frac{4\sin\theta}{\cos^2\theta}$$

$$\frac{1+2\sin\theta+\sin^2\theta-1+2\sin\theta-\sin^2\theta}{1-\sin^2\theta}=\frac{4\sin\theta}{\cos^2\theta}$$

$$\frac{4\sin\theta}{\cos^2\theta}=\frac{4\sin\theta}{\cos^2\theta}$$

4A.) (5 pts) Solve for θ on the interval $[0,\ 2\pi)$:

4A. $\dfrac{\pi}{6},\ \dfrac{11\pi}{6},\ \dfrac{\pi}{2},\ \dfrac{3\pi}{2}$

$$\left(2\cos\theta-\sqrt{3}\right)\left(\sin^2\theta-1\right)=0$$

$2\cos\theta-\sqrt{3}=0$ $\qquad$ $\sin^2\theta-1=0$

$\cos\theta=\dfrac{\sqrt{3}}{2}$ $\qquad$ $\sin^2\theta=1 \quad\Rightarrow\quad \sin\theta=\pm1$

$\theta=\dfrac{\pi}{6},\ \dfrac{11\pi}{6}$ $\qquad$ $\theta=\dfrac{\pi}{2},\ \dfrac{3\pi}{2}$

4B.) (5 pts) Solve for θ on the interval $[0,\ 2\pi)$:

4B. $\theta=\dfrac{\pi}{2},\ \dfrac{2\pi}{3},\ \dfrac{4\pi}{3},\ \dfrac{3\pi}{2}$

$$\cot x+2\cos x\cot x=0$$

$$\cot x\left(1+2\cos x\right)=0$$

$\cot x=0$ $\qquad$ $1+2\cos x=0$

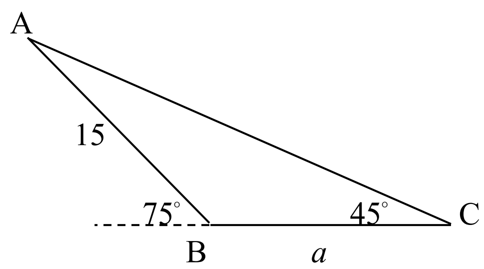
$\dfrac{\cos x}{\sin x}=0$ $\qquad$ $\cos\theta=-\dfrac{1}{2}$

$\cos x=0$ $\qquad$ $\theta=\dfrac{2\pi}{3},\ \dfrac{4\pi}{3}$

$x=\dfrac{\pi}{2},\dfrac{3\pi}{2}$

5A.) (4 pts) In the figure below, find the following.

a: $\frac{15\sqrt{2}}{2}$



$m\angle A$: 30 degrees

Since 75° is outside the triangle, we need to find the angle inside the triangle. Note that 75° and the angle inside the triangle creates a straight line, which is 180°. This means $m\angle B = 180^\circ - 75^\circ = 105^\circ$.

Now that we have angles B and C, we can find angle A by subtracting from 180°.
$m\angle A = 180^\circ - 105^\circ - 45^\circ = 30^\circ$. Now we can use the Law of Sines to find *a*.

$$\frac{\sin 45^\circ}{15} = \frac{\sin 30^\circ}{a}$$

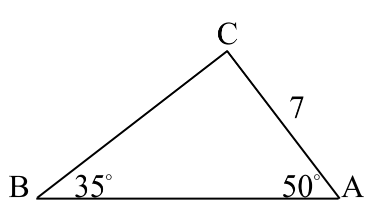
$$a \sin 45^\circ = 15 \sin 30^\circ$$

When we solve for b, we need to put the exact values in from the unit circle since the instructions specifically told us to write the exact value.

$$a = \frac{15 \sin 30^\circ}{\sin 45^\circ} = \frac{15\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)} = 15\left(\frac{1}{2}\right)\left(\frac{2}{\sqrt{2}}\right) = \frac{15}{\sqrt{2}} = \frac{15\sqrt{2}}{2}$$

5B.) (4 pts) In the figure below, find the following:

a: 9.35



$m\angle C$: 95 degrees

Since we are given angles A and B, we can find angle C by subtracting from 180°.
$m\angle C = 180^\circ - 35^\circ - 50^\circ = 95^\circ$. Now we can use the Law of Sines to find *a*.

$$\frac{\sin 35^\circ}{7} = \frac{\sin 50^\circ}{a}$$

$$a \sin 35^\circ = 7 \sin 50^\circ$$

$$a = \frac{7 \sin 50^\circ}{\sin 35^\circ} \approx 9.35$$

6A.) (4 pts) Given the vector $u = \langle -3, 8 \rangle$ and $v = \langle -9, -6 \rangle$, find the dot product $u \cdot v$.

6A. –21

Use the formula $u \cdot v = a_1 a_2 + b_1 b_2 = (-3)(-9) + (8)(-6) = -21$

6B.) (4 pts) Given the vector $u = \langle -7, 3 \rangle$ and $v = \langle 4, 8 \rangle$, find the dot product $u \cdot v$.

6B. –4

Use the formula $u \cdot v = a_1 a_2 + b_1 b_2 = (-7)(4) + (3)(8) = -4$

7A.) (3 pts) Which equation's graph is an ellipse? (No partial credit on this one.)

An ellipse will have x and y both squared. Their coefficients will be both positive and different.
Choice B is correct here.

(A) $2x^2 + 3x - 2y^2 + 4y = 0$

(B) $3x^2 + 7x + 4y^2 - 5y = 0$

(C) $2y^2 + 3x - 6y = 0$

(D) $9y + 8x - 4 = 0$

7B.) (3 pts) Which equation's graph is a parabola? (No partial credit on this one.)

A parabola will have either an x squared or y squared but not both. Choice C is correct here.

(A) $3x^2 + 8x + 4y^2 + 2y = 0$

(B) $7x^2 - 4x - y^2 + 5y = 0$

(C) $3x^2 + 5x + 7y = 0$

(D) $4y - 9x + 3 = 0$

8A.) (4 pts) Find the exact value: $1 + \tan^2\left(\frac{\pi}{6}\right) - \csc^2\left(\frac{\pi}{4}\right)$

8A. $-\frac{2}{3}$

$$1 + \left(\tan\left(\frac{\pi}{6}\right)\right)^2 - \left(\frac{1}{\sin\left(\frac{\pi}{4}\right)}\right)^2 \quad \Rightarrow \quad 1 + \left(\frac{\sqrt{3}}{3}\right)^2 - \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} \quad \Rightarrow \quad 1 + \frac{3}{9} - \frac{1}{\left(\frac{2}{4}\right)}$$

$$\Rightarrow \quad 1 + \frac{1}{3} - 2 = -\frac{2}{3}$$

8B.) (4 pts) Find the exact value: $3\cot^2\left(\frac{\pi}{3}\right) + 2\cos\left(\frac{\pi}{6}\right)$

8B. $1 + \sqrt{3}$

$$3\left(\frac{1}{\tan\left(\frac{\pi}{3}\right)}\right)^2 + 2\cos\left(\frac{\pi}{6}\right) \quad \Rightarrow \quad 3 \cdot \frac{1}{\left(\sqrt{3}\right)^2} + 2\left(\frac{\sqrt{3}}{2}\right) \quad \Rightarrow \quad \frac{3}{3} + \sqrt{3} \quad \Rightarrow \quad 1 + \sqrt{3}$$

9A.) (5 pts) Find the following EXACT values if you are given $\sec\theta = -\sqrt{26}/5$ and $180^\circ < \theta < 270^\circ$.

$\sin\theta = -\dfrac{\sqrt{26}}{26}$ $\quad\csc\theta = -\sqrt{26}$

$\tan\theta = \dfrac{1}{5}$ $\quad\cot\theta = 5$

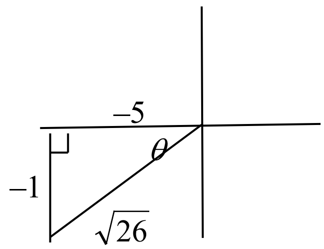
$\cos\theta = -\dfrac{5\sqrt{26}}{26}$

$$a^2 + b^2 = c^2$$

$$(-5)^2 + b^2 = \left(\sqrt{26}\right)^2$$

$$25 + b^2 = 26$$

$$b^2 = 1, \text{ so } \ b = \pm 1$$



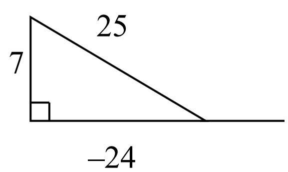
9B.) (5 pts) Find the following given:

$\sin\theta = \dfrac{7}{25}$ and $\tan\theta < 0$.

$\cos\theta:\ -\dfrac{24}{25}$ $\quad\sec\theta:\ -\dfrac{25}{24}$

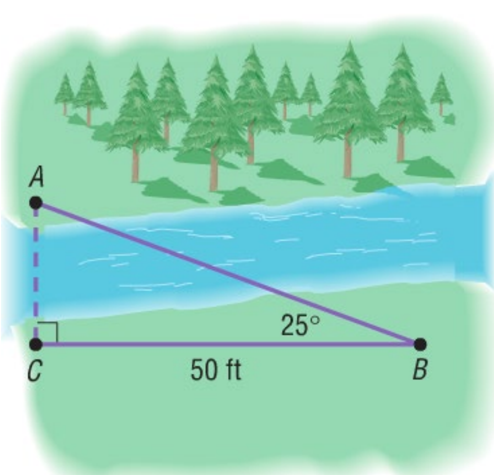
$\tan\theta:\ -\dfrac{7}{24}$ $\quad\cot\theta:\ -\dfrac{24}{7}$

$\csc\theta:\ \dfrac{25}{7}$



10A.) (4 pts) Find the distance from **A to C** in the figure below:

10A. $23.32\ ft$

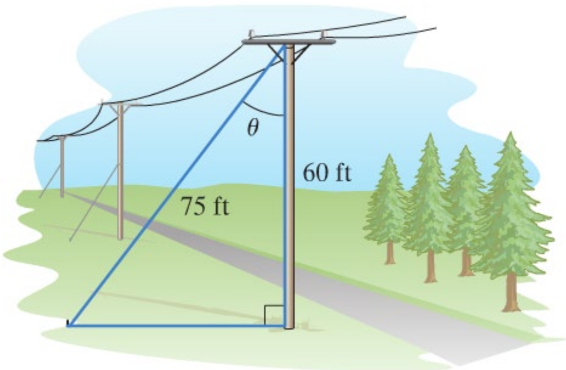


$$\tan 25^\circ = \frac{x}{50}$$

$$x = 50\tan 25^\circ$$

$$x = 23.32\ ft$$

10B.) (4 pts) Find the angle between the wire and the pole to the nearest degree. 10B. 37°



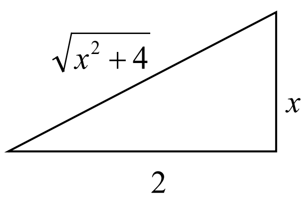
$$\cos\theta = \frac{60}{75}$$

$$\cos\theta = 0.8$$

$$\theta = \cos^{-1}(0.8)$$

$$\theta = 36.87^{\circ}$$

11A.) (5 pts) Use a right triangle to write $\sec\left(\sin^{-1}\left(\frac{x}{\sqrt{x^2+4}}\right)\right)$ as an algebraic expression. Assume that x is positive and that the given inverse trigonometric function is defined for the expression in x. 11A. $\frac{\sqrt{x^2+4}}{2}$



$$a^2 + x^2 = \left(\sqrt{x^2+4}\right)^2$$

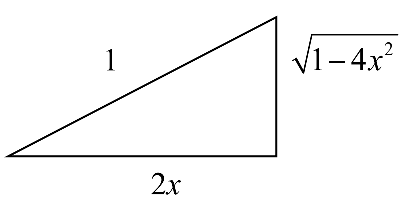
$$a^2 + x^2 = x^2 + 4$$

$$a^2 = 4$$

$$a = 2$$

$$\sec\left(\sin^{-1}\left(\frac{x}{\sqrt{x^2+4}}\right)\right) = \frac{\sqrt{x^2+4}}{2}$$

11B.) (5 pts) Use a right triangle to write $\sin\left(\cos^{-1}(2x)\right)$ as an algebraic expression. Assume that x is positive and that the given inverse trigonometric function is defined for the expression in x. 11B. $\sqrt{1-4x^2}$



$$(2x)^2 + b^2 = 1^2$$

$$4x^2 + b^2 = 1$$

$$b^2 = 1 - 4x^2$$

$$b = \sqrt{1-4x^2}$$

$$\sin\left(\cos^{-1}(2x)\right) = \frac{\sqrt{1-4x^2}}{1}$$

12A.) (5 pts) Verify the identity: $\sin 2\theta = \dfrac{2\cot\theta}{1+\cot^2\theta}$

$$\sin 2\theta = \frac{\dfrac{2\cos\theta}{\sin\theta}}{1+\dfrac{\cos^2\theta}{\sin^2\theta}}$$

$$\sin 2\theta = \frac{\dfrac{2\cos\theta}{\sin\theta}}{\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}}$$

$$\sin 2\theta = \frac{\dfrac{2\cos\theta}{\sin\theta}}{\dfrac{1}{\sin^2\theta}}$$

$$\sin 2\theta = \frac{2\cos\theta}{\sin\theta}\cdot\frac{\sin^2\theta}{1}$$

$$\sin 2\theta = 2\cos\theta\sin\theta$$

$$\sin 2\theta = \sin 2\theta$$

12B.) (5 pts) Verify the identity: $\cot\theta - \sin 2\theta = \cot\theta\cos 2\theta$

$$\frac{\cos\theta}{\sin\theta} - 2\sin\theta\cos\theta = \cot\theta\cos 2\theta$$

$$\frac{\cos\theta}{\sin\theta} - \frac{2\sin\theta\cos\theta}{1}\cdot\left(\frac{\sin\theta}{\sin\theta}\right) = \cot\theta\cos 2\theta$$

$$\frac{\cos\theta}{\sin\theta} - \frac{2\sin^2\theta\cos\theta}{\sin\theta} = \cot\theta\cos 2\theta$$

$$\frac{\cos\theta - 2\sin^2\theta\cos\theta}{\sin\theta} = \cot\theta\cos 2\theta$$

$$\frac{\cos\theta\left(1-2\sin^2\theta\right)}{\sin\theta} = \cot\theta\cos 2\theta$$

$$\frac{\cos\theta}{\sin\theta}\cdot\frac{1-2\sin^2\theta}{1} = \cot\theta\cos 2\theta$$

Use identities: $\dfrac{\cos\theta}{\sin\theta} = \cot\theta$ and $1-2\sin^2\theta = \cos 2\theta$

$$\cot\theta\cos 2\theta = \cot\theta\cos 2\theta$$

13A.) (5 pts) Use a sum or difference formula to fully simplify. Evaluate when possible: $\cos\left(\theta-\frac{3\pi}{2}\right)$

$$\cos(x-y)=\cos x\cos y+\sin x\sin y$$

$$\cos\left(\theta-\frac{3\pi}{2}\right)=\cos\theta\cos\left(\frac{3\pi}{2}\right)+\sin\theta\sin\left(\frac{3\pi}{2}\right)$$

$$\cos\left(\theta-\frac{3\pi}{2}\right)=\cos\theta\cdot(0)+\sin\theta\cdot(-1)$$

$$\cos\left(\theta-\frac{3\pi}{2}\right)=-\sin\theta$$

13B.) (5 pts) Use a sum or difference formula to fully simplify. Evaluate when possible: $\dfrac{\sin\left(x+\frac{3\pi}{2}\right)}{\cos(\pi-x)}$.

$$\frac{\sin(x+y)}{\cos(x-y)}=\frac{\sin x\cos y+\cos x\sin y}{\cos x\cos y+\sin x\sin y}$$

$$\frac{\sin\left(x+\frac{3\pi}{2}\right)}{\cos(\pi-x)}=\frac{\sin x\cos\left(\frac{3\pi}{2}\right)+\cos x\sin\left(\frac{3\pi}{2}\right)}{\cos\pi\cos x+\sin\pi\sin x}$$

$$\frac{\sin\left(x+\frac{3\pi}{2}\right)}{\cos(\pi-x)}=\frac{\sin x\cdot(0)+\cos x\cdot(-1)}{(-1)\cdot\cos x+(0)\cdot\sin x}$$

$$\frac{\sin\left(x+\frac{3\pi}{2}\right)}{\cos(\pi-x)}=\frac{-\cos x}{-\cos x}=1$$

14A.) (4 pts) Convert the rectangular equation $x^2y=4$ into a **polar** equation that expresses r in terms of θ.

14A. $r=\sqrt[3]{\dfrac{4}{\cos^2\theta\sin\theta}}$

$$(r\cos\theta)^2\cdot r\sin\theta=4$$

$$r^2\cos^2\theta\cdot r\sin\theta=4$$

$$r^3\cos^2\theta\sin\theta=4$$

$$r^3=\frac{4}{\cos^2\theta\sin\theta}\quad\Rightarrow\quad r=\sqrt[3]{\frac{4}{\cos^2\theta\sin\theta}}\quad\text{or}\quad r=\sqrt[3]{4\sec^2\theta\csc\theta}$$

14B.) (4 pts) Convert the rectangular equation $(x-2)^2+y^2=4$ into a **polar** equation that expresses r in terms of θ.

14B. $r=4\cos\theta$

First we want to expand $(x-2)^2+y^2=4 \Rightarrow (x-2)(x-2)+y^2=4 \Rightarrow x^2-4x+4+y^2=4$

$$x^2-4x+4+y^2=4$$

$$x^2-4x+y^2=0$$

$$x^2+y^2=4x$$

$$r^2=4r\cos\theta$$

$$r^2-4r\cos\theta=0$$

$$r(r-4\cos\theta)=0$$

$$\cancel{r=0} \quad \text{or} \quad r-4\cos\theta=0 \Rightarrow r=4\cos\theta$$