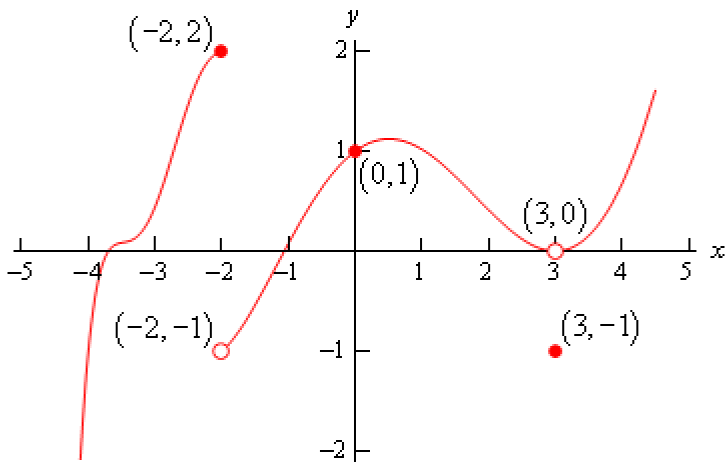
NAME:______________________________

MATH 181   FINAL EXAM SAMPLE

NOTE: The actual exam will only have 14 questions. The different parts of each question (part A, B, etc.) are variations. Know how to do all the variations on this exam.

1A/B.) (6 pts) Find the following by using the graph of $f(x)$ below. If it doesn't exist, write DNE.
NOTE: Problem 1 on the actual test will not have this many parts



a.) Find $f(-2)$ ______________________

b.) Find $f(3)$ ______________________

c.) Find $\lim_{x\to 3^+} f(x)$ ______________________

d.) Find $\lim_{x\to 3^-} f(x)$ ______________________

e.) Find $\lim_{x\to 3} f(x)$ ______________________

f.) Find $\lim_{x\to -2^+} f(x)$ ______________________

g.) Find $\lim_{x\to -2^-} f(x)$ ______________________

h.) Find $\lim_{x\to -2} f(x)$ ______________________

i.) Find $\lim_{x\to -1} f(x)$ ______________________

j.) Which statement is **false**? (Choose all that apply.)

A.) $\lim_{x\to 3} f(x)$ exists

B.) $f(x)$ is differentiable at $x = 3$

C.) $\lim_{x\to -2} f(x) = DNE$

D.) $f(x)$ is continuous at $x = 3$

E.) $\lim_{x\to 0} f(x)$ exists

2A.) (6 pts) Evaluate the limit. If the limit does not exist, indicate DNE.

i.) $\lim_{x\to -1} \dfrac{\sqrt{x+10}-3}{x+1}$

i.______________________

ii.) $\lim_{x\to\infty} \dfrac{7x^2+\sqrt{2}}{14x^3-3x^2+5}$ ii.________________________

2B.) (6 pts) Evaluate the limit. If the limit does not exist, indicate DNE.

i.) $\lim_{x\to 3} \dfrac{x^2-11x+24}{x-3}$ i.________________________

ii.) $\lim_{x\to\infty} \dfrac{\sqrt{4x^2+3x-5}}{7-4x}$ ii.________________________

2C.) (6 pts) Evaluate the limit. If the limit does not exist, indicate DNE.

i.) $\lim_{x\to 4} \dfrac{2x^2-9x+4}{x-2}$ i.________________________

ii.) $\lim_{x\to-\infty} \dfrac{2x^2+3}{3+4x+5x^2}$

ii.________________________

3A. (5 pts) Differentiate: $y = 2\sqrt{3x^7 - \sin x}$

3A.________________________________

3B. (5 pts) Differentiate: $y = \cos\left(\tan x - 2x^8\right)$

3B.________________________________

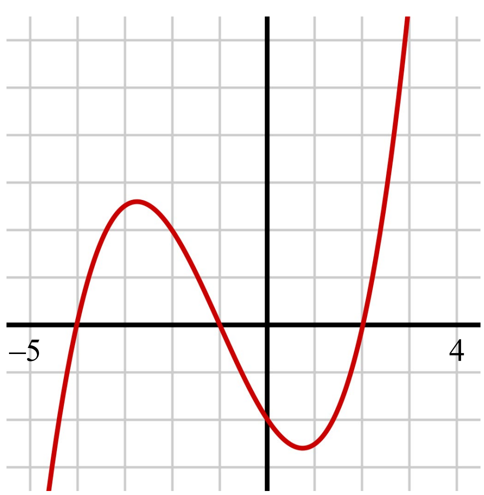
4A.) (4 pts) The function f is defined by $f(x)=\dfrac{-\cos x}{1+\sin x}$ on the interval $[0,2\pi)$. What point(s) (x, y) on the graph of f have the property that the line tangent to f at (x, y) has a slope of $\dfrac{1}{2}$?

4A.______________________

4B.) (4 pts) The function f is defined by $f(x)=\dfrac{2x}{x^2+9}$. What point(s) (x, y) on the graph of f have the property that the line tangent to f at (x, y) has a slope of 0?

4B.______________________

5A.) (3 pts) The graph of a differentiable function f is shown in the figure below. If $g(x)=\int_{-4}^{x} f(t)\,dt$ Which of the following is true?



(A) $g(-1)<g'(-1)<g''(-1)$

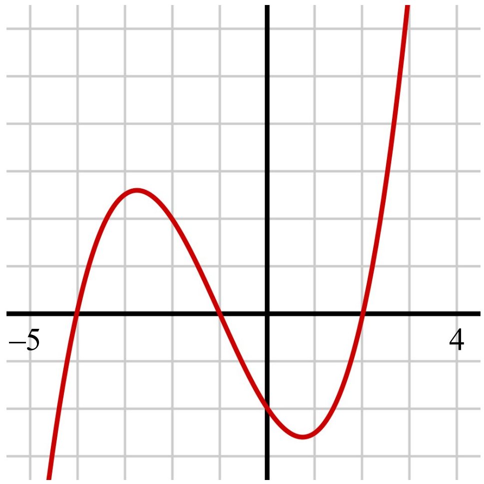
(B) $g''(-1)<g'(-1)<g(-1)$

(C) $g(-1)<g''(-1)<g'(-1)$

(D) $g''(-1)<g(-1)<g'(-1)$

(E) $g'(-1)<g(-1)<g''(-1)$

5B.) (3 pts) The graph of a differentiable function f is shown in the figure below. If $g(x)=\int_{-1}^{x} f(t)\,dt$ Which of the following is true?



(A) $g(2)<g'(2)<g''(2)$

(B) $g''(2)<g'(2)<g(2)$

(C) $g(2)<g''(2)<g'(2)$

(D) $g''(2)<g(2)<g'(2)$

(E) $g'(2)<g(2)<g''(2)$

6A.) (6 pts) Given $f'(x)=8x^3-32x$, find the critical numbers, interval(s) of increasing and decreasing, and where the relative maximum and minimum occur.

Critical #s:_______________

Increasing:_______________

Decreasing:_______________

The relative max occurs at x =_____

The relative min occurs at x =_____

6B.) (6 pts) Given $f'(x)=\dfrac{4(x-2)}{3x^{\frac{2}{3}}}$, find the critical numbers, interval(s) of increasing and decreasing, and where the relative maximum and minimum occur.

Critical #s:_______________

Increasing:_______________

Decreasing:_______________

The relative max occurs at x=______

The relative min occurs at x =_____

7A.) (4 pts) Evaluate the definite integral: $\int_0^1 \frac{2x}{\left(x^2+1\right)^4}\,dx$ 7A.______________________

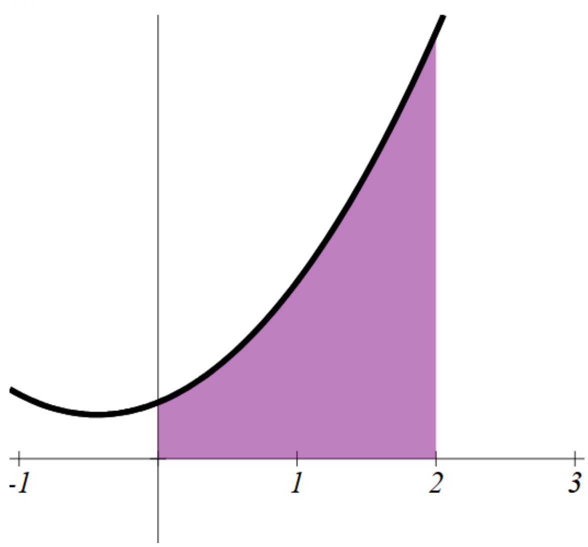
7B.) (4 pts) Evaluate the integral: $\int_0^{2\pi} \frac{\cos\theta}{\sqrt{4+3\sin\theta}}\,d\theta$ 7B.______________________

8A.) (4 pts) Find the solution to the differential equation $\frac{dy}{d\theta} = \sqrt{2}\cos\theta + \sec^2\theta$ with the initial condition $y\left(\frac{\pi}{4}\right) = -1$ 8A.______________________

8B.) (4 pts) Find the solution to the differential equation $\frac{dy}{dt} = 6\sin(3t)$ with the initial condition $y\left(\frac{\pi}{6}\right) = 6$

8B.______________________

9A.) (4 pts) The graph below shows the parabola $f(x) = 8x^2 + 7x + 7$ together with a shaded region.



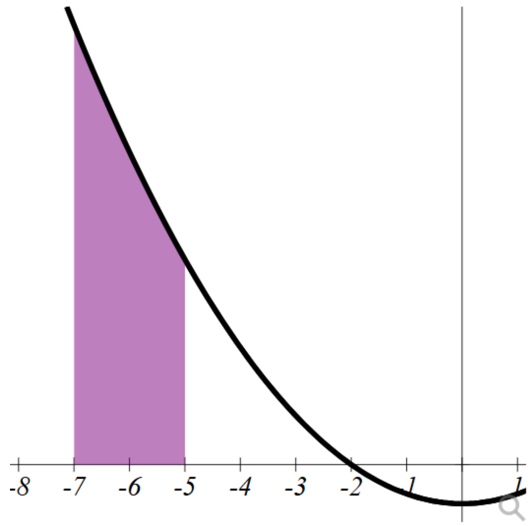
a.) Set up an integral to calculate the area of the shaded region.

____

b.) Evaluate the integral to find the exact area of the shaded region.

$A =$ _______________

9B.) (4 pts) The graph below shows the parabola $f(x) = x^2 - 4$ together with a shaded region.



a.) Set up an integral to calculate the area of the shaded region.

____

b.) Evaluate the integral to find the exact area of the shaded region.

$A =$ _______________

10A.) (4 pts) A particle moves along the x-axis. The velocity of the particle at time t is $-\frac{1}{2}t^2\left(t^2 - 9\right)$. What is the total distance traveled by the particle from time $t = 0$ to $t = 3$?

10A.______________________

10B.) (4 pts) A particle moves along the x-axis. The velocity of the particle at time t is $8\sqrt{t}-t^2$. What is the total distance traveled by the particle from time $t=0$ to $t=4$?

10B.____________________

11A.) (4 pts) Differentiate $y=\sqrt{x^5}-\csc x+\ln(4x)$

11A.__________________________

11B.) (4 pts) Differentiate $y=\dfrac{1}{\sqrt[3]{x^2}}+\cot x+e^{5x}$

11B.__________________________

12A.) (4 pts) Find the exact x-coordinate where there is a horizontal tangent line to the graph of $f(x)=\left(x-3\right)\cdot e^{-2x}$.

12A.____________________________

12B.) (4 pts) Find the exact x-coordinate where there is a horizontal tangent line to the graph of $f(x)=\left(x+1\right)\cdot 3^{x}$.

12A.____________________________

13A.) (4 pts) Use implicit differentiation to find $\dfrac{dy}{dx}$ for the curve

$\cos(xy)=1+\sin y$

13A.______________________

13B.) (4 pts) Use implicit differentiation to find $\frac{dy}{dx}$ for the curve

$$x^2\left(x^2+y^2\right)=y^3$$

13B.____________________

14A.) (4 pts) The amount of a chemical in a solution at time t is modeled by the equation: $C(t)=\frac{4t}{t^2+3}$, $t>0$ where t is measured in minutes. Find the time t when the concentration $C(t)$ is maximized.

14A.____________________

14B.) (4 pts) The cost $C(x)$, in thousands, of producing x units of a product is given by the equation: $C(x)=\frac{5x^2}{x-50}$. Find the number of units x that minimizes the cost.

14B.____________________

FORMULA SHEET

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

$$\frac{d}{dx}[\cos x] = -\sin x \qquad \frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}\left[e^u\right] = e^u \cdot u' \qquad \frac{d}{dx}\left[a^u\right] = (\ln a) \cdot a^u \cdot u'$$

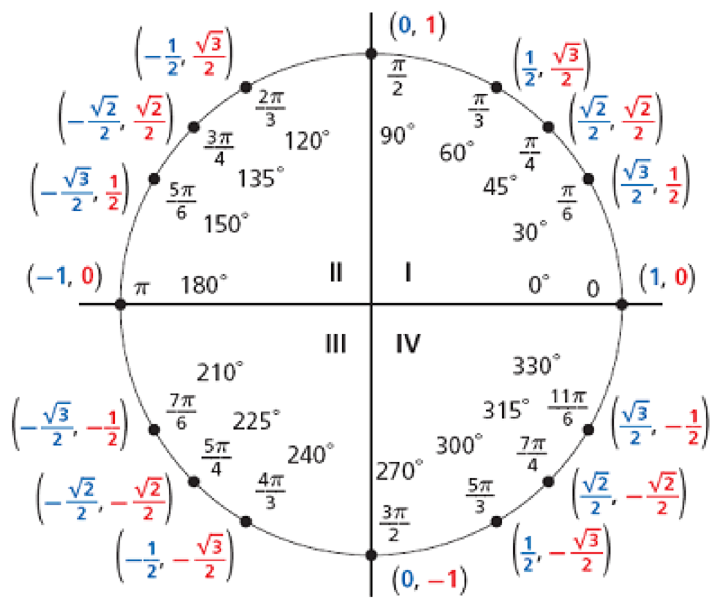
$$\frac{d}{dx}[\ln u] = \frac{u'}{u} \qquad \frac{d}{dx}[\log_a u] = \frac{u'}{u \ln a}$$

$$\frac{d}{dx}[f \cdot g] = f \cdot g' + g \cdot f' \qquad \frac{d}{dx}\left[\frac{f}{g}\right] = \frac{g \cdot f' - f \cdot g'}{g^2}$$

$$\frac{d}{dx}\left[x^n\right] = n \cdot x^{n-1}$$

Chain Rule: $y' = f'(u) \cdot u'$

The Unit Circle



$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \qquad \int \frac{1}{u} \, dx = \ln|u| + C, \quad u \neq 0 \qquad \int e^{kx} dx = \frac{1}{k} e^{kx} + C \qquad \int a^{kx} dx = \frac{a^{kx}}{k \ln a} + C$$

$$\int \sin kx \, dx = -\frac{1}{k} \cos kx + C \qquad \int \cos kx \, du = \frac{1}{k} \sin kx + C \qquad \int \sec^2 kx \, dx = \frac{1}{k} \tan kx + C$$

$$\int \sec kx \cdot \tan kx \, dx = \frac{1}{k} \sec kx + C \qquad \int \csc^2 kx \, dx = -\frac{1}{k} \cot kx + C \qquad \int \csc kx \cdot \cot kx \, dx = -\frac{1}{k} \csc kx + C$$

$$\int \tan u \, du = -\ln|\cos u| + C \qquad \int \cot u \, du = \ln|\sin u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C \qquad \int \csc u \, du = -\ln|\csc u + \cot u| + C$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad \sin^2\theta = 1 - \cos^2\theta \qquad \cos^2\theta = 1 - \sin^2\theta$$