NAME:_____   **/ 60 =** ☐ **%**

MATH 127   TEST 1   SAMPLE

NOTE: The actual exam will only have 14 questions. The different parts of each question (part A, B, etc.) are variations. Know how to do all the variations on this exam.

1A.) (4 pts) To measure two distances on the Earth, we must account for the curvature of the Earth. We measure along a circle with a center C at the center of the Earth (see below). The radius of the Earth is 4000 miles. If $\theta = 21°$, find the distance between A and B to the nearest mile. You may write your answer in terms of π. Also find the area of the sector ABC.
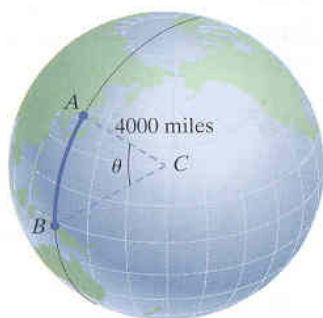
Arc Length:_____

Area:_____

1B.) (4 pts) The minute hand of a clock is 6 inches long. How far does its tip travel in 20 minutes? (Hint: in 20 minutes the hand covers 120 degrees). What is the area of the sector swept swept by the minute hand?   Round all answers to the nearest tenth.

Arc length:_____

Area:_____

2A.)   (4 pts) A wind machine used to generate electricity has blades that are 8 feet in length. The propeller is rotating at 400 revolutions every 2 **minutes**. Find the angular speed, in radians per **second**, as well as the linear speed, in feet per **second,** of the tips of the blades. You may write your answers in terms of π.

angular speed:_____

linear speed:_____

2B.) (4 pts) An object is traveling around a circle with a radius   angular speed:_____
of 5 meters. The object is rotating at 1/3 radians every 20 **seconds**.
Find the angular speed, in radians per **minute**, as well as the linear
speed, in meters per **minute,** of the object.   linear speed:_____

3A. (4 pts) Convert $61°42'21''$ to a decimal in degrees.   3A._____
        Round to two places.

3B. (4 pts) Convert $40.24°$ to $D°M'S''$ form.   3B._____
        Round to the nearest second.

4A.) (4 pts) Find the EXACT value: $\sec^2\left(\dfrac{\pi}{3}\right) - \tan^2\left(\dfrac{\pi}{4}\right)$ 4A._____

4B.) (4 pts) Find the EXACT value: $3\csc\dfrac{\pi}{3}+\cot\dfrac{\pi}{4}$ 4B._____

5A.) (5 points) Find the following EXACT values if you are given $\sec\theta=\dfrac{3}{2}$ and $270°\le\theta\le360°$. Rationalize all roots.

$\sin\theta:$_____ $\csc\theta:$_____
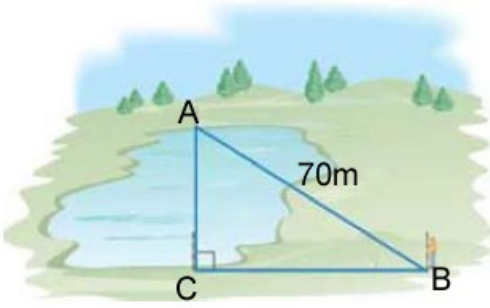
$\tan\theta:$_____ $\cot\theta:$_____

$\cos\theta:$_____

5B.) (5 points) Find the following EXACT values if you are given $\tan\theta=\dfrac{8}{15}$ and $\sin\theta<0$.

$\sin\theta:$_____ $\csc\theta:$_____

$\cos\theta:$_____ $\sec\theta:$_____

$\cot\theta:$_____

6A.) (4 pts) In the picture below, the bearing from B to A is given as $N55.15°W$, and the distance from B to A is 70m. Find the distance across the lake, from A to C to the nearest meter. Find the distance from C to B to the nearest meter.

A to C:_____

70m

C to B:_____

6B.) (4 pts) A semi leaves its present location and travels along a bearing of $S48.89°E$ for 73 miles. How far south and east of its original position is it? Round to the nearest whole number.

South:_____

East:_____

7A.) (4 pts) The Burj Khalifa is currently the tallest freestanding structure. A person is standing 619 feet from the base of the building. The angle of depression from the top of the building to the person is $77.013°$. Approximate the height of the Burj Khalifa to the nearest ft.

7A._____

7B.) (4 pts) Suppose you are headed toward a plateau 50 meters high. If the angle of elevation to the top of the plateau is twenty degrees, how far are you from the base of the plateau? Round to two decimal places.

7B._____

8A.) (5 pts) Find the <u>exact</u> value of $\cos\left(\dfrac{11\pi}{6}\right)$ using reference

    angles. Indicate the ref. angle and draw in standard position.
    To get full credit, you must show the three steps as in the notes.
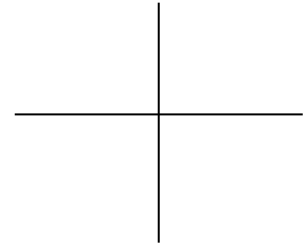
Value:_____

Ref. angle:_____

8B.) (5 pts) Find the <u>exact</u> value of $\sin\left(\dfrac{5\pi}{4}\right)$ using reference

    angles. Indicate the ref. angle and draw in standard position.
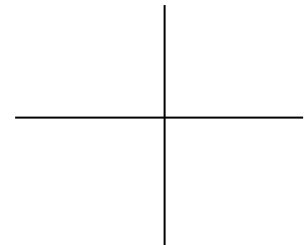   To get full credit, you must show the three steps as in the notes.

Value:_____

Ref. angle:_____

9A.) (4 pts) Find the exact value and reference angle for $\tan\left(-585°\right)$.
    To get full credit, you must show the three steps as in the notes.

Value:_____

Ref. Angle:_____

9B.) (4 pts) Find the exact value and reference angle for $\sec\left(-510°\right)$.
    To get full credit, you must show the three steps as in the notes.

Value:_____

Ref. Angle:_____

10A.) (6 points) Use the following equation to answer the questions: $y = -3\sin\left(\dfrac{\pi}{8}x + \dfrac{\pi}{2}\right)$

    i.) Find the period.   10i._____

    ii.) Find the amplitude.   10ii._____

    iii.) Find the phase shift.   10iii._____

    iv.) Graph the function over one period.

10B.) (6 points) Use the following equation to answer the questions: $y = -\dfrac{1}{2}\cos\left(2x - \dfrac{\pi}{2}\right)$

    i.) Find the period.   10i._____

    ii.) Find the amplitude.   10ii._____

    iii.) Find the phase shift.   10iii._____

    iv.) Graph the function over one period.

11A.) (4 pts) Find the following and graph over two full periods: $y = -\tan\left(\dfrac{\pi}{4}x\right)$

Period:_____

Phase shift:_____

11B.) (4 pts) Find the following and graph over two full periods: $y = \cot\left(3x + \dfrac{\pi}{4}\right)$

Period:_____

Phase shift:_____

12A.) (4 pts) Find the exact value if possible: $\cos^{-1}\left(\cos\left(\sqrt{8}\right)\right)$.   12A._____

(Assume $\sqrt{8}$ is in radians.)

12B.) (4 pts) Find the exact value if possible: $\sin^{-1}\left(\sin\left(\dfrac{3\pi}{4}\right)\right)$.   12B._____

13A.) (4 pts) Find the exact value of $\sec\left(\tan^{-1}\left(-\dfrac{3}{\sqrt{7}}\right)\right)$.

13A._____

13B.) (4 pts) Find the exact value of $\cot\left(\cos^{-1}\left(-\dfrac{\sqrt{3}}{3}\right)\right)$.

13B._____

14A.) (4 pts) Use a right triangle to write $\sin\left(\cos^{-1}(u)\right)$ as an algebraic expression. Assume that u is positive and that the given inverse trigonometric function is defined for the expression in u.

14A._____

14B.) (4 pts) Use a right triangle to write $\tan\left(\sin^{-1}(4u)\right)$ as an algebraic expression. Assume that u is positive and that the given inverse trigonometric function is defined for the expression in u.

14B._____

FORMULA SHEET

$$s = r\theta \qquad A = \frac{1}{2}r^2\theta \qquad \omega = \frac{\theta}{t} \qquad v = r\omega$$

$\cos^{-1}(\cos x) = x \quad if \quad 0 \le x \le \pi$

$\cos(\cos^{-1} x) = x \quad if \quad -1 \le x \le 1$

$\sin^{-1}(\sin x) = x \quad if \quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$\sin(\sin^{-1} x) = x \quad if \quad -1 \le x \le 1$

$\tan^{-1}(\tan x) = x \quad if \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$

$\tan(\tan^{-1} x) = x \quad if \quad -\infty < x < \infty$

Complementary Angle Theorem
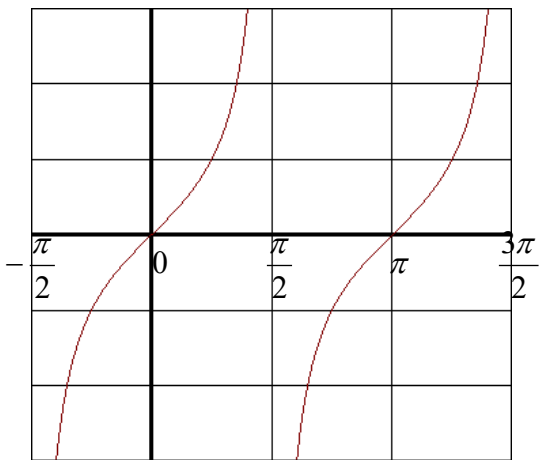
$\sin\theta = \cos(90 - \theta) \qquad \csc\theta = \sec(90 - \theta)$

$\cos\theta = \sin(90 - \theta) \qquad \sec\theta = \csc(90 - \theta)$
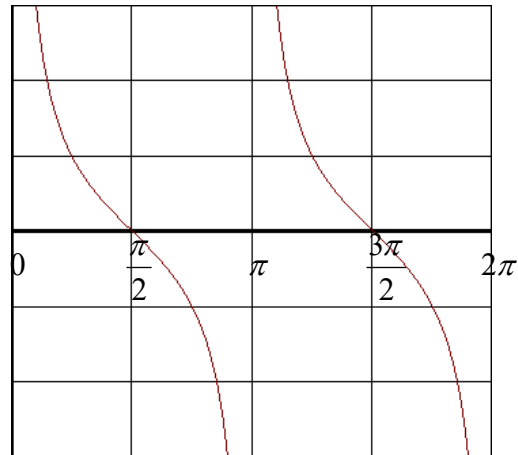
$\tan\theta = \cot(90 - \theta) \qquad \cot\theta = \tan(90 - \theta)$

SOHCAHTOA

Graph of $y = \tan x$

Graph of $y = \cot x$

Sine/Cosine Graph formulas

$$\text{Period} = \frac{2\pi}{B} \qquad \text{Amplitude} = |A| \qquad \text{Phase Shift} = \frac{C}{B} \qquad \text{Q.P.} = \frac{period}{4}$$

Tangent/Cotangent Graph Formulas

$$\text{Period} = \frac{\pi}{B} \qquad\qquad \text{Phase Shift} = \frac{C}{B}$$

$$\text{H.P.} = \frac{period}{2}$$