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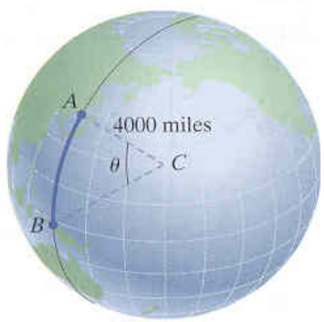
MATH 127 TEST 1 SAMPLE

NOTE: The actual exam will only have 14 questions. The different parts of each question (part A, B, etc.) are variations. Know how to do all the variations on this exam.

1A.) (4 pts) To measure two distances on the Earth, we must account for the curvature of the Earth. We measure along a circle with a center C at the center of the Earth (see below). The radius of the Earth is 4000 miles. If $\theta = 21^\circ$, find the distance between A and B to the nearest mile. You may write your answer in terms of π. Also find the area of the sector ABC.



$$\theta = 21^\circ \cdot \frac{\pi}{180^\circ} = \frac{7\pi}{60}$$
$$S = r\theta$$
$$S = 4000 \cdot \frac{7\pi}{60} = \frac{1400\pi}{3}$$
or $S = 1466.08$ miles

Arc Length: $\frac{1400\pi}{3}$ or 1466.08 miles

Area: $\frac{2800000\pi}{3}$ or 2932153.14 sq miles

$$A = \frac{1}{2}r^2\theta$$
$$A = \frac{1}{2}4000^2 \cdot \frac{7\pi}{60} = \frac{2800000\pi}{3}$$ or 2932153.14 sq. miles

1B.) (4 pts) The minute hand of a clock is 6 inches long. How far does its tip travel in 20 minutes? (Hint: in 20 minutes the hand covers 120 degrees). What is the area of the sector swept swept by the minute hand? Round all answers to the nearest tenth.

Arc length: $12.6\ in$

Area: $37.7\ in^2$

$$\theta = 120^\circ \cdot \frac{\pi}{180^\circ} = \frac{2\pi}{3}$$
$$S = r\theta$$
$$S = 6 \cdot \frac{2\pi}{3} = 4\pi = 12.6 \text{ inches}$$

$$A = \frac{1}{2}r^2\theta$$
$$A = \frac{1}{2}6^2 \cdot \frac{2\pi}{3} = 12\pi = 37.7\ in^2$$

2A.) (4 pts) A wind machine used to generate electricity has blades that are 8 feet in length. The propeller is rotating at 400 revolutions every 2 **minutes**. Find the angular speed, in radians per **second**, as well as the linear speed, in feet per **second,** of the tips of the blades. You may write your answers in terms of π.

angular speed: $\frac{20\pi}{3} rad/\sec$

linear speed: $\frac{160\pi}{3}\ feet/\sec$

$$\omega = \frac{400\ rev}{2\ \min} \cdot \frac{2\pi\ rad}{1\ rev} \cdot \frac{1\ \min}{60\ \sec} = \frac{20\pi}{3} rad/\sec$$

$$v = r\omega = 8 \cdot \frac{20\pi}{3} = \frac{160\pi}{3}\ feet/\sec$$

2B.) (4 pts) An object is traveling around a circle with a radius of 5 meters. The object is rotating at 1/3 radians every 20 **seconds**. Find the angular speed, in radians per **minute**, as well as the linear speed, in meters per **minute,** of the object.

angular speed: $1\ rad/\min$

linear speed: $5\ meters/\min$

$$\omega = \frac{1/3\ rad}{20\ \sec} \cdot \frac{60\ \sec}{1\ \min} = 1\ rad/\min$$

$$v = r\omega = 5 \cdot 1 = 5\ meters/\min$$

3A. (4 pts) Convert $61^\circ 42' 21''$ to a decimal in degrees. Round to two places. 3A. 61.71°

$$61^\circ 42' 21'' = 61^\circ + 42' + 21''$$

$$42' \cdot \frac{1^\circ}{60'} = 0.7^\circ$$

$$21'' \cdot \frac{1^\circ}{3600''} = 0.0058\overline{3}^\circ$$

$$61^\circ 42' 21'' = 61^\circ + 0.7^\circ + 0.00583^\circ = 61.71^\circ$$

3B. (4 pts) Convert 40.24° to $D^\circ M' S''$ form. Round to the nearest second. 3B. $40^\circ 14' 24''$

$$40.24^\circ = 40^\circ + 0.24^\circ$$

$$0.24^\circ \cdot \frac{60'}{1^\circ} = 14.4' \qquad 14.4' = 14' + 0.4'$$

$$0.4' \cdot \frac{60''}{1'} = 24''$$

$$40.24^\circ = 40^\circ + 14' + 24'' = 40^\circ 14' 24''$$

4A.) (4 pts) Find the EXACT value: $\sec^2\left(\frac{\pi}{3}\right) - \tan^2\left(\frac{\pi}{4}\right)$ 4A. 3

$$\frac{1}{\cos^2\left(\frac{\pi}{3}\right)} - \tan^2\left(\frac{\pi}{4}\right) = \frac{1}{\left(\frac{1}{2}\right)^2} - 1^2 = 4 - 1 = 3$$

4B.) (4 pts) Find the EXACT value: $3\csc\frac{\pi}{3}+\cot\frac{\pi}{4}$ 4B. $2\sqrt{3}+1$

$$3\cdot\frac{1}{\sin\frac{\pi}{3}}+\frac{1}{\tan\frac{\pi}{4}} = 3\cdot\frac{1}{\frac{\sqrt{3}}{2}}+\frac{1}{1} = \frac{6}{\sqrt{3}}+1 = \frac{6}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}+1 = 2\sqrt{3}+1$$

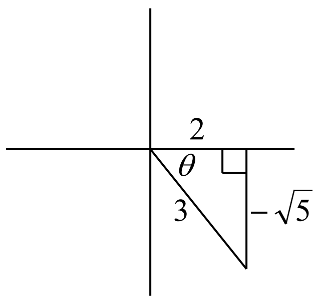
5A.) (5 points) Find the following EXACT values if you are given $\sec\theta=\frac{3}{2}$ and $270^\circ \le \theta \le 360^\circ$. Rationalize all roots.

$\sin\theta=-\frac{\sqrt{5}}{3}$ $\csc\theta=-\frac{3\sqrt{5}}{5}$

$\tan\theta=-\frac{\sqrt{5}}{2}$ $\cot\theta=-\frac{2\sqrt{5}}{5}$

$$a^2+b^2=c^2$$
$$(-2)^2+b^2=3^2$$
$b^2=5$, so $b=\pm\sqrt{5}$

$\cos\theta=\frac{2}{3}$



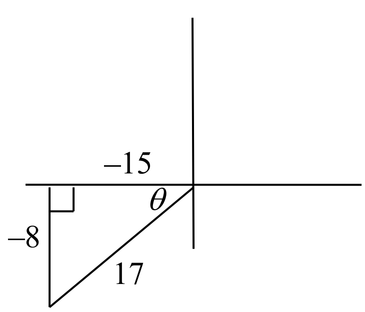
5B.) (5 points) Find the following EXACT values if you are given $\tan\theta=\frac{8}{15}$ and $\sin\theta<0$.

$\sin\theta=-\frac{8}{17}$ $\csc\theta=-\frac{17}{8}$

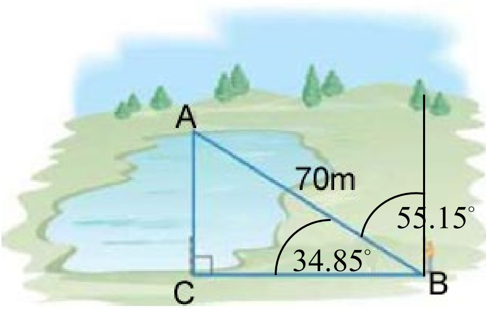
$\cos\theta=-\frac{15}{17}$ $\sec\theta=-\frac{17}{15}$

$$a^2+b^2=c^2$$
$$(-15)^2+(-8)^2=c^2$$
$c^2=289$, so $b=\pm17$

$\cot\theta=\frac{15}{8}$



6A.) (4 pts) In the picture below, the bearing from B to A is given as $N55.15^\circ W$, and the distance from B to A is 70m. Find the distance across the lake, from A to C to the nearest meter. Find the distance from C to B to the nearest meter.



$$\sin 34.85 = \frac{AC}{70} \qquad \cos 34.85 = \frac{CB}{70}$$

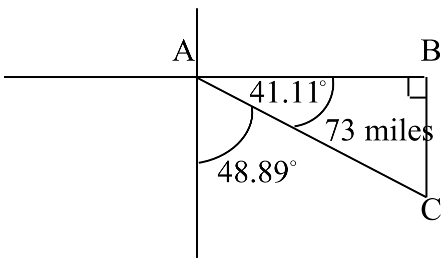
$$AC = 70\sin 34.85 \qquad CB = 70\cos 34.85$$

$$AC \approx 40 \ m \qquad CB \approx 57 \ m$$

A to C: 40 m

C to B: 57 m

6B.) (4 pts) A semi leaves its present location and travels along a bearing of $S48.89^\circ E$ for 73 miles. How far south and east of its original position is it? Round to the nearest whole number.



$$\sin 41.11 = \frac{BC}{73} \qquad \cos 41.11 = \frac{AB}{73}$$

$$BC = 73\sin 41.11 \qquad AB = 73\cos 41.11$$

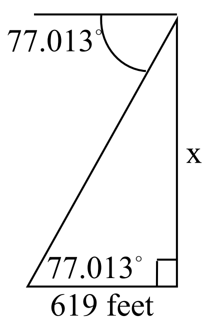
$$BC \approx 48 \ miles \qquad AB \approx 55 \ miles$$

South: 48 miles

East: 55 miles

7A.) (4 pts) The Burj Khalifa is currently the tallest freestanding structure. A person is standing 619 feet from the base of the building. The angle of depression from the top of the building to the person is 77.013°. Approximate the height of the Burj Khalifa to the nearest foot.

7A. 2684 feet



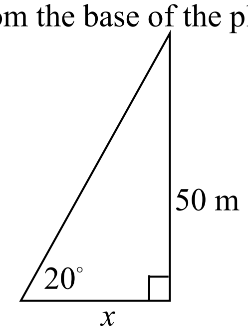
$$\tan 77.013^\circ = \frac{x}{619}$$

$$x = 619\tan 77.013^\circ$$

$$x \approx 2684 \text{ feet}$$

7B.) (4 pts) Suppose you are headed toward a plateau 50 meters high. If the angle of elevation to the top of the plateau is twenty degrees, how far are you from the base of the plateau? Round to two decimal places.

7B. 137.37 meters



$$\tan 20^\circ = \frac{50}{x}$$

$$x\tan 20^\circ = 50$$

$$x = \frac{50}{\tan 20^\circ} \approx 137.37 \text{ meters}$$

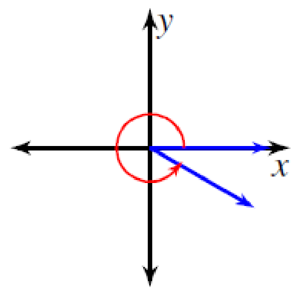
8A.) (5 pts) Find the <u>exact</u> value of $\cos\left(\frac{11\pi}{6}\right)$ using reference angles. Indicate the ref. angle and draw in standard position. To get full credit, you must show the three steps as in the notes.

Value: $\frac{\sqrt{3}}{2}$

1.) R.A. = $2\pi - \frac{11\pi}{6} = \frac{\pi}{6}$

2.) $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

3.) $\cos\left(\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}$



Ref. angle: $\frac{\pi}{6}$

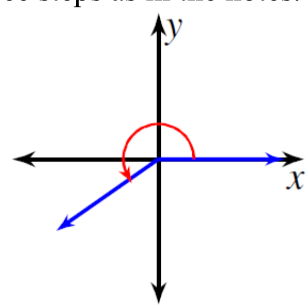
8B.) (5 pts) Find the <u>exact</u> value of $\sin\left(\frac{5\pi}{4}\right)$ using reference angles. Indicate the ref. angle and draw in standard position. To get full credit, you must show the three steps as in the notes.

Value: $-\frac{\sqrt{2}}{2}$

1.) R.A. = $\frac{5\pi}{4} - \pi = \frac{\pi}{4}$

2.) $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

3.) $\sin\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$



Ref. angle: $\frac{\pi}{4}$

9A.) (4 pts) Find the exact value and reference angle for $\tan(-585^\circ)$. To get full credit, you must show the three steps as in the notes.

Value: -1

$-585^\circ + 360^\circ + 360^\circ = 135^\circ$, so $\tan(-585^\circ) = \tan(135^\circ)$

1.) R.A. = $180^\circ - 135^\circ = 45^\circ$

Ref. Angle: 45°

2.) $\tan(45^\circ) = 1$

3.) $\tan(-585^\circ) = -1$

9B.) (4 pts) Find the exact value and reference angle for $\sec(-510^\circ)$. To get full credit, you must show the three steps as in the notes.

Value: $-2\sqrt{3}/3$

$-510^\circ + 360^\circ + 360^\circ = 210^\circ$, so $\sec(-510^\circ) = \sec(210^\circ)$

1.) R.A. = $210^\circ - 180^\circ = 30^\circ$

Ref. Angle: 30°

2.) $\sec(30^\circ) = 1/(\cos 30^\circ) = 1/(\sqrt{3}/2) = 2/\sqrt{3} = 2\sqrt{3}/3$

3.) $\sec(-210^\circ) = -2\sqrt{3}/3$

10A.) (6 points) Use the following equation to answer the questions: $y = -3\sin\left(\frac{\pi}{8}x + \frac{\pi}{2}\right)$

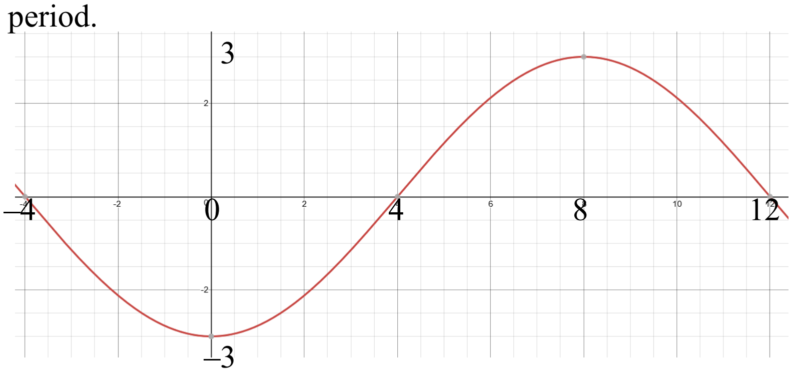
i.) Find the period. $\quad$ Period $= \frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{8}} = 2\pi \cdot \frac{8}{\pi} = 16$ $\quad$ 10i. $\quad$ 16

ii.) Find the amplitude. $\quad$ Amplitude $= |A| = |-3| = 3$ $\quad$ 10ii. $\quad$ 3

iii.) Find the phase shift. $\quad$ P.S. $= \frac{C}{B} = -\frac{\frac{\pi}{2}}{\frac{\pi}{8}} = -\frac{\pi}{2} \cdot \frac{8}{\pi} = -4$ $\quad$ 10iii. $\quad$ -4

iv.) Graph the function over one period.

Q.P.$= \frac{period}{4} = \frac{16}{4} = 4$



10B.) (6 points) Use the following equation to answer the questions: $y = -\frac{1}{2}\cos\left(2x - \frac{\pi}{2}\right)$

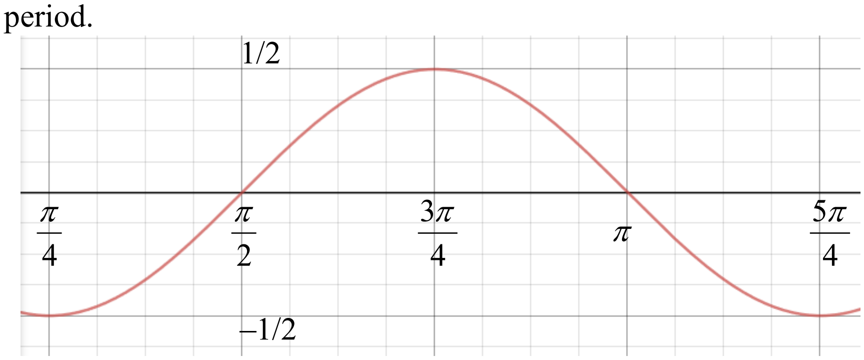
i.) Find the period. $\quad$ Period $= \frac{2\pi}{B} = \frac{2\pi}{2} = \pi$ $\quad$ 10i. $\quad$ π

ii.) Find the amplitude. $\quad$ Amplitude $= |A| = \left|-\frac{1}{2}\right| = \frac{1}{2}$ $\quad$ 10ii. $\quad$ $\frac{1}{2}$

iii.) Find the phase shift $\quad$ P.S. $= \frac{C}{B} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$ $\quad$ 10iii. $\quad$ $\frac{\pi}{4}$

iv.) Graph the function over one period.

Q.P.$= \frac{period}{4} = \frac{\pi}{4}$



11A.) (4 pts) Graph over two full periods: $y = -\tan\left(\frac{\pi}{4}x\right)$

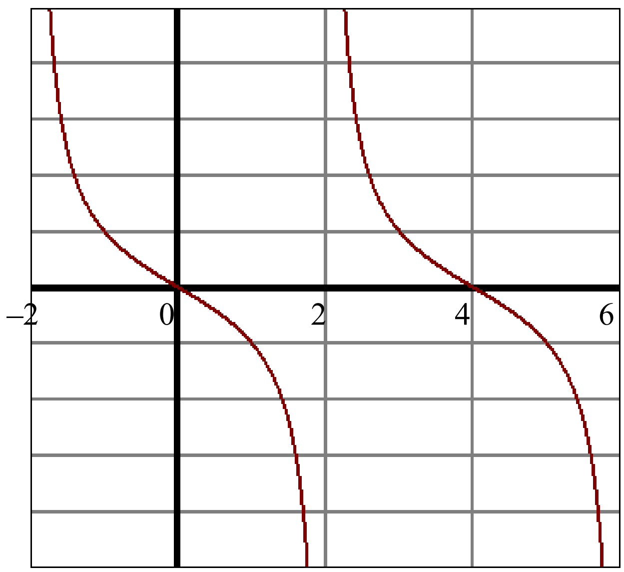
First, we will find the period, phase shift, and half-point using the given formulas.

$$\text{Period} = \frac{\pi}{B} = \frac{\pi}{\frac{\pi}{4}} = \frac{\pi}{1}\cdot\frac{4}{\pi} = 4, \quad \text{Half point} = \frac{\textit{period}}{2} = \frac{4}{2} = 2$$

$$\text{Phase shift} = \frac{C}{B} = \frac{0}{1} = 0$$

These are all the key points, including the phase shift which is the second key point. The graph follows.

$0-2=-2$	P.S. 0	$0+2=2$	$2+2=4$	$4+2=6$



11B.) (4 pts) Graph over two full periods: $y = \cot\left(3x + \frac{\pi}{4}\right)$

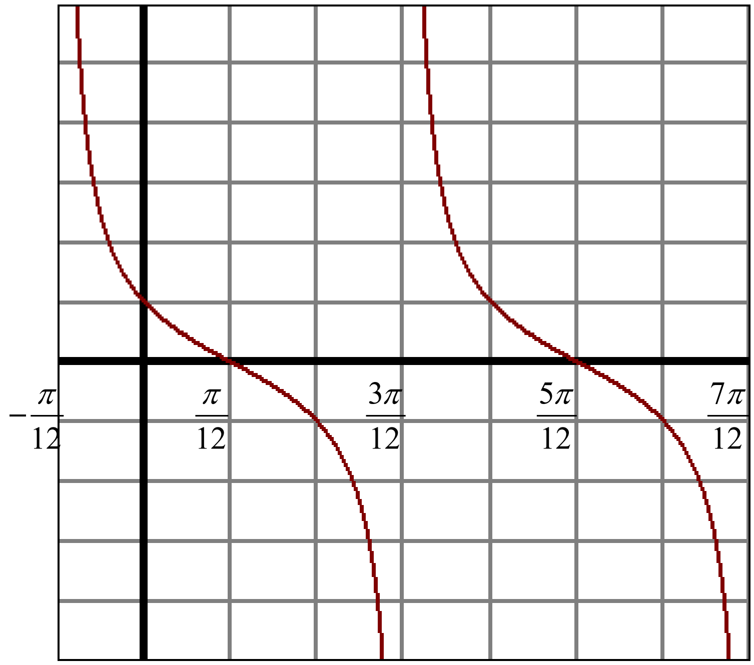
First, we will find the period, phase shift, and half-point using the given formulas.

$$\text{Period} = \frac{\pi}{B} = \frac{\pi}{3}, \quad \text{Half point} = \frac{\textit{period}}{2} = \frac{\frac{\pi}{3}}{2} = \frac{\pi}{6}$$

$$\text{Phase shift} = \frac{C}{B} = -\frac{\frac{\pi}{4}}{3} = -\frac{\pi}{12}$$ (We will use $-\frac{2\pi}{12}$ as the half-point for same denominators)

These are all the key points, including the phase shift which is the first key point. The graph follows.

P.S. $-\frac{\pi}{12}$	$-\frac{\pi}{12}+\frac{2\pi}{12}=\frac{\pi}{12}$	$\frac{\pi}{12}+\frac{2\pi}{12}=\frac{3\pi}{12}$	$\frac{3\pi}{12}+\frac{2\pi}{12}=\frac{5\pi}{12}$	$\frac{5\pi}{12}+\frac{2\pi}{12}=\frac{7\pi}{12}$



12A.) (4 pts) Find the exact value if possible: $\cos^{-1}\left(\cos\left(\sqrt{8}\right)\right)$. 12A. $\sqrt{8}$

(Assume $\sqrt{8}$ is in radians.)

Since $\sqrt{8}$ is in the interval $[0, \pi]$, the inverse cosine and cosine cancel.

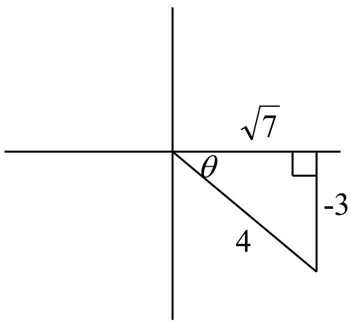
$$\cos^{-1}\left(\cos\left(\sqrt{8}\right)\right) = \sqrt{8}$$

12B.) (4 pts) Find the exact value if possible: $\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right)$. 12B. $\frac{\pi}{4}$

Since $\frac{3\pi}{4}$ is not in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the inverse sine and sine do not cancel. You need to find the reference angle. So R.A. = $\pi - \frac{3\pi}{4} = \frac{\pi}{4}$. Because $\frac{3\pi}{4}$ is in the second quadrant, $\sin\left(\frac{3\pi}{4}\right)$ would be positive, which is also the case for $\sin\left(\frac{\pi}{4}\right)$. This means that $\sin\left(\frac{3\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right)$, so our problem now becomes $\sin^{-1}\left(\sin\left(\frac{\pi}{4}\right)\right)$. Since $\frac{\pi}{4}$ is in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, we can say that $\sin^{-1}\left(\sin\left(\frac{\pi}{4}\right)\right) = \frac{\pi}{4}$.

13A.) (4 pts) Find the exact value of $\sec\left(\tan^{-1}\left(-\frac{3}{\sqrt{7}}\right)\right)$. 13A. $\frac{4\sqrt{7}}{7}$

The inverse trig function will tell you where to draw the triangle, and in this case we have an inverse tangent. The inverse tangent's range will tell us where we can draw the triangle. From the last section, the range for the inverse tangent is $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$. This corresponds to the first and fourth quadrant. Since the fraction inside the inverse is negative, the only quadrant the triangle can be drawn in is the fourth quadrant. We know that the opposite side is -3 and the adjacent is $\sqrt{7}$. The Pythagorean Theorem will give us the adjacent side: 4.

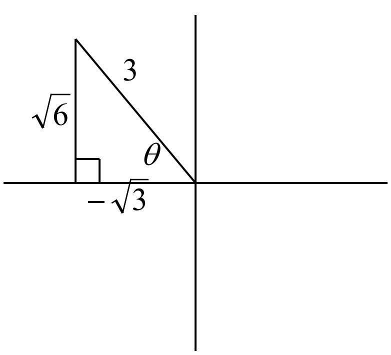


The cosecant on the outside of our problem tells us how to write our answer.

$$\sec\left(\tan^{-1}\left(-\frac{3}{\sqrt{7}}\right)\right) = \frac{4}{\sqrt{7}} = \frac{4\sqrt{7}}{7}.$$

13B.) (4 pts) Find the exact value of $\cot\left(\cos^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right)$. 13B. $-\frac{\sqrt{2}}{2}$

The inverse trig function will tell you where to draw the triangle, and in our case there is an inverse cosine. The inverse cosine's range will tell us where we can draw the triangle. From the last section, the range for the inverse cosine is $0 \le y \le \pi$. This corresponds to the first and second quadrant. Since the fraction inside the inverse is negative, the only quadrant the triangle can be drawn in is the second quadrant. We know that the adjacent side is $-\sqrt{3}$ and the hypotenuse is 3. The Pythagorean Theorem will give us the opposite side: $\sqrt{6}$.



The tangent on the outside of our problem tells us how to write our answer.

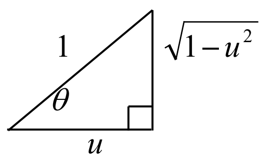
$$\cot\left(\cos^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right) = -\frac{\sqrt{3}}{\sqrt{6}} = -\sqrt{\frac{1}{2}} = -\frac{\sqrt{2}}{2}$$

14A.) (4 pts) Use a right triangle to write $\sin\left(\cos^{-1}(u)\right)$ as an algebraic expression. Assume that u is positive and that the given inverse trigonometric function is defined for the expression in u.

14A. $\sqrt{1-u^2}$

We can rewrite our problem as: $\sin\left(\cos^{-1}\left(\frac{u}{1}\right)\right)$ We know that the adjacent side is u and the hypotenuse is 1.

We can use the Pythagorean theorem to find the hypotenuse: $1^2 = u^2 + b^2$. So we have $b = \sqrt{1-u^2}$



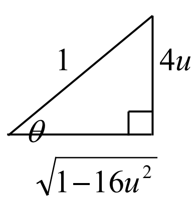
The sine on the outside of our problem tells us how to write our answer. Therefore, $\sin\left(\cos^{-1}(u)\right) = \sqrt{1-u^2}$

14B.) (4 pts) Use a right triangle to write $\tan\left(\sin^{-1}(4u)\right)$ as an algebraic expression. Assume that u is positive and that the given inverse trigonometric function is defined for the expression in u.

14B. $\frac{4u}{\sqrt{1-16u^2}}$

We can rewrite our problem as: $\tan\left(\sin^{-1}\left(\frac{4u}{1}\right)\right)$ We know that the adjacent side is $4u$ and the hypotenuse is 1.

We can use the Pythagorean theorem to find the hypotenuse: $1^2 = (4u)^2 + b^2$. So we have $b = \sqrt{1-16u^2}$



So, $\tan\left(\sin^{-1}(4u)\right) = \frac{4u}{\sqrt{1-16u^2}}$