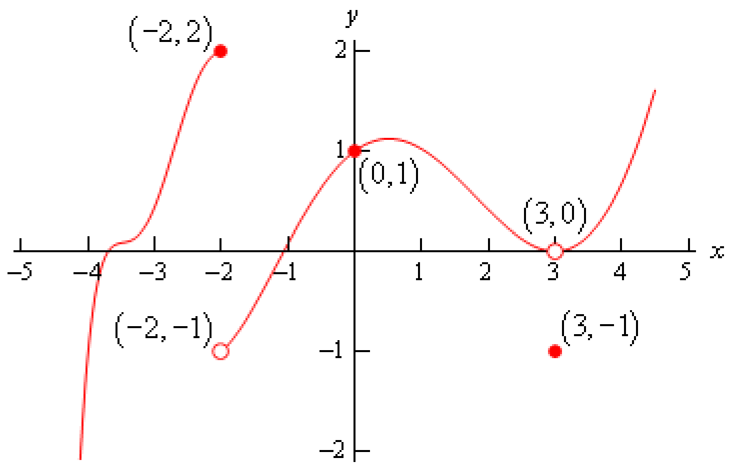
NAME:____________KEY________________

MATH 181 FINAL EXAM SAMPLE

NOTE: The actual exam will only have 12 questions. The different parts of each question (part A, B, etc.) are variations. Know how to do all the variations on this exam.

1A/B.) (6 pts) Find the following by using the graph of *f*(x) below. If it doesn't exist, write DNE.
NOTE: Problem 1 on the actual test will not have this many parts **Section 2.2, 2.4**



a.) Find $f(-2)$ 2

b.) Find $f(3)$ –1

c.) Find $\lim_{x\to 3^+} f(x)$ 0

d.) Find $\lim_{x\to 3^-} f(x)$ 0

e.) Find $\lim_{x\to 3} f(x)$ 0

f.) Find $\lim_{x\to -2^+} f(x)$ –1

g.) Find $\lim_{x\to -2^-} f(x)$ 2

h.) Find $\lim_{x\to -2} f(x)$ DNE

i.) Find $\lim_{x\to -1} f(x)$ 0

j.) At what value(s) is f(x) discontinuous?

–2, 3

2A.) (6 pts) Evaluate the limit. If the limit does not exist, indicate DNE. **Section 2.2, 2.6**

i.) $\lim_{x\to 0} \frac{\sqrt{4+x}-2}{x}$ i. $\frac{1}{4}$

$$\lim_{x\to 0} \frac{\sqrt{4+x}-2}{x}\cdot\frac{\sqrt{4+x}+2}{\sqrt{4+x}+2} = \lim_{x\to 0} \frac{4+x-4}{x\left(\sqrt{4+x}+2\right)} = \lim_{x\to 0} \frac{1}{\left(\sqrt{4+x}+2\right)} = \frac{1}{\left(\sqrt{4+0}+2\right)} = \frac{1}{4}$$

ii.) $\lim\limits_{x\to\infty} \dfrac{7x^2+\sqrt{2}}{14x^3-3x^2+5}$ ii. 0

$$\lim_{x\to\infty} \frac{\frac{7x^2}{x^3}+\frac{\sqrt{2}}{x^3}}{\frac{14x^3}{x^3}-\frac{3x^2}{x^3}+\frac{5}{x^3}} = \lim_{x\to\infty} \frac{\frac{7}{x}+\frac{\sqrt{2}}{x^3}}{14-\frac{3}{x}+\frac{5}{x^3}} = \frac{0+0}{14-0+0}=0$$

2B.) (6 pts) Evaluate the limit. If the limit does not exist, indicate DNE. **Section 2.2, 2.6**

i.) $\lim\limits_{x\to 3} \dfrac{x^2-11x+24}{x-3}$ i. -5

$$\lim_{x\to 3} \frac{(x-3)(x-8)}{x-3} = \lim_{x\to 3} x-8 = 3-8=-5$$

ii.) $\lim\limits_{x\to\infty} \dfrac{\sqrt{4x^2+3x-5}}{7-4x}$ ii. $-\dfrac{1}{2}$

$$\lim_{x\to\infty} \frac{\frac{\sqrt{4x^2+3x-5}}{\sqrt{x^2}}}{\frac{7}{x}-\frac{4x}{x}} = \lim_{x\to\infty} \frac{\sqrt{\frac{4x^2+3x-5}{x^2}}}{\frac{7}{x}-4} = \lim_{x\to\infty} \frac{\sqrt{4+\frac{3}{x}-\frac{5}{x^2}}}{\frac{7}{x}-4} = \frac{\sqrt{4+0-0}}{0-4}=\frac{2}{-4}=-\frac{1}{2}$$

2C.) (10 pts) Evaluate the limit. If the limit does not exist, indicate DNE. **Section 2.2, 2.6**

i.) $\lim\limits_{x\to 4} \dfrac{2x^2-9x+4}{x-2}$ i. 0

$$\lim_{x\to 4} \frac{(x-4)(2x-1)}{x-2} = \frac{(4-4)(2(4)-1)}{4-2}=0$$

ii.) $\lim_{x\to\infty} \dfrac{2x^2+3}{3+4x+5x^2}$ ii. $\dfrac{2}{5}$

$$\lim_{x\to\infty} \frac{\frac{2x^2}{x^2}+\frac{3}{x^2}}{\frac{3}{x^2}+\frac{4x}{x^2}+\frac{5x^2}{x^2}} = \lim_{x\to\infty} \frac{2+\frac{3}{x^2}}{\frac{3}{x^2}+\frac{4}{x}+5} = \frac{2+0}{0+0+5} = \frac{2}{5}$$

3A.) (4 pts) Differentiate $y=\sqrt{x^5}-\csc x+\ln(4x)$ **Section 3.3, 3.5** 3A. $y' = \frac{5}{2}x^{\frac{3}{2}}+\csc x\cot x+\frac{1}{x}$

$$y = x^{\frac{5}{2}}-\csc x+\ln(4x)$$

$$y' = \frac{5}{2}x^{\frac{3}{2}}+\csc x\cot x+\frac{4}{4x}$$

$$y' = \frac{5}{2}x^{\frac{3}{2}}+\csc x\cot x+\frac{1}{x}$$

3B.) (4 pts) Differentiate $y=\dfrac{1}{\sqrt[3]{x^2}}+\cot x+e^{5x}$ **Section 3.3, 3.5** 3B. $y' = -\dfrac{2}{3x^{\frac{5}{3}}}-\csc^2 x+5e^{5x}$

$$y = x^{-\frac{2}{3}}+\cot x+e^{5x}$$

$$y' = -\frac{2}{3}x^{-\frac{5}{3}}-\csc^2 x+e^{5x}(5)$$

$$y' = -\frac{2}{3x^{\frac{5}{3}}}-\csc^2 x+5e^{5x}$$

4A.) (4 pts) Differentiate $f(x) = x^2e^{3x}$ **Section 3.3, 3,5, 3.8** 4A. $f'(x) = xe^{3x}(3x+2)$

$$f'(x) = x^2e^{3x}(3) + e^{3x}(2x)$$

$$f'(x) = 3x^2e^{3x} + 2xe^{3x}$$

$$f'(x) = xe^{3x}(3x+2)$$

4B.) (4 pts) Differentiate $f(x) = x^2 \tan x$ **Section 3.3, 3.5, 3.8** 4B. $f'(x) = x^2 \sec^2 x + 2x \tan x$

$$f'(x) = x^2 \sec^2 x + \tan x(2x)$$

$$f'(x) = x^2 \sec^2 x + 2x \tan x$$

4C.) (4 pts) Differentiate $f(x) = \dfrac{3x-5}{x+7}$ **Section 3.3, 3.5, 3.8** 4C. $f'(x) = \dfrac{26}{(x+7)^2}$

$$f'(x) = \frac{(x+7)(3) - (3x-5)(1)}{(x+7)^2}$$

$$f'(x) = \frac{3x+21-3x+5}{(x+7)^2}$$

$$f'(x) = \frac{26}{(x+7)^2}$$

5A.) (4 pts) Use implicit differentiation to find $\frac{dy}{dx}$ for the curve

$\cos(xy) = 1 + \sin y$

$$-\sin(xy)\left(x\frac{dy}{dx} + y(1)\right) = 0 + \cos y\frac{dy}{dx}$$
$$-x\frac{dy}{dx}\sin(xy) - y\sin(xy) = \cos y\frac{dy}{dx}$$
$$-y\sin(xy) = \cos y\frac{dy}{dx} + x\frac{dy}{dx}\sin(xy)$$
$$-y\sin(xy) = \frac{dy}{dx}(\cos y + x\sin(xy))$$
$$\frac{-y\sin(xy)}{\cos y + x\sin(xy)} = \frac{dy}{dx}$$

5A. $\frac{dy}{dx} = \frac{-y\sin(xy)}{\cos y + x\sin(xy)}$

Section 3.6, 3.7

5B.) (4 pts) Use implicit differentiation to find $\frac{dy}{dx}$ for the curve

$x^2(x^2 + y^2) = y^3$

$$x^4 + x^2y^2 = y^3$$
$$4x^3 + x^2 \cdot 2y\frac{dy}{dx} + y^2(2x) = 3y^2\frac{dy}{dx}$$
$$4x^3 + 2xy^2 = 3y^2\frac{dy}{dx} - 2x^2y\frac{dy}{dx}$$
$$4x^3 + 2xy^2 = \frac{dy}{dx}(3y^2 - 2x^2y)$$
$$\frac{4x^3 + 2xy^2}{3y^2 - 2x^2y} = \frac{dy}{dx}$$

5B. $\frac{dy}{dx} = \frac{4x^3 + 2xy^2}{3y^2 - 2x^2y}$

Section 3.6, 3.7

5C.) (4 pts) Use implicit differentiation to find $\frac{dy}{dx}$ for the curve

$e^y = xy$

$$e^y \cdot \frac{dy}{dx} = x\frac{dy}{dx} + y(1)$$
$$e^y \cdot \frac{dy}{dx} - x\frac{dy}{dx} = y$$
$$\frac{dy}{dx}(e^y - x) = y \quad \Rightarrow \quad \frac{dy}{dx} = \frac{y}{e^y - x} \quad \text{or} \quad \frac{dy}{dx} = \frac{y}{xy - x}$$

5C. $\frac{dy}{dx} = \frac{y}{e^y - x}$

Section 3.6, 3.7

6A. (5 pts) Differentiate: $y = 3(\sin(2x))^4$

6A. $y' = 24\sin^3(2x)\cos(2x)$
Section 3.6

$$y' = 3\cdot 4(\sin(2x))^3 \cdot \frac{d}{dx}[\sin(2x)]$$

$$y' = 3\cdot 4(\sin(2x))^3 \cdot \cos(2x)\cdot \frac{d}{dx}[2x]$$

$$y' = 3\cdot 4(\sin(2x))^3 \cdot \cos(2x)\cdot 2$$

$$y' = 24\sin^3(2x)\cos(2x)$$

6B. (5 pts) Differentiate: $y = 2e^{\cos(5x)}$

6B. $y' = -10\sin(5x)e^{\cos(5x)}$
Section 3.6

$$y' = 2\cdot e^{\cos(5x)} \cdot \frac{d}{dx}[\cos(5x)]$$

$$y' = 2\cdot e^{\cos(5x)} \cdot -\sin(5x)\cdot \frac{d}{dx}[5x]$$

$$y' = 2\cdot e^{\cos(5x)} \cdot -\sin(5x)\cdot 5$$

$$y' = -10\sin(5x)e^{\cos(5x)}$$

7A.) (6 pts) Find y'' (second derivative) if $y = \ln(x^2+5)^2$.

6A. $y'' = \dfrac{20-4x^2}{(x^2+5)^2}$
Section 3.3, 3.6, 3.8

$$y' = 2\cdot \frac{2x}{x^2+5}$$

$$y' = \frac{4x}{x^2+5}$$

$$y'' = \frac{(x^2+5)(4) - 4x(2x)}{(x^2+5)^2}$$

$$y'' = \frac{4x^2+20-8x^2}{(x^2+5)^2}$$

$$y'' = \frac{20-4x^2}{(x^2+5)^2}$$

7B.) (6 pts) Find y'' (second derivative) if $y = \cos^4 x$.

6B. $y'' = 12\sin^2 x\cos^2 x - 4\cos^4 x$

Section 3.3, 3.5, 3.6

$y = (\cos x)^4$

$y' = 4(\cos x)^3(-\sin x)$

$y' = -4\sin x\cos^3 x$

$y'' = -4\sin x(3\cos^2 x)(-\sin x) + \cos^3 x(-4\cos x)$

$y'' = 12\sin^2 x\cos^2 x - 4\cos^4 x$

8A.) (6 pts) Use $f(x) = 2x^4 - 16x^2$ to find the relative extrema and interval(s) of increasing and decreasing. **Section 4.3**

Increasing: $(-2,\ 0)\cup(2,\ \infty)$

Decreasing: $(-\infty,\ -2)\cup(0,\ 2)$

Relative Max: (0, 0)

Relative Min: $(-2,\ 32)$, $(2,\ -32)$

$f'(x) = 8x^3 - 32x$

$f'(x) = 8x(x^2 - 4)$

$f'(x) = 8x(x+2)(x-2)$

$x = 0,\ -2,\ 2$

–	+	–	+

–2 0 2

8B.) (6 pts) Use $f(x) = x^{\frac{1}{3}}(x-8)$ to find the relative extrema and interval(s) of increasing and decreasing. **Section 4.3**

Increasing: $(2,\ \infty)$

Decreasing: $(-\infty,\ 0)\cup(0,\ 2)$

Relative Max: None

Relative Min: $(2,\ -7.56)$

$f(x) = x^{\frac{4}{3}} - 8x^{\frac{1}{3}}$

$f'(x) = \frac{4}{3}x^{\frac{1}{3}} - \frac{8}{3}x^{-\frac{2}{3}}$

$f'(x) = \frac{4}{3}x^{\frac{1}{3}} - \frac{8}{3x^{\frac{2}{3}}}$

$0 = \frac{4}{3}x^{\frac{1}{3}} - \frac{8}{3x^{\frac{2}{3}}} \Rightarrow \frac{4x^{\frac{1}{3}}}{3} = \frac{8}{3x^{\frac{2}{3}}} \Rightarrow 12x = 24 \Rightarrow x = 2$ (0 is also a critical #)

–	–	+

0 2

9A.) (4 pts) Evaluate the integral: $\int \frac{1}{3\theta^2} - 2\sec\theta\tan\theta + e^{2\theta} - 4 \ d\theta$

9A. $-\frac{1}{3\theta} - 2\sec\theta + \frac{1}{2}e^{2\theta} - 4\theta + C$

Section 4.8

$$\int \frac{1}{3}\theta^{-2} - 2\sec\theta\tan\theta + e^{2\theta} - 4 \ d\theta$$

$$\frac{1}{3}\cdot\frac{\theta^{-1}}{-1} - 2\sec\theta + \frac{1}{2}e^{2\theta} - 4\theta + C$$

$$-\frac{1}{3\theta} - 2\sec\theta + \frac{1}{2}e^{2\theta} - 4\theta + C$$

9B.) (4 pts) Evaluate the integral: $\int \frac{1}{5\cdot\sqrt[3]{\theta}} - 4\sec^2\theta + 3\cdot 2^{3\theta} + 5 \ d\theta$

9B. $\frac{3}{10}\theta^{\frac{2}{3}} - 4\tan\theta + \frac{2^{3\theta}}{\ln 2} + 5\theta + C$

Section 4.8

$$\int \frac{1}{5}\theta^{-\frac{1}{3}} - 4\sec^2\theta + 3\cdot 2^{3\theta} + 5 \ d\theta$$

$$\frac{1}{5}\cdot\frac{\theta^{\frac{2}{3}}}{\frac{2}{3}} - 4\tan\theta + 3\cdot\frac{2^{3\theta}}{3\ln 2} + 5\theta + C$$

$$\frac{3}{10}\theta^{\frac{2}{3}} - 4\tan\theta + \frac{2^{3\theta}}{\ln 2} + 5\theta + C$$

10A.) (4 pts) Evaluate the integral: $\int \frac{2x^3}{\sqrt{3+x^4}} \ dx$

10A. $\sqrt{3+x^4} + C$

Section 5.5

Let $u = 3 + x^4$

$$du = 4x^3 \ dx$$

$$\frac{du}{4x^3} = dx$$

$$\int \frac{2x^3}{\sqrt{u}}\cdot\frac{du}{4x^3} \quad \Rightarrow \quad \frac{1}{2}\int u^{-\frac{1}{2}} \ du \quad \Rightarrow \quad \frac{1}{2}\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C \quad \Rightarrow \quad u^{\frac{1}{2}} + C \quad \Rightarrow \quad \sqrt{3+x^4} + C$$

10B.) (4 pts) Evaluate the integral: $\int \frac{3x^4}{7}\sin\left(\pi - \frac{x^5}{5}\right)\, dx$

10B. $\frac{3}{7}\cos\left(\pi - \frac{x^5}{5}\right) + C$

Section 5.5

Let $u = \pi - \frac{1}{5}x^5$

$du = -x^4\ dx$

$\frac{du}{-x^4} = dx$

$$\int \frac{3x^4}{7}\sin(u)\ \frac{du}{-x^4} \quad \Rightarrow \quad -\frac{3}{7}\int \sin(u)\ du \quad \Rightarrow \quad \frac{3}{7}\cos(u) + C \quad \Rightarrow \quad \frac{3}{7}\cos\left(\pi - \frac{x^5}{5}\right) + C$$

10C.) (4 pts) Evaluate the integral: $\int \left(\frac{x^2 + 4x + 10}{x^3 + 6x^2 + 30x - 4}\right) dx$

10C. $\frac{1}{3}\ln\left|x^3 + 6x^2 + 30x - 4\right| + C$

Section 5.5

Let $u = x^3 + 6x^2 + 30x - 4$

$du = 3x^2 + 12x + 30\ dx$

$\frac{du}{3x^2 + 12x + 30} = dx$

$$\int \left(\frac{x^2 + 4x + 10}{u}\right) \frac{du}{3(x^2 + 4x + 10)} \quad \Rightarrow \quad \frac{1}{3}\int \frac{1}{u}\ du \quad \Rightarrow \quad \frac{1}{3}\ln|u| + C \quad \Rightarrow$$

$$\frac{1}{3}\ln\left|x^3 + 6x^2 + 30x - 4\right| + C$$

10D.) (4 pts) Evaluate the integral: $\int \frac{2dx}{3x^{\frac{2}{3}}\left(1 + x^{\frac{1}{3}}\right)}$

10D. $2\ln\left|1 + x^{\frac{1}{3}}\right| + C$

Section 5.5

Let $u = 1 + x^{\frac{1}{3}} \quad \Rightarrow \quad du = \frac{1}{3}x^{-\frac{2}{3}}dx \quad \Rightarrow \quad du = \frac{1}{3x^{\frac{2}{3}}}dx \quad \Rightarrow \quad 3x^{\frac{2}{3}}du = dx$

$$\int \frac{2}{3x^{\frac{2}{3}}(u)} \cdot 3x^{\frac{2}{3}}du \quad \Rightarrow \quad 2\int \frac{1}{u}\ du \quad \Rightarrow \quad 2\ln|u| + C \quad \Rightarrow \quad 2\ln\left|1 + x^{\frac{1}{3}}\right| + C$$

11A.) (5 pts) Evaluate the definite integral: $\displaystyle\int_0^{\frac{\pi}{2}} \frac{40\cos\theta}{(4+\sin\theta)^2}\, d\theta$ 11A. 2

Section 5.6

Let $u = 4+\sin\theta \quad \Rightarrow \quad du = \cos\theta \; d\theta \quad \Rightarrow \quad \dfrac{du}{\cos\theta} = d\theta$

$$\int \frac{40\cos\theta}{u^2} \cdot \frac{du}{\cos\theta} \quad \Rightarrow \quad 40\int u^{-2}\, du \quad \Rightarrow \quad 40\cdot\frac{u^{-1}}{-1}+C \quad \Rightarrow \quad -\frac{40}{4+\sin\theta}\bigg|_0^{\frac{\pi}{2}}$$

$$\Rightarrow \quad -\frac{40}{4+\sin\left(\frac{\pi}{2}\right)}+\frac{40}{4+\sin(0)} \quad \Rightarrow \quad -\frac{40}{4+1}+\frac{40}{4+0} = -8+10 = 2$$

11B.) (5 pts) Evaluate the definite integral: $\displaystyle\int_0^{\frac{\pi}{4}} \frac{e^{\tan\theta}}{\cos^2\theta}\, d\theta$ 11B. $e-1$

Section 5.6

Let $u = \tan\theta \quad \Rightarrow \quad du = \sec^2\theta \; d\theta \quad \Rightarrow \quad \dfrac{du}{\sec^2\theta} = d\theta \quad \Rightarrow \quad \cos^2\theta \; du = d\theta$

$$\int \frac{e^u}{\cos^2\theta} \cdot \cos^2\theta \; du \quad \Rightarrow \quad \int e^u du \quad \Rightarrow \quad e^{\tan\theta}\Big|_0^{\frac{\pi}{4}}$$

$$\Rightarrow \quad e^{\tan\frac{\pi}{4}} - e^{\tan 0} \quad \Rightarrow \quad e^1 - e^0 \quad \Rightarrow \quad e-1$$

11C.) (5 pts) Evaluate the definite integral: $\displaystyle\int_0^{2} \theta^2\sqrt{\theta^3+1}\; d\theta$ 11C. $\dfrac{52}{9}$

Section 5.6

Let $u = \theta^3+1 \quad \Rightarrow \quad du = 3\theta^2 \; d\theta \quad \Rightarrow \quad \dfrac{du}{3\theta^2} = d\theta$

$$\int \theta^2\sqrt{u} \cdot \frac{du}{3\theta^2} \quad \Rightarrow \quad \frac{1}{3}\int u^{\frac{1}{2}}\, du \quad \Rightarrow \quad \frac{1}{3}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\; du \quad \Rightarrow \quad \frac{2}{9}\left(\theta^3+1\right)^{\frac{3}{2}}\bigg|_0^{2}$$

$$\Rightarrow \quad \frac{2}{9}\left(2^3+1\right)^{\frac{3}{2}} - \frac{2}{9}\left(0^3+1\right)^{\frac{3}{2}} \quad \Rightarrow \quad \frac{2}{9}(9)^{\frac{3}{2}} - \frac{2}{9}(1)^{\frac{3}{2}} \quad \Rightarrow \quad \frac{2}{9}(27) - \frac{2}{9}(1) = \frac{52}{9}$$

12.) (6 pts) The following problems deal with related rates or optimization. CHOOSE and SOLVE **ONE** of these problems. Indicate which one you want me to grade. (The actual test will have only 3 problems to choose from) **Parts A – D, F are from Section 3.10. Parts E & G are from section 4.6**

A.) The radius of a right circular cylinder is increasing at a rate of 5cm/min. If the height is always twice the radius, find the rate the volume is changing when the height is 8 cm.

A. $480\pi\ cm^3/\min$

Given: $\frac{dr}{dt} = 5\ cm/\min$, $h = 8\ cm$, $\frac{dV}{dt} = ?$

$V = \pi r^2 h$

$V = \pi r^2 (2r)$

$V = 2\pi r^3$

$\frac{dV}{dt} = 6\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 6\pi (4)^2 (5)$

$\frac{dV}{dt} = 480\pi\ cm^3/\min$

B.) When a circular plate of metal is heated in an oven, its radius increases at a rate of 0.02 cm/min. At what rate is the plate's area increasing when the radius is 40 cm?

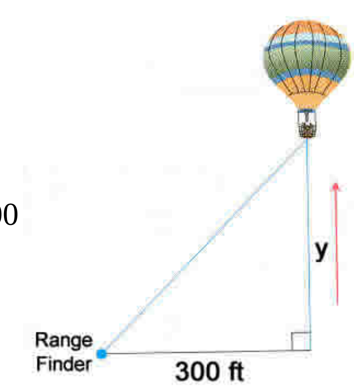
B. $1.6\pi\ or\ 5.03\ cm^2/\min$

Given: $\frac{dr}{dt} = 0.02\ cm/\min$, $r = 40\ cm$, $\frac{dA}{dt} = ?$

$A = \pi r^2 \quad \Rightarrow \quad \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \quad \Rightarrow \quad \frac{dA}{dt} = 2\pi (40)(0.02) = 1.6\pi \approx 5.03\ cm^2/\min$

C.) A hot air balloon is rising straight up from a level field is being tracked by a range finder 300 feet from the liftoff point. The diagonal distance between the range finder and the balloon is increasing at a rate of 16 feet per minute. Find that rate the balloon is rising when the balloon's height is 400 feet.

C. $2\ ft/\min$



Given:

$\frac{dz}{dt} = 16\ ft/\min$, $\frac{dy}{dt} = ?$, $y = 400\ ft \quad \Rightarrow \quad 300^2 + 400^2 = z^2 \quad \Rightarrow \quad z = 500$

$300^2 + y^2 = z^2$

$0 + 2y\frac{dy}{dt} = 2z\frac{dz}{dt} \quad \Rightarrow \quad 2(400)\frac{dy}{dt} = 2(500)(16)$

$800\frac{dy}{dt} = 16000 \quad \Rightarrow \quad \frac{dy}{dt} = 20\ ft/\min$

D.) A right triangle with a hypotenuse of 30 feet has its base increasing at a rate of 5 ft/sec. Find the rate the height is decreasing when the base is 18 ft.

D. $\frac{dy}{dt} = -3.75 \;\; ft/\sec$

Given:

$\frac{dx}{dt} = 5 \;\; ft/\sec$, $\frac{dy}{dt} = ?$, $x = 18 \;\; ft \;\; \Rightarrow \;\; 18^2 + y^2 = 30^2 \;\; \Rightarrow \;\; y = 24$

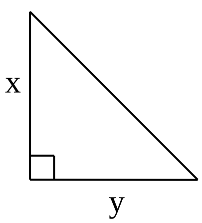
$x^2 + y^2 = 30^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \quad \Rightarrow \quad 2(18)(5) + 2(24)\frac{dy}{dt} = 0$

$48\frac{dy}{dt} = -180 \quad \Rightarrow \quad \frac{dy}{dt} = -3.75 \;\; ft/\sec$

E.) What is the maximum area of a right triangle whose legs have a combined length of 10cm? Note: the legs are x and y. The area of this triangle is $A = \frac{1}{2}xy$.

E. $\frac{25}{2}$



$x + y = 10 \quad \Rightarrow \quad y = 10 - x \quad \Rightarrow \quad A = \frac{1}{2}x(10 - x) \quad \Rightarrow$

$A = 5x - \frac{1}{2}x^2 \quad \Rightarrow \quad A' = 5 - x \quad \Rightarrow \quad 0 = 5 - x \quad \Rightarrow \quad x = 5, \;\; y = 5$

$A = \frac{1}{2}(5)(5) = \frac{25}{2}$

F.) The edges of a cube are expanding at a rate of 5 cm/sec. How fast is the Surface area changing when each edge is 4.5cm?

F. $270 \;\; cm^2/\sec$

Given: $\frac{dx}{dt} = 5 \;\; cm/\sec$, $x = 4.5 \;\; cm$, $\frac{dS}{dt} = ?$

$S = 6x^2$

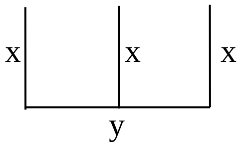
$\frac{dS}{dt} = 12x\frac{dx}{dt}$

$\frac{dS}{dt} = 12(4.5)(5) = 270 \;\; cm^2/\sec$

G.) A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a river, so no fencing is needed along the river. The field will be divided as shown. What dimensions of the field produce the largest area?

G. 400, 1200

First label the drawing:



$$3x + y = 2400 \quad \Rightarrow \quad y = 2400 - 3x$$

$$A = xy$$

$$A = x(2400 - 3x)$$

$$A = 2400x - 3x^2$$

$$A' = 2400 - 6x$$

$$0 = 2400 - 6x \quad \Rightarrow \quad x = 400 \quad \Rightarrow \quad y = 2400 - 3(400) \quad \Rightarrow \quad y = 1200$$