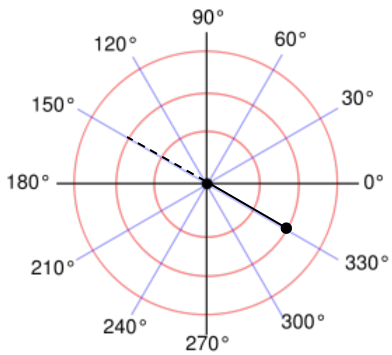
NAME: <u>KEY</u>

MATH 127 TEST 3 SAMPLE

PLEASE SHOW ALL WORK!

1A.) (4 pts) Plot $\left(-2,\ -210^\circ\right)$ on the polar grid provided. Then find an equivalent point such that:



a.) $-360^\circ \le \theta < 0,\ r > 0$ $\qquad \left(2,\ -30^\circ\right)$

$$\left(2,\ -210^\circ + 180^\circ\right) = \left(2,\ -30^\circ\right)$$

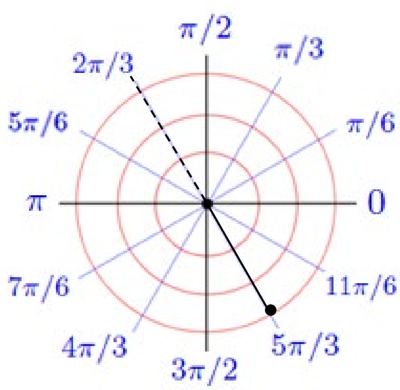
b.) $0 \le \theta < 360^\circ,\ r < 0$ $\qquad \left(-2,\ 150^\circ\right)$

$$\left(-2,\ -210^\circ + 360^\circ\right) = \left(-2,\ 150^\circ\right)$$

c.) $360 \le \theta < 720^\circ,\ r > 0$ $\qquad \left(2,\ 690^\circ\right)$

$$\left(2,\ -210^\circ + 180^\circ\right) = \left(2,\ -30^\circ\right)$$
$$\left(2,\ -30^\circ + 360^\circ\right) = \left(2,\ 330^\circ\right)$$
$$\left(2,\ 330^\circ + 360^\circ\right) = \left(2,\ 690^\circ\right)$$

1B.) (4 pts) Plot $\left(-3,\ \frac{2\pi}{3}\right)$ on the polar grid provided. Then find an equivalent point such that:



a.) $-2\pi \le \theta < 0,\ r > 0$ $\qquad \left(3,\ -\frac{\pi}{3}\right)$

$$\left(3,\ \frac{2\pi}{3} - \pi\right) = \left(3,\ -\frac{\pi}{3}\right)$$

b.) $0 \le \theta < 2\pi,\ r < 0$ $\qquad \left(-3,\ \frac{2\pi}{3}\right)$

$\left(-3,\ \frac{2\pi}{3}\right)$ Original point meets conditions.

c.) $2\pi \le \theta < 4\pi,\ r > 0$ $\qquad \left(3,\ \frac{11\pi}{3}\right)$

$$\left(3,\ \frac{2\pi}{3} + \pi\right) = \left(3,\ \frac{5\pi}{3}\right) \Rightarrow \left(3,\ \frac{5\pi}{3} + 2\pi\right) = \left(3,\ \frac{11\pi}{3}\right)$$

2A.) (4 pts) Convert $\left(-3,\ 3\sqrt{3}\right)$ into polar coordinates with r > 0 and $0 \le \theta \le 2\pi$.

2A. $\left(6,\ \frac{2\pi}{3}\right)$

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{(-3)^2 + \left(3\sqrt{3}\right)^2}$

$r = \sqrt{9+27}$

$r = \sqrt{36} = 6$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) + \pi$

$\theta = \tan^{-1}\left(\frac{3\sqrt{3}}{-3}\right) + \pi$

$\theta = -\frac{\pi}{3} + \pi = \frac{2\pi}{3}$

2B.) (4 pts) Convert $\left(-4,\ \frac{5\pi}{6}\right)$ into rectangular coordinates.

2B. $\left(2\sqrt{3},\ -2\right)$

$x = r\cos\theta$

$x = -4\cos\frac{5\pi}{6}$

$x = -4\left(-\frac{\sqrt{3}}{2}\right)$

$x = 2\sqrt{3}$

$y = r\sin\theta$

$y = -4\sin\frac{5\pi}{6}$

$y = -4\left(\frac{1}{2}\right)$

$y = -2$

3A.) (4 pts) Convert $4\cos^2\theta + 9\sin^2\theta = \frac{36}{r^2}$ into a **rectangular** equation. Show all work for full credit.

$r^2\left(4\cos^2\theta + 9\sin^2\theta\right) = 36$

$4r^2\cos^2\theta + 9r^2\sin^2\theta = 36$

$4\left(r\cos\theta\right)^2 + 9\left(r\sin\theta\right)^2 = 36$

$4x^2 + 9y^2 = 36$

a.) $4x + 9y = 36$

b.) $4x^2 + 9y^2 = 36$

c.) $4r^2\cos^2\theta + 9r^2\sin^2\theta = 36$

d.) $4y^2 + 9x^2 = 36$

e.) None of the above

In our answer, $4x^2 + 9y^2 = 36$, we see that both the x and y are squared and their coefficients are both positive and different. This means that the graph will be an ellipse.

3B.) (4 pts) Convert $r\sin\theta - 4 = r^2\cos^2\theta - 4r\cos\theta$ into a **rectangular** equation. Show all work for full credit.

$y - 4 = x^2 - 4x$

$y = x^2 - 4x + 4$

$y = (x-2)^2$

a.) $x = (y-2)^2$

b.) $x^2 - 4x - 4 = y$

c.) $y = (x-2)^2$

d.) $r\sin\theta = r^2\cos^2\theta - 4r\cos\theta + 4$

e.) None of the above

This graph is a parabola since only one variable is squared.

4A.) (4 pts) Convert $x^2 = 6y$ into a **polar** equation. Solve for *r*.

4A. $r = \dfrac{6\sin\theta}{\cos^2\theta}$

$(r\cos\theta)^2 = 6r\sin\theta$

$r^2\cos^2\theta - 6r\sin\theta = 0$

$r(r\cos^2\theta - 6\sin\theta) = 0$

$\cancel{r=0}$ or $r\cos^2\theta - 6\sin\theta = 0$

$r\cos^2\theta - 6\sin\theta = 0$

$r\cos^2\theta = 6\sin\theta$

$r = \dfrac{6\sin\theta}{\cos^2\theta}$ or $r = 6\sec\theta\tan\theta$

4B.) (4 pts) Convert $3x + y = 7$ into a **polar** equation. Solve for *r*.

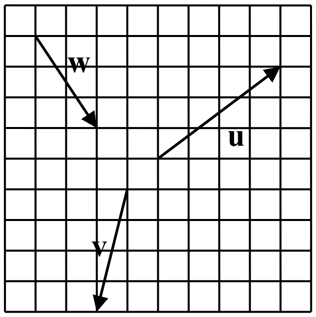
4B. $r = \dfrac{7}{3\cos\theta + \sin\theta}$

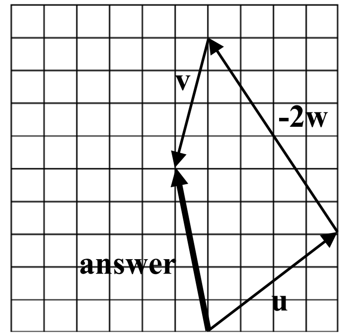
$3r\cos\theta + r\sin\theta = 7$

$r(3\cos\theta + \sin\theta) = 7$

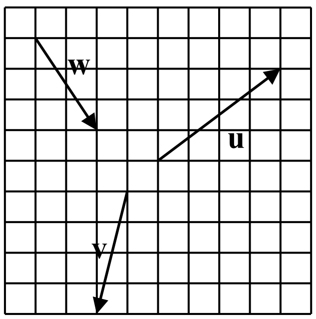
$$r = \frac{7}{3\cos\theta + \sin\theta}$$

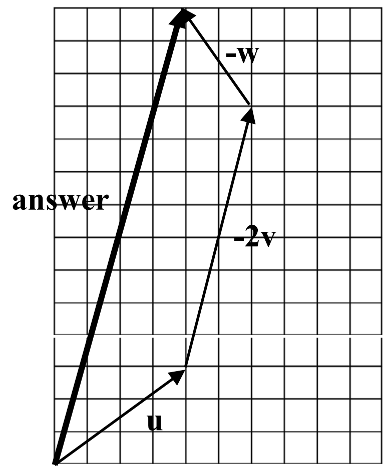
5A.) (4 pts) Use the following vectors to draw **u** – **2w** + **v**.





5B.) (4 pts) Use the following vectors to draw **u** – **w** – **2v**.





6A.) (5 points) Given $v = -2\boldsymbol{i} + 3\boldsymbol{j}$ and $w = 6\boldsymbol{i} + 4\boldsymbol{j}$, find the following:

i. $\|v\| = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$ — 6i. $\sqrt{13}$

ii. $\|w\| = \sqrt{6^2 + 4^2} = \sqrt{52} = 2\sqrt{13}$ — 6ii. $2\sqrt{13}$

iii. $v \cdot w = (-2)(6) + (3)(4) = 0$ — 6iii. 0

iv. $2v - 3w$ — 6iv. $-22\boldsymbol{i} - 6\boldsymbol{j}$

$$\begin{aligned} &2(-2\boldsymbol{i} + 3\boldsymbol{j}) - 3(6\boldsymbol{i} + 4\boldsymbol{j}) \\ &= -4\boldsymbol{i} + 6\boldsymbol{j} - 18\boldsymbol{i} - 12\boldsymbol{j} \\ &= -22\boldsymbol{i} - 6\boldsymbol{j} \end{aligned}$$

v.) Unit vector u in the same direction as v. — 6v. $u = \frac{-2\sqrt{13}}{13}\boldsymbol{i} + \frac{3\sqrt{13}}{13}\boldsymbol{j}$

$$u = \frac{-2i + 3j}{\sqrt{13}} \quad \Rightarrow \quad u = \frac{-2}{\sqrt{13}}\boldsymbol{i} + \frac{3}{\sqrt{13}}\boldsymbol{j} \quad \Rightarrow \quad u = \frac{-2\sqrt{13}}{13}\boldsymbol{i} + \frac{3\sqrt{13}}{13}\boldsymbol{j}$$

6B.) (5 points) Given $v = \frac{1}{2}\boldsymbol{i} - \frac{7}{2}\boldsymbol{j}$ and $w = 12\boldsymbol{i} + 5\boldsymbol{j}$, find the following:

i. $\|v\| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{7}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{49}{4}} = \sqrt{\frac{50}{4}} = \sqrt{\frac{25}{2}} = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$ 6i. $\frac{5\sqrt{2}}{2}$

ii. $\|w\| = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$ 6ii. 13

iii. $v \cdot w = \left(\frac{1}{2}\right)(12) + \left(-\frac{7}{2}\right)(5) = 6 - \frac{35}{2} = -\frac{23}{2}$ 6iii. $-\frac{23}{2}$

iv. $4v - w$ 6iv. $-10\boldsymbol{i} - 19\boldsymbol{j}$

$$4\left((1/2)\boldsymbol{i} - (7/2)\boldsymbol{j}\right) - (12\boldsymbol{i} + 5\boldsymbol{j})$$
$$= 2\boldsymbol{i} - 14\boldsymbol{j} - 12\boldsymbol{i} - 5\boldsymbol{j}$$
$$= -10\boldsymbol{i} - 19\boldsymbol{j}$$

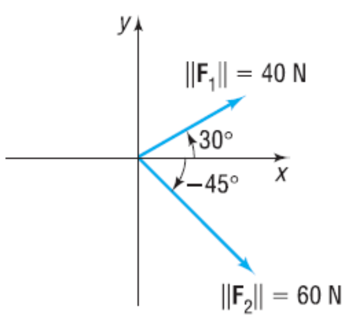
v.) Unit vector u in the same direction as w. 6v. $u = \frac{12}{13}\boldsymbol{i} + \frac{5}{13}\boldsymbol{j}$

$$u = \frac{12i + 5j}{13} \quad \Rightarrow \quad u = \frac{12}{13}\boldsymbol{i} + \frac{5}{13}\boldsymbol{j}$$

7A.) (6 pts) Two forces of magnitude 40 Newtons (N) and 60 Newtons act on an object at angles of 30 degrees and -45 degrees with the positive axis, as shown in the figure. Find the direction and magnitude of the resultant force; that is, find $F_1 + F_2$.

Magnitude: 80.27 N

Direction: -16.23°



$$F_1 = 40\cos 30^\circ\,\hat{i} + 40\sin 30^\circ\,\hat{j}$$
$$F_2 = 60\cos(-45^\circ)\hat{i} + 60\sin(-45^\circ)\hat{j}$$

$$F_1 = 34.64\hat{i} + 20\hat{j}$$
$$+\ F_2 = 42.43\hat{i} - 42.43\hat{j}$$
$$F_1 + F_2 = 77.07\hat{i} - 22.43\hat{j}$$

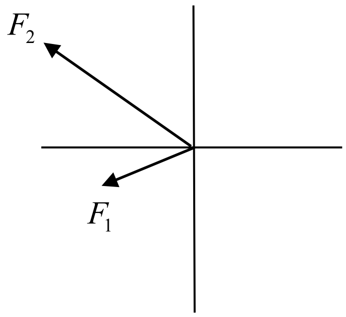
$$\|F_1 + F_2\| = \sqrt{77.07^2 + (-22.43)^2} = 80.27$$

$$\theta = \tan^{-1}\left(\frac{-22.43}{77.07}\right) = -16.23$$

7B.) (6 pts) One force of 4.12 pounds acts on an object at an angle of 194.04 degrees. Another force of 10 pounds acts on the same object at an angle of 143.13 degrees. Find the magnitude and direction of the resultant force.

Magnitude: 13

Direction: 157.4°



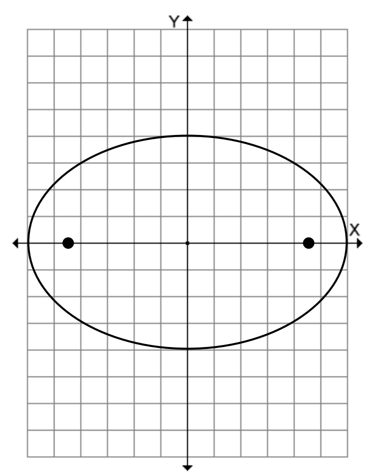
$$F_1 = 4.12\cos 194.04^\circ\,\hat{i} + 4.12\sin 194.04^\circ\,\hat{j}$$
$$F_2 = 10\cos\left(143.13^\circ\right)\hat{i} + 60\sin\left(143.13^\circ\right)\hat{j}$$

$$\begin{aligned} F_1 &= -4\hat{i} - \hat{j} \\ + \quad F_2 &= -8\hat{i} + 6\hat{j} \\ \hline F_1 + F_2 &= -12\hat{i} + 5\hat{j} \end{aligned}$$

$$\left\|F_1 + F_2\right\| = \sqrt{\left(-12\right)^2 + 5^2} = 13$$

$$\theta = \tan^{-1}\left(\frac{5}{-12}\right) + 180^\circ = 157.4^\circ$$

8A.) (8 pts) Use the equation $\dfrac{x^2}{36} + \dfrac{y^2}{16} = 1$ to find the following and graph.



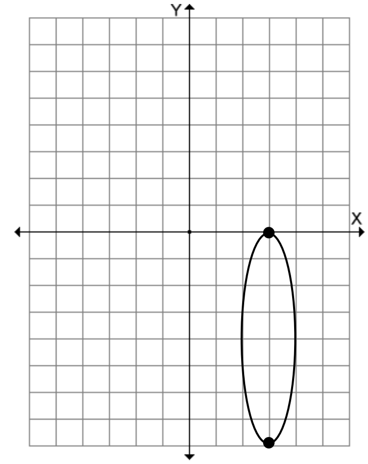
$$a = 6, \;\; b = 4$$

$$c = \sqrt{6^2 - 4^2} = 2\sqrt{5}$$

Foci: $\left(\pm 2\sqrt{5},\ 0\right)$ Vertices: $\left(\pm 6,\ 0\right)$ Eccentricity: $\dfrac{2\sqrt{5}}{6} = \dfrac{\sqrt{5}}{3} \approx 0.75$ Center: (0, 0)

Length of major axis: 2(6) = 12 Length of minor axis: 2(4) = 8

8B.) (8 pts) Use the equation $16x^2 + y^2 - 96x + 8y + 144 = 0$ to find the following and graph.



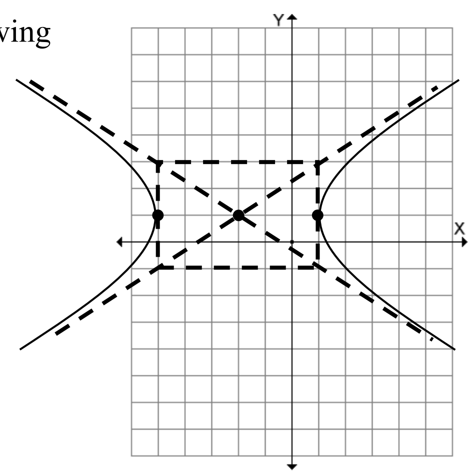
$$16x^2 - 96x + y^2 + 8y = -144$$
$$16(x^2 - 6x + 9) + y^2 + 8y + 16 = -144 + 9(16) + 16$$
$$16(x-3)^2 + (y+4)^2 = 16$$

$$\frac{(x-3)^2}{1} + \frac{(y+4)^2}{16} = 1 \qquad a = 4, \quad b = 1, \quad c = \sqrt{4^2 - 1^2} = \sqrt{15} \approx 3.9$$

Foci: $(3,\ -4 \pm \sqrt{15})$ Vertices: $(3,\ 0),\ (3,\ -8)$ Eccentricity: $\frac{\sqrt{15}}{4} \approx 0.97$ Center: $(3,\ -4)$

Length of major axis: $2(4) = 8$ Length of minor axis: $2(1) = 2$

9A.) (9 pts) Use the equation $4x^2 - 9y^2 + 16x + 18y - 29 = 0$ to find the following and graph.



$$4x^2 + 16x - 9y^2 + 18y = 29$$
$$4(x^2 + 4x + 4) - 9(y^2 - 2y + 1) = 29 + 4(4) + 1(-9)$$
$$4(x+2)^2 - 9(y-1)^2 = 36$$

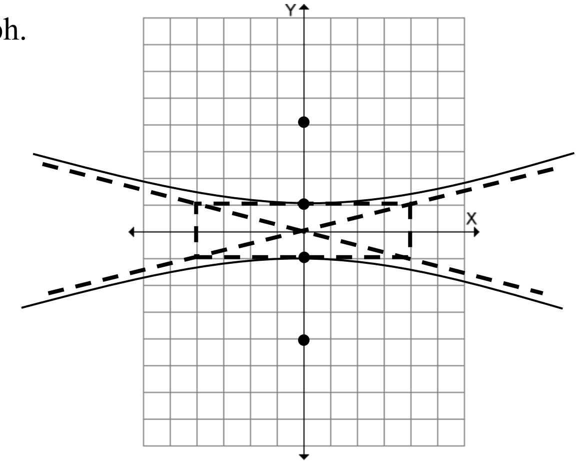
$$\frac{(x+2)^2}{9} - \frac{(y-1)^2}{4} = 1 \qquad a = 3, \ b = 2, \quad c = \sqrt{3^2 + 2^2} = \sqrt{13} \approx 3.6$$

Foci: $(-2 \pm \sqrt{13},\ 1)$ Vertices: $(-5,\ 1),\ (1,\ 1)$ Eccentricity: $\frac{\sqrt{13}}{3} \approx 1.2$

Asymptotes: $(y-1) = \pm\frac{2}{3}(x+2)$ Center: $(-2,\ 1)$

Length of transverse axis: $2(3) = 6$ Length of conjugate axis: $2(2) = 4$

9B.) (9 pts) Use the equation $16y^2 - x^2 = 16$ to find the following and graph.



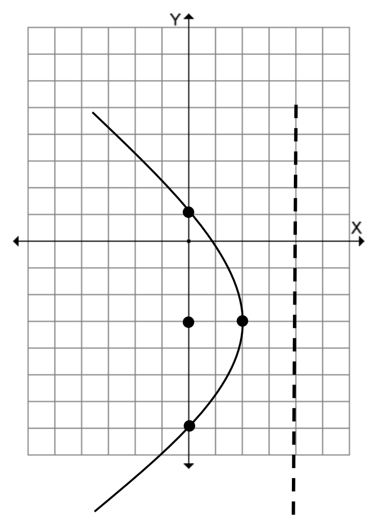
$$\frac{y^2}{1} - \frac{x^2}{16} = 1 \qquad a = 1,\ b = 4, \quad c = \sqrt{1^2 + 4^2} = \sqrt{17} \approx 4.1$$

Foci: $\left(0,\ \pm\sqrt{17}\right)$ Vertices: $\left(0,\ \pm 1\right)$ Eccentricity: $\frac{\sqrt{17}}{1} \approx 4.1$

Asymptotes: $y = \pm\frac{1}{4}x$ Center: (0, 0)

Length of transverse axis: 2(1) = 2 Length of conjugate axis: 2(4) = 8

10A.) (6 pts) Use the equation $y^2 + 8x + 6y = 7$ to find the following and graph.



$$y^2 + 6y = -8x + 7$$

$$y^2 + 6y + 9 = -8x + 7 + 9$$

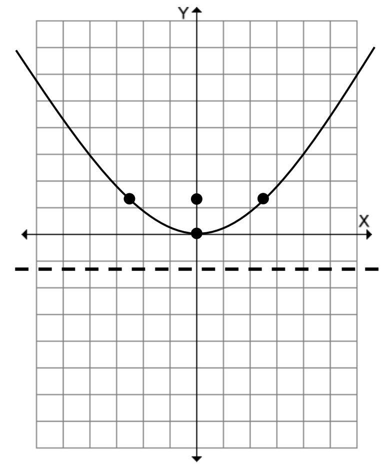
$$(y+3)^2 = -8x + 16$$

$$(y+3)^2 = -8(x-2)$$

$$-4a = -8, \quad a = 2$$

Directrix: $x = 4$ Focus: $(0,\ -3)$ Focal Width: $|-8| = 8$ Vertex: $(2,\ -3)$

10B.) (6 points) Use the equation $x^2 = 5y$ to find the following and graph.



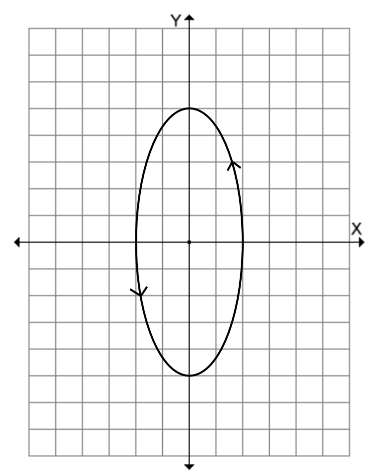
$x^2 = 5y$

$4a = 5$

$a = \frac{5}{4} = 1.25$

Directrix: $y = -\frac{5}{4}$ Focus: $\left(0,\ \frac{5}{4}\right)$ Focal Width: $|5| = 5$ Vertex: (0, 0)

11A.) (6 pts) Fill in the table, graph, and eliminate the parameter:

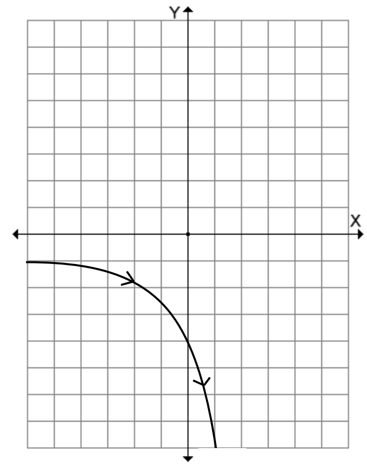


	$x = 2\cos t$	$y = 5\sin t$	(x, y)
0	$2\cos 0 = 2$	$5\sin 0 = 0$	$(2,\ 0)$
$\frac{\pi}{2}$	$2\cos\frac{\pi}{2} = 0$	$5\sin\frac{\pi}{2} = 5$	$(0,\ 5)$
π	$2\cos\pi = -2$	$5\sin\pi = 0$	$(-2,\ 0)$
$\frac{3\pi}{2}$	$2\cos\frac{3\pi}{2} = 0$	$5\sin\frac{3\pi}{2} = -5$	$(0,\ -5)$
2π	$2\cos 2\pi = 2$	$5\sin 2\pi = 0$	$(2,\ 0)$

Eliminate the parameter: $\frac{x^2}{4} + \frac{y^2}{25} = 1$

11B.) (6 pts) Fill in the table, graph, and eliminate the parameter:

	$x = t + 2$	$y = \frac{8}{t}$	(x, y)
-8	$-8+2=-6$	$\frac{8}{-8}=-1$	$(-6, -1)$
-6	$-6+2=-4$	$\frac{8}{-6}=-\frac{4}{3}$	$\left(-4, -\frac{4}{3}\right)$
-4	$-4+2=-2$	$\frac{8}{-4}=-2$	$(-2, -2)$
-2	$-2+2=0$	$\frac{8}{-2}=-4$	$(0, -4)$
-1	$-1+2=1$	$\frac{8}{-1}=-8$	$(1, -8)$



$x = t + 2$

$x - 2 = t$

$y = \frac{8}{x-2}$

Eliminate the parameter: $y = \frac{8}{x-2}$