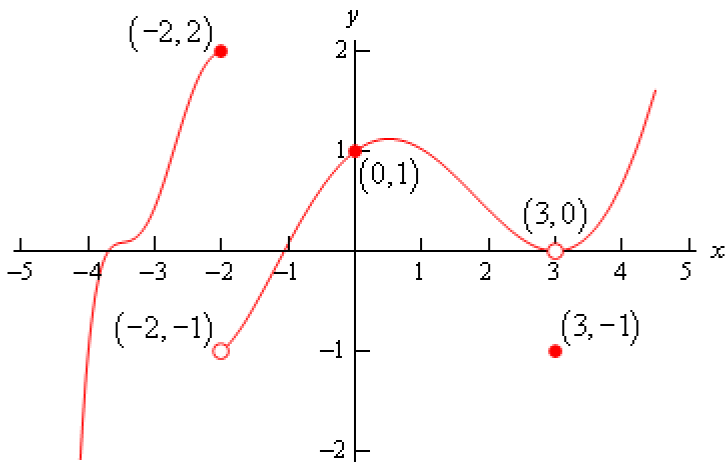
NAME: KEY

MATH 181 FINAL EXAM SAMPLE

NOTE: The actual exam will only have 14 questions. The different parts of each question (part A, B, etc.) are variations. Know how to do all the variations on this exam.

1A/B.) (6 pts) Find the following by using the graph of f(x) below. If it doesn't exist, write DNE.

NOTE: Problem 1 on the actual test will not have this many parts



a.) Find $f(-2)$ 2

b.) Find $f(3)$ –1

c.) Find $\lim_{x\to 3^+} f(x)$ 0

d.) Find $\lim_{x\to 3^-} f(x)$ 0

e.) Find $\lim_{x\to 3} f(x)$ 0

f.) Find $\lim_{x\to -2^+} f(x)$ –1

g.) Find $\lim_{x\to -2^-} f(x)$ 2

h.) Find $\lim_{x\to -2} f(x)$ DNE

i.) Find $\lim_{x\to -1} f(x)$ 0

j.) Which statement is **false**? (Choose all that apply)

A.) $\lim_{x\to 3} f(x)$ exists

B.) $f(x)$ is differentiable at $x = 3$

C.) $\lim_{x\to -2} f(x) = DNE$

D.) $f(x)$ is continuous at $x = 3$

E.) $\lim_{x\to 0} f(x)$ exists

2A.) (6 pts) Evaluate the limit. If the limit does not exist, indicate DNE.

i.) $\lim_{x\to -1} \dfrac{\sqrt{x+10}-3}{x+1}$ i. $\dfrac{1}{6}$

$$\lim_{x\to -1} \frac{\sqrt{x+10}-3}{x+1}\cdot\frac{\sqrt{x+10}+3}{\sqrt{x+10}+3} = \lim_{x\to -1} \frac{x+10-9}{(x+1)\left(\sqrt{x+10}+3\right)} = \lim_{x\to 0} \frac{x+10-9}{(x+1)\left(\sqrt{x+10}+3\right)} =$$

$$\lim_{x\to -1} \frac{1}{\left(\sqrt{x+10}+3\right)} = \frac{1}{\left(\sqrt{-1+10}+3\right)} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$$

ii.) $\lim\limits_{x\to\infty} \dfrac{7x^2+\sqrt{2}}{14x^3-3x^2+5}$ ii. 0

$$\lim_{x\to\infty} \frac{\frac{7x^2}{x^3}+\frac{\sqrt{2}}{x^3}}{\frac{14x^3}{x^3}-\frac{3x^2}{x^3}+\frac{5}{x^3}} = \lim_{x\to\infty} \frac{\frac{7}{x}+\frac{\sqrt{2}}{x^3}}{14-\frac{3}{x}+\frac{5}{x^3}} = \frac{0+0}{14-0+0}=0$$

2B.) (6 pts) Evaluate the limit. If the limit does not exist, indicate DNE.

i.) $\lim\limits_{x\to 3} \dfrac{x^2-11x+24}{x-3}$ i. -5

$$\lim_{x\to 3} \frac{(x-3)(x-8)}{x-3} = \lim_{x\to 3} x-8 = 3-8=-5$$

ii.) $\lim\limits_{x\to\infty} \dfrac{\sqrt{4x^2+3x-5}}{7-4x}$ ii. $-\dfrac{1}{2}$

$$\lim_{x\to\infty} \frac{\frac{\sqrt{4x^2+3x-5}}{\sqrt{x^2}}}{\frac{7}{x}-\frac{4x}{x}} = \lim_{x\to\infty} \frac{\sqrt{\frac{4x^2+3x-5}{x^2}}}{\frac{7}{x}-4} = \lim_{x\to\infty} \frac{\sqrt{4+\frac{3}{x}-\frac{5}{x^2}}}{\frac{7}{x}-4} = \frac{\sqrt{4+0-0}}{0-4}=\frac{2}{-4}=-\frac{1}{2}$$

2C.) (10 pts) Evaluate the limit. If the limit does not exist, indicate DNE.

i.) $\lim\limits_{x\to 4} \dfrac{2x^2-9x+4}{x-2}$ i. 0

$$\lim_{x\to 4} \frac{(x-4)(2x-1)}{x-2} = \frac{(4-4)(2(4)-1)}{4-2}=0$$

ii.) $\lim_{x\to\infty} \dfrac{2x^2+3}{3+4x+5x^2}$ ii. $\dfrac{2}{5}$

$$\lim_{x\to\infty} \frac{\frac{2x^2}{x^2}+\frac{3}{x^2}}{\frac{3}{x^2}+\frac{4x}{x^2}+\frac{5x^2}{x^2}} = \lim_{x\to\infty} \frac{2+\frac{3}{x^2}}{\frac{3}{x^2}+\frac{4}{x}+5} = \frac{2+0}{0+0+5}=\frac{2}{5}$$

3A. (5 pts) Differentiate: $y = 2\sqrt{3x^7 - \sin x}$ 3A. $y' = \dfrac{21x^6 - \cos x}{\sqrt{3x^7 - \sin x}}$

$$y' = 2\cdot\frac{1}{2}\left(3x^7 - \sin x\right)^{\frac{-1}{2}}\cdot\frac{d}{dx}\left[3x^7 - \sin x\right]$$

$$y' = \frac{1}{\sqrt{3x^7 - \sin x}}\cdot\left(21x^6 - \cos x\right)$$

$$y' = \frac{21x^6 - \cos x}{\sqrt{3x^7 - \sin x}}$$

3B. (5 pts) Differentiate: $y = \cos\left(\tan x - 2x^8\right)$ 3B. $y' = -\sin\left(\tan x - 2x^8\right)\cdot\left(\sec^2 x - 16x^7\right)$

$$y' = -\sin\left(\tan x - 2x^8\right)\cdot\frac{d}{dx}\left[\tan x - 2x^8\right]$$

$$y' = -\sin\left(\tan x - 2x^8\right)\cdot\left(\sec^2 x - 16x^7\right)$$

4A.) (4 pts) The function f is defined by $f(x) = \dfrac{-\cos x}{1+\sin x}$ on the interval $\left[0, 2\pi\right)$. What point(s) (x, y) on the graph of f have the property that the line tangent to f at (x, y) has a slope of $\dfrac{1}{2}$? 4A. $\left(\pi/2, 0\right)$

$$f'(x) = \frac{(1+\sin x)(\sin x)-(-\cos x)(\cos x)}{(1+\sin x)^2} = \frac{\sin x + \sin^2 x + \cos^2 x}{(1+\sin x)^2} = \frac{\sin x + 1}{(1+\sin x)^2} = \frac{1}{1+\sin x}$$

$$\frac{1}{1+\sin x} = \frac{1}{2} \quad\Rightarrow\quad 1+\sin x = 2 \quad\Rightarrow\quad \sin x = 1 \quad\Rightarrow\quad x = \frac{\pi}{2} \quad\Rightarrow\quad f\left(\frac{\pi}{2}\right) = \frac{-\cos\left(\pi/2\right)}{1+\sin\left(\pi/2\right)} = \frac{-(0)}{1+1} = 0$$

4B.) (4 pts) The function f is defined by $f(x) = \dfrac{2x}{x^2+9}$. What point(s) (x, y) on the graph of f have the property that the line tangent to f at (x, y) has a slope of 0?

4B. $\left(3, \frac{1}{3}\right), \left(-3, -\frac{1}{3}\right)$

$$f'(x) = \frac{(x^2+9)(2)-(2x)(2x)}{(x^2+9)^2} = \frac{2x^2+18-4x^2}{(x^2+9)^2} = \frac{18-2x^2}{(x^2+9)^2}$$

$$\frac{18-2x^2}{(x^2+9)^2} = \frac{0}{1} \quad \Rightarrow \quad 18-2x^2=0 \quad \Rightarrow \quad 2(9-x^2)=0 \quad \Rightarrow \quad 2(3+x)(3-x)=0 \quad \Rightarrow \quad x=3,-3$$

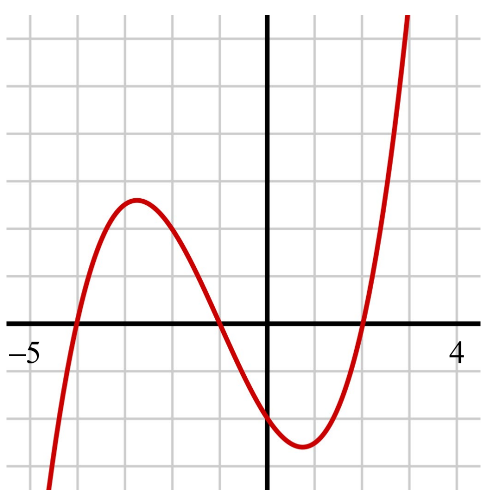
$$f(3) = \frac{2(3)}{(3)^2+9} = \frac{6}{18} = \frac{1}{3} \qquad f(-3) = \frac{2(-3)}{(-3)^2+9} = \frac{-6}{18} = -\frac{1}{3}$$

5A.) (3 pts) The graph of a differentiable function f is shown in the figure below. If $g(x) = \int_{-4}^{x} f(t)\,dt$ Which of the following is true?

$g(-1) = \int_{-4}^{-1} f(t)\,dt$ represents the area under the curve $f(x)$ between -4 and -1.

$g'(x) = \dfrac{d}{dx}\left[\int_{-4}^{x} f(t)\,dt\right] = f(x)$ and $g''(x) = f'(x)$

From the graph, we see that the area under the curve is positive. This means that $g(-1) > 0$.
The graph crosses the x-axis at –1, therefore $f(-1) = 0$ and so $g'(-1) = 0$.
The graph slants to the left as it crosses –1, which means that $f'(x) < 0$ and so $g''(x) < 0$



(A) $g(-1) < g'(-1) < g''(-1)$

(B) $g''(-1) < g'(-1) < g(-1)$

(C) $g(-1) < g''(-1) < g'(-1)$

(D) $g''(-1) < g(-1) < g'(-1)$

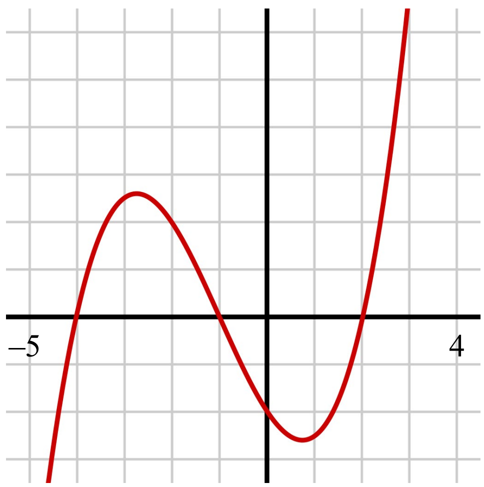
(E) $g'(-1) < g(-1) < g''(-1)$

5B.) (3 pts) The graph of a differentiable function f is shown in the figure below. If $g(x)=\int_{-1}^{x} f(t)\,dt$ Which of the following is true?

$g(2)=\int_{-1}^{2} f(t)\,dt$ represents the area above the curve $f(x)$ between -1 and 2.

$g'(x)=\frac{d}{dx}\left[\int_{-1}^{x} f(t)\,dt\right]=f(x)$ and $g''(x)=f'(x)$

From the graph, we see that the area between the curve and the x-axis is negative. This means that $g(2)<0$.
The graph crosses the x-axis at 2, therefore $f(2)=0$ and so $g'(2)=0$.
The graph slants to the right as it crosses 2, which means that $f'(x)>0$ and so $g''(x)>0$



(A) $g(2)<g'(2)<g''(2)$

(B) $g''(2)<g'(2)<g(2)$

(C) $g(2)<g''(2)<g'(2)$

(D) $g''(2)<g(2)<g'(2)$

(E) $g'(2)<g(2)<g''(2)$

6A.) (6 pts) Given $f'(x)=8x^3-32x$, find the critical numbers, interval(s) of increasing and decreasing, and where the relative maximum and minimum occur.

$f'(x)=8x^3-32x$

$f'(x)=8x(x^2-4)$

$f'(x)=8x(x+2)(x-2)$

$x=0,\ -2,\ 2$

–	+	–	+
	–2	0	2

Critical #s: $0,\ -2,\ 2$

Increasing: $(-2,\ 0)\cup(2,\ \infty)$

Decreasing: $(-\infty,\ -2)\cup(0,\ 2)$

The relative max occurs at $x=$ 0

The relative min occurs at $x=\pm 2$

6B.) (6 pts) Given $f'(x) = \dfrac{4(x-2)}{3x^{\frac{2}{3}}}$, find the critical numbers, interval(s) of increasing and decreasing, and where the relative maximum and minimum occur.

Critical #s: 0, 2

Increasing: $(2,\ \infty)$

Decreasing: $(-\infty,\ 0)\cup(0,\ 2)$

There is no relative max.

The relative min occurs at $x = 2$

One critical number is x = 0 since it causes the derivative to be undefined. Next, set the derivative equal to zero.

$$0 = \frac{4(x-2)}{3x^{\frac{2}{3}}} \quad \Rightarrow \quad 0 = 4(x-2) \quad \Rightarrow \quad x = 2$$

−	−	+

0   2

7A.) (4 pts) Evaluate the definite integral: $\displaystyle\int_0^1 \frac{2x}{\left(x^2+1\right)^4}\,dx$

7A. $\dfrac{7}{24}$

Let $u = x^2 + 1$

$du = 2x\ dx$

$\dfrac{du}{2x} = dx$

$$\int \frac{2x}{u^4}\cdot\frac{du}{2x} \quad \Rightarrow \quad \int u^{-4}\,du \quad \Rightarrow \quad \cdot\frac{u^{-3}}{-3} \quad \Rightarrow \quad -\frac{1}{3u^3} \quad \Rightarrow \quad -\frac{1}{3\left(x^2+1\right)^3}\Bigg|_0^1$$

$$-\frac{1}{3\left(x^2+1\right)^4}\Bigg|_0^1 = -\frac{1}{3\left(1^2+1\right)^3} + \frac{1}{3\left(0^2+1\right)^3} = -\frac{1}{24} + \frac{1}{3} = \frac{7}{24}$$

7B.) (4 pts) Evaluate the integral: $\displaystyle\int_0^{2\pi} \frac{\cos\theta}{\sqrt{4+3\sin\theta}}\,d\theta$

7B. 0

Let $u = 4 + 3\sin\theta$

$du = 3\cos\theta\ d\theta$

$\dfrac{du}{3\cos\theta} = d\theta$

$$\int \frac{\cos\theta}{\sqrt{u}} \cdot \frac{du}{3\cos\theta} \Rightarrow \frac{1}{3}\int u^{\frac{-1}{2}}\, du \Rightarrow \frac{1}{3}\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{2}} \Rightarrow \frac{2}{3}\sqrt{u} \Rightarrow \frac{2}{3}\sqrt{4+3\sin\theta}\Big|_0^{2\pi}$$

$$\frac{2}{3}\sqrt{4+3\sin\theta}\Big|_0^{2\pi} = \frac{2}{3}\sqrt{4+3\sin(2\pi)} - \frac{2}{3}\sqrt{4+3\sin(0)} = \frac{2}{3}\sqrt{4+3(0)} - \frac{2}{3}\sqrt{4+3(0)} = 0$$

8A.) (4 pts) Find the solution to the differential equation $\frac{dy}{d\theta} = \sqrt{2}\cos\theta + \sec^2\theta$ with the initial condition $y\left(\frac{\pi}{4}\right) = -1$

8A. $y(\theta) = \sqrt{2}\sin\theta + \tan\theta - 3$

$$dy = \sqrt{2}\cos\theta + \sec^2\theta\ d\theta \qquad y\left(\frac{\pi}{4}\right) = -1$$

$$\int dy = \int \sqrt{2}\cos\theta + \sec^2\theta\ d\theta \qquad -1 = \sqrt{2}\sin\left(\frac{\pi}{4}\right) + \tan\left(\frac{\pi}{4}\right) + C$$

$$y(\theta) = \sqrt{2}\sin\theta + \tan\theta + C \qquad -1 = \sqrt{2}\left(\frac{\sqrt{2}}{2}\right) + 1 + C \Rightarrow -1 = 1 + 1 + C \Rightarrow C = -3$$

8B.) (4 pts) Find the solution to the differential equation $\frac{dy}{dt} = 6\sin(3t)$ with the initial condition $y\left(\frac{\pi}{6}\right) = 6$

8A. $y(\theta) = -2\cos(3t) + 6$

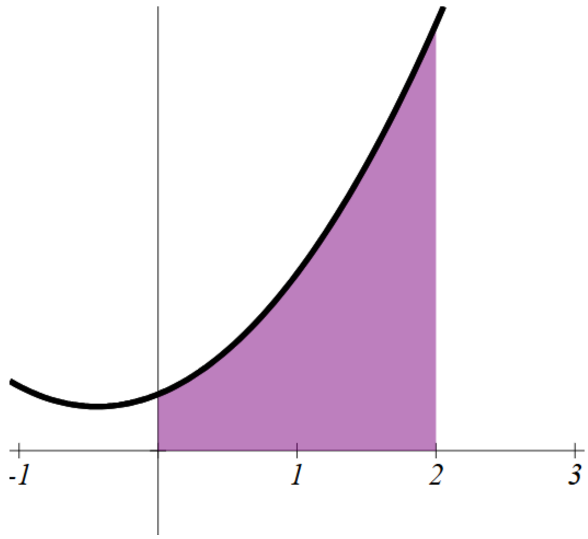
$$dy = 6\sin(3t)\ dt \qquad y\left(\frac{\pi}{6}\right) = 6$$

$$\int dy = \int 6\sin(3t)\ dt \qquad 6 = -2\cos\left(3\cdot\frac{\pi}{6}\right) + C$$

$$y(\theta) = 6\cdot -\frac{1}{3}\cos(3t) + C \qquad 6 = -2\cos\left(\frac{\pi}{2}\right) + C$$

$$y(\theta) = -2\cos(3t) + C \qquad 6 = -2(0) + C \Rightarrow C = 6$$

9A.) (4 pts) The graph below shows the parabola $f(x) = 8x^2 + 7x + 7$ together with a shaded region.



a.) Set up an integral to calculate the area of the shaded region.

$$\int_0^2 8x^2 + 7x + 7 \; dx$$

b.) Evaluate the integral to find the exact area of the shaded region.

$$A = \frac{148}{3}$$

$$\int_0^2 8x^2 + 7x + 7 \; dx = 8 \cdot \frac{x^3}{3} + 7 \cdot \frac{x^2}{2} + 7x \Big|_0^2 = \left(\frac{8(2)^3}{3} + \frac{7(2)^2}{2} + 7(2) \right) - \left(\frac{8(0)^3}{3} + \frac{7(0)^2}{2} + 7(0) \right)$$

$$= \left(\frac{64}{3} + 14 + 14 \right) - (0) = \frac{148}{3}$$

9B.) (4 pts) The graph below shows the parabola $f(x) = x^2 - 4$ together with a shaded region.

a.) Set up an integral to calculate the area of the shaded region.

$$\int_{-7}^{-5} x^2 - 4 \; dx$$

b.) Evaluate the integral to find the exact area of the shaded region.

$$A = \frac{194}{3}$$

$$\int_{-7}^{-5} x^2 - 4 \; dx = \frac{x^3}{3} - 4x \Big|_{-7}^{-5} = \left(\frac{(-5)^3}{3} - 4(-5) \right) - \left(\frac{(-7)^3}{3} - 4(-7) \right)$$

$$= \left(-\frac{125}{3} + 20 \right) - \left(-\frac{343}{3} + 28 \right) = \left(-\frac{65}{3} \right) - \left(-\frac{259}{3} \right) = \frac{194}{3}$$

10A.) (4 pts) A particle moves along the x-axis. The velocity of the particle at time t is $-\frac{1}{2}t^2\left(t^2-9\right)$. What is the total distance traveled by the particle from time $t=0$ to $t=3$?

10A. $\frac{81}{5}$

We are given the velocity function $v(t)=-\frac{1}{2}t^2\left(t^2-9\right)$. We need to find the position function. To do this, we need to integrate. Integrating from 0 to 3 will give us the total distance traveled:

$$\text{Total distance} = \int_0^3 -\frac{1}{2}t^2\left(t^2-9\right)\,dt$$

$$=\int_0^3 -\frac{1}{2}t^4+\frac{9}{2}t^2\,dt = -\frac{1}{2}\cdot\frac{t^5}{5}+\frac{9}{2}\cdot\frac{t^3}{3}\Bigg|_0^3 = \left(-\frac{3^5}{10}+\frac{9(3)^3}{6}\right)-\left(-\frac{0^5}{10}+\frac{9(0)^3}{6}\right)=\left(-\frac{243}{10}+\frac{81}{2}\right)-(0)=\frac{81}{5}$$

10B.) (4 pts) A particle moves along the x-axis. The velocity of the particle at time t is $8\sqrt{t}-t^2$. What is the total distance traveled by the particle from time $t=0$ to $t=4$?

10B. $\frac{64}{3}$

We are given the velocity function $v(t)=8\sqrt{t}-t^2$. We need to find the position function. To do this, we need to integrate. Integrating from 0 to 4 will give us the total distance traveled:

$$\text{Total distance} = \int_0^4 8\sqrt{t}-t^2\,dt$$

$$=\int_0^4 8t^{\frac{1}{2}}-t^2\,dt = 8\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}}-\frac{t^3}{3}\Bigg|_0^4 = \left(\frac{16}{3}(4)^{\frac{3}{2}}-\frac{4^3}{3}\right)-\left(\frac{16}{3}(0)^{\frac{3}{2}}-\frac{0^3}{3}\right)=\left(\frac{128}{3}-\frac{64}{3}\right)-(0)=\frac{64}{3}$$

11A.) (4 pts) Differentiate $y=\sqrt{x^5}-\csc x+\ln(4x)$

11A. $y'=\frac{5}{2}x^{\frac{3}{2}}+\csc x\cot x+\frac{1}{x}$

$$y=x^{\frac{5}{2}}-\csc x+\ln(4x)$$

$$y'=\frac{5}{2}x^{\frac{3}{2}}+\csc x\cot x+\frac{4}{4x}$$

$$y'=\frac{5}{2}x^{\frac{3}{2}}+\csc x\cot x+\frac{1}{x}$$

11B.) (4 pts) Differentiate $y = \frac{1}{\sqrt[3]{x^2}} + \cot x + e^{5x}$

11B. $y' = -\frac{2}{3x^{\frac{5}{3}}} - \csc^2 x + 5e^{5x}$

$$y = x^{-\frac{2}{3}} + \cot x + e^{5x}$$

$$y' = -\frac{2}{3}x^{-\frac{5}{3}} - \csc^2 x + e^{5x}(5)$$

$$y' = -\frac{2}{3x^{\frac{5}{3}}} - \csc^2 x + 5e^{5x}$$

12A.) (4 pts) Find the exact x-coordinate where there is a horizontal tangent line to the graph of $f(x) = (x-3)\cdot e^{-2x}$.

12A. $x = \frac{7}{2}$

$$f'(x) = (x-3)\cdot e^{-2x}(-2) + e^{-2x}(1)$$

$$f'(x) = e^{-2x}\left(-2(x-3)+1\right)$$

$$f'(x) = \frac{-2x+6+1}{e^{2x}}$$

$$f'(x) = \frac{7-2x}{e^{2x}}$$

$$f'(x) = 0$$

$$\frac{7-2x}{e^{2x}} = \frac{0}{1}$$

$$7-2x = 0$$

$$x = \frac{7}{2}$$

12B.) (4 pts) Find the exact x-coordinate where there is a horizontal tangent line to the graph of $f(x) = (x+1)\cdot 3^x$.

12B. $x = -1 - \frac{1}{\ln(3)}$

$$f'(x) = (x+1)\cdot 3^x \ln(3) + 3^x(1)$$

$$f'(x) = 3^x\left((x+1)\ln(3)+1\right)$$

$$f'(x) = 3^x\left(x\ln(3)+\ln(3)+1\right)$$

$$f'(x) = 0$$

$$f'(x) = 3^x\left(x\ln(3)+\ln(3)+1\right)$$

$$3^x = 0 \quad \Rightarrow \quad \ln\left(3^x\right) = \ln(0) \quad \Rightarrow \quad \text{DNE}$$

$$x\ln(3)+\ln(3)+1 = 0$$

$$x\ln(3) = -\ln(3)-1 \quad \Rightarrow \quad x = -\frac{\ln(3)}{\ln(3)} - \frac{1}{\ln(3)}$$

$$x = -1 - \frac{1}{\ln(3)}$$

13A.) (4 pts) Use implicit differentiation to find $\frac{dy}{dx}$ for the curve

13A. $\frac{dy}{dx} = \frac{-y\sin(xy)}{\cos y + x\sin(xy)}$

$\cos(xy) = 1 + \sin y$

$$-\sin(xy)\left(x\frac{dy}{dx} + y(1)\right) = 0 + \cos y\frac{dy}{dx}$$

$$-x\frac{dy}{dx}\sin(xy) - y\sin(xy) = \cos y\frac{dy}{dx}$$

$$-y\sin(xy) = \cos y\frac{dy}{dx} + x\frac{dy}{dx}\sin(xy)$$

$$-y\sin(xy) = \frac{dy}{dx}(\cos y + x\sin(xy))$$

$$\frac{-y\sin(xy)}{\cos y + x\sin(xy)} = \frac{dy}{dx}$$

13B.) (4 pts) Use implicit differentiation to find $\frac{dy}{dx}$ for the curve

13B. $\frac{dy}{dx} = \frac{4x^3 + 2xy^2}{3y^2 - 2x^2y}$

$x^2(x^2 + y^2) = y^3$

$$x^4 + x^2y^2 = y^3$$

$$4x^3 + x^2 \cdot 2y\frac{dy}{dx} + y^2(2x) = 3y^2\frac{dy}{dx}$$

$$4x^3 + 2xy^2 = 3y^2\frac{dy}{dx} - 2x^2y\frac{dy}{dx}$$

$$4x^3 + 2xy^2 = \frac{dy}{dx}(3y^2 - 2x^2y)$$

$$\frac{4x^3 + 2xy^2}{3y^2 - 2x^2y} = \frac{dy}{dx}$$

14A.) (4 pts) The amount of a chemical in a solution at time t is modeled by the equation: $C(t) = \frac{4t}{t^2+3}$, $t > 0$ where t is measured in minutes. Find the time t when the concentration $C(t)$ is maximized.

14A. $t = \sqrt{3}$ minutes

For maximizing and minimizing problems, we must set the derivative equal to zero:

$$C'(t) = \frac{(t^2+3)(4) - (4t)(2t)}{(t^2+3)^2} \Rightarrow C'(t) = \frac{4t^2 + 12 - 8t^2}{(t^2+3)^2} \Rightarrow C'(t) = \frac{12 - 4t^2}{(t^2+3)^2} \Rightarrow 0 = \frac{12 - 4t^2}{(t^2+3)^2}$$

$$\Rightarrow 0 = 12 - 4t^2 \Rightarrow 4t^2 = 12 \Rightarrow t^2 = 3 \Rightarrow t = \sqrt{3}$$ (We reject any negative time.)

14B.) (4 pts) The cost $C(x)$, in thousands, of producing x units of a product is given by the equation: $C(x)=\frac{5x^2}{x-50}$. Find the number of units x that minimizes the cost.

14B. 100 units

For maximizing and minimizing problems, we must set the derivative equal to zero:

$$C'(x)=\frac{(x-50)(10x)-(5x^2)(1)}{(x-50)^2} \Rightarrow C'(x)=\frac{10x^2-500x-5x^2}{(x-50)^2} \Rightarrow C'(x)=\frac{5x^2-500x}{(x-50)^2} \Rightarrow$$

$$0=\frac{5x^2-500x}{(x-50)^2} \Rightarrow 5x^2-500x=0 \Rightarrow 5x(x-100)=0 \Rightarrow x=100$$

(We reject $x=0$ since we need to at least sell something.)