NAME:______________KEY_____________

MATH 181 TEST 1 SAMPLE

NOTE: The actual exam will only have 14 questions. The different parts of each question (part A, B, etc.) are variations. Know how to do all the variations on this exam.

1A.) (5 pts) Find the equation for the tangent to the curve $y = x^3 - 2x^2$ at the point (2, 0). Use $\lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}$.

1A. $y = 4x - 8$

Section 1.3, 2.1

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$
$$\lim_{h\to 0}\frac{(2+h)^3-2(2+h)^2-0}{h}$$
$$\lim_{h\to 0}\frac{h^3+6h^2+12h+8-2(h^2+4h+4)}{h}$$
$$\lim_{h\to 0}\frac{h^3+6h^2+12h+8-2h^2-8h-8}{h}$$
$$\lim_{h\to 0}\frac{h^3+4h^2+4h}{h}$$
$$\lim_{h\to 0}\frac{h(h^2+4h+4)}{h}$$
$$\lim_{h\to 0} h^2+4h+4 = 0^2+4(0)+4 = 4$$

$$y = mx + b$$
$$0 = 4(2) + b$$
$$-8 = b$$
$$y = 4x - 8$$

1B.) (5 pts) Find the equation for the tangent to the curve $y = \sqrt{x-1}$ at the point (5, 2). Use $\lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}$.

1B. $y = \frac{1}{4}x + \frac{3}{4}$

Section 1.3, 2.1

$$\lim_{h\to 0}\frac{f(5+h)-f(5)}{h}$$
$$\lim_{h\to 0}\frac{\sqrt{5+h-1}-2}{h}$$
$$\lim_{h\to 0}\frac{\sqrt{h+4}-2}{h}$$
$$\lim_{h\to 0}\frac{\sqrt{h+4}-2}{h}\cdot\frac{\sqrt{h+4}+2}{\sqrt{h+4}+2}$$
$$\lim_{h\to 0}\frac{h+4-4}{h(\sqrt{h+4}+2)}$$
$$\lim_{h\to 0}\frac{1}{\sqrt{h+4}+2} = \frac{1}{\sqrt{0+4}+2} = \frac{1}{4}$$

$$y = mx + b$$
$$2 = \frac{1}{4}(5) + b$$
$$2 = \frac{5}{4} + b$$
$$\frac{3}{4} = b$$
$$y = \frac{1}{4}x + \frac{3}{4}$$

1C.) (5 pts) Find the equation for the tangent to the curve $y=\frac{4}{x-1}$ at the point (5, 1). Use $\lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}$.

1C. $y=-\frac{1}{4}x+\frac{9}{4}$

Section 1.3, 2.1

$$\lim_{h\to 0}\frac{f(5+h)-f(5)}{h}$$

$$\lim_{h\to 0}\frac{\frac{4}{5+h-1}-1}{h}$$

$$\lim_{h\to 0}\frac{\frac{4}{4+h}-\frac{4+h}{4+h}}{h}$$

$$\lim_{h\to 0}\frac{\frac{4-4-h}{4+h}}{h}$$

$$\lim_{h\to 0}\frac{\frac{-h}{4+h}}{h}$$

$$\lim_{h\to 0}\frac{-h}{4+h}\cdot\frac{1}{h}$$

$$\lim_{h\to 0}\frac{-1}{4+h}=\frac{-1}{4+0}=-\frac{1}{4}$$

$$y=mx+b$$

$$1=5\left(-\frac{1}{4}\right)+b$$

$$1=-\frac{5}{4}+b$$

$$\frac{9}{4}=b$$

$$y=-\frac{1}{4}x+\frac{9}{4}$$

2A.) (4 pts) Find the limit: $\lim_{x\to 0}\left(\frac{\sin^2 x}{1-\cos x}\right)$ **Section 1.4**

2A. 2

$$\lim_{x\to 0}\left(\frac{\sin^2 x}{1-\cos x}\right)$$

$$\lim_{x\to 0}\left(\frac{1-\cos^2 x}{1-\cos x}\right)$$

$$\lim_{x\to 0}\frac{(1+\cos x)(1-\cos x)}{1-\cos x}$$

$$\lim_{x\to 0}\ 1+\cos x=1+\cos 0=1+1=2$$

Tangent and Cotangent Identities

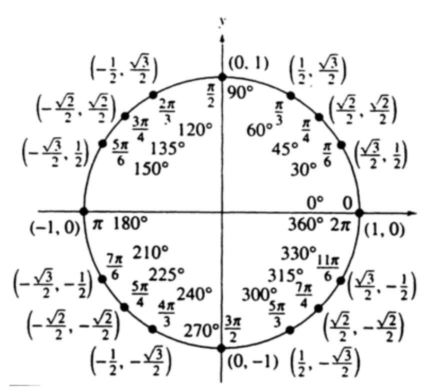
$\tan\theta=\frac{\sin\theta}{\cos\theta}$ $\qquad\cot\theta=\frac{\cos\theta}{\sin\theta}$

Reciprocal Identities

$\csc\theta=\frac{1}{\sin\theta}$ $\qquad\sin\theta=\frac{1}{\csc\theta}$

$\sec\theta=\frac{1}{\cos\theta}$ $\qquad\cos\theta=\frac{1}{\sec\theta}$

$\cot\theta=\frac{1}{\tan\theta}$ $\qquad\tan\theta=\frac{1}{\cot\theta}$



2B.) (4 pts) Find the limit: $\lim_{x\to\frac{\pi}{2}}\left(\frac{\cos x}{\cot x}\right)$ **Section 1.4** 2B. 1

$$\lim_{x\to\frac{\pi}{2}}\left(\frac{\cos x}{\cot x}\right) = \lim_{x\to\frac{\pi}{2}}\left(\frac{\cos x}{\frac{\cos x}{\sin x}}\right) = \lim_{x\to\frac{\pi}{2}}\left(\frac{\cos x}{1}\cdot\frac{\sin x}{\cos x}\right) = \lim_{x\to\frac{\pi}{2}} \sin x = \sin\frac{\pi}{2} = 1$$

3A.) (4 pts) Find the exact value: $\lim_{x\to\frac{5}{2}}\left(\frac{2x^2-3x-5}{2x^2-7x+5}\right)$ 3A. $\frac{7}{3}$

Section 1.4

$$\lim_{x\to\frac{5}{2}}\left(\frac{2x^2-3x-5}{2x^2-7x+5}\right) = \lim_{x\to\frac{5}{2}}\frac{(2x-5)(x+1)}{(2x-5)(x-1)} = \lim_{x\to\frac{5}{2}}\frac{x+1}{x-1} = \frac{\frac{5}{2}+1}{\frac{5}{2}-1} = \frac{\frac{7}{2}}{\frac{3}{2}} = \frac{7}{2}\cdot\frac{2}{3} = \frac{7}{3}$$

3B.) (4 pts) Find the exact value: $\lim_{x\to\frac{3}{2}}\left(\frac{3-2x}{4x^2-9}\right)$ 3B. $-\frac{1}{6}$

Section 1.4

$$\lim_{x\to\frac{3}{2}}\left(\frac{3-2x}{4x^2-9}\right) = \lim_{x\to\frac{3}{2}}\frac{3-2x}{(2x+3)(2x-3)} = \lim_{x\to\frac{3}{2}}\frac{-\left(-3+2x\right)}{(2x+3)(2x-3)} = \lim_{x\to\frac{3}{2}}\frac{-1}{(2x+3)} = \frac{-1}{2x+3} = \frac{-1}{2\left(\frac{3}{2}\right)+3} = -\frac{1}{6}$$

4A.) (4 pts) Find the exact value: $\lim_{h \to -2} \left(\frac{h+2}{\sqrt{h^2+5}-3} \right)$ **Section 1.4** 4A. $-\frac{3}{2}$

$$\lim_{h \to -2} \frac{h+2}{\sqrt{h^2+5}-3} \cdot \frac{\sqrt{h^2+5}+3}{\sqrt{h^2+5}+3} = \lim_{h \to -2} \frac{(h+2)\left(\sqrt{h^2+5}+3\right)}{h^2+5-9} = \lim_{h \to -2} \frac{(h+2)\left(\sqrt{h^2+5}+3\right)}{h^2-4}$$

$$= \lim_{h \to -2} \frac{(h+2)\left(\sqrt{h^2+5}+3\right)}{(h+2)(h-2)} = \lim_{h \to -2} \frac{\sqrt{h^2+5}+3}{h-2} = \frac{\sqrt{(-2)^2+5}+3}{-2-2} = \frac{3+3}{-4} = \frac{6}{-4} = -\frac{3}{2}$$

4B.) (4 pts) Find the exact value: $\lim_{h \to 0} \left(\frac{\sqrt{5+h}-\sqrt{5}}{h} \right)$ **Section 1.4** 4B. $\frac{\sqrt{5}}{10}$

$$\lim_{h \to 0} \frac{\sqrt{5+h}-\sqrt{5}}{h} \cdot \frac{\sqrt{5+h}+\sqrt{5}}{\sqrt{5+h}+\sqrt{5}} = \lim_{h \to 0} \frac{5+h-5}{h\left(\sqrt{5+h}+\sqrt{5}\right)} = \lim_{h \to 0} \frac{h}{h\left(\sqrt{5+h}+\sqrt{5}\right)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{5+h}+\sqrt{5}} = \frac{1}{\sqrt{5+0}+\sqrt{5}} = \frac{1}{2\sqrt{5}} = \frac{1}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{10}$$

5A.) (6 points) Find $\lim_{x \to -3} (3-4x)$. Then use the $\varepsilon - \delta$ definition of a limit to prove your answer. Please show all steps for full credit. **Section 1.5**

$$\lim_{x \to -3} (3-4x) = 3-4(-3) = 15$$

<u>Proof:</u>

For each $\varepsilon > 0$ there exists a $\delta > 0$ such that if $0 < |x-(-3)| < \delta$, then $|3-4x-15| < \varepsilon$.

$|-4x-12| < \varepsilon$

$|-4(x+3)| < \varepsilon$

$4|x+3| < \varepsilon$ so $|x+3| < \frac{\varepsilon}{4}$. Hence, let $\delta = \frac{\varepsilon}{4}$.

Hence, if $0 < |x+3| < \delta = \frac{\varepsilon}{4}$

$|x+3| < \frac{\varepsilon}{4}$

$4|x+3| < \varepsilon$

$|-4(x+3)| < \varepsilon$

$|-4x-12| < \varepsilon$, or $|3-4x-15| < \varepsilon$

So $|f(x)-L| < \varepsilon$

5B.) (6 points) Find $\lim_{x\to 2}\left(\frac{3x}{2}-5\right)$. Then use the $\varepsilon-\delta$ definition of a limit to prove your answer. Please show all steps for full credit. **Section 1.5**

$$\lim_{x\to 2}\left(\frac{3x}{2}-5\right)=\frac{3(2)}{2}-5=3-5=-2$$

<u>Proof:</u>

For each $\varepsilon>0$ there exists a $\delta>0$ such that if $0<|x-2|<\delta$, then $\left|\frac{3x}{2}-5-(-2)\right|<\varepsilon$.

$\left|\frac{3x}{2}-3\right|<\varepsilon$

$\left|\frac{3}{2}(x-2)\right|<\varepsilon$

$\frac{3}{2}|x-2|<\varepsilon$ so $|x-2|<\frac{2\varepsilon}{3}$. Hence, let $\delta=\frac{2\varepsilon}{3}$.

Hence, if $0<|x-2|<\delta=\frac{2\varepsilon}{3}$

$|x-2|<\frac{2\varepsilon}{3}$

$\frac{3}{2}|x-2|<\varepsilon$

$\left|\frac{3}{2}(x-2)\right|<\varepsilon$

$\left|\frac{3x}{2}-3\right|<\varepsilon$, or $\left|\frac{3x}{2}-5-(-2)\right|<\varepsilon$

So $|f(x)-L|<\varepsilon$

6A.) (4 pts) Use the following information to answer the questions:

$f(x)=\sqrt{x-3}$, L = 2, $x_0=7$, $\varepsilon=1$

i.) Find an open interval about x_0 on which the inequality $|f(x)-L|<\varepsilon$ holds.

6i. $(4,\ 12)$

Section 1.5

$\left|\sqrt{x-3}-2\right|<1$

$-1<\sqrt{x-3}-2<1$

$1<\sqrt{x-3}<3$

$1<x-3<9$

$4<x<12$

$(4,\ 12)$

ii.) Give a value for $\delta > 0$ such that for all x satisfying $0 < |x - x_0| < \delta$ the inequality $|f(x) - L| < \varepsilon$ holds:

6ii. $\delta = 3$

Section 1.5

$0 < |x - 7| < \delta$

$|x - 7| < \delta$

$-\delta < x - 7 < \delta$

$7 - \delta < x < 7 + \delta$

$7 - \delta = 4$ and $7 + \delta = 12$

$\delta = 3$ and $\delta = 5$

6B.) (4 pts) Use the following information to answer the questions:

$f(x) = \dfrac{x}{4}$, L = 2, $x_0 = 8$, $\varepsilon = 0.01$

i.) Find an open interval about x_0 on which the inequality $|f(x) - L| < \varepsilon$ holds.

6i. $(7.96,\ 8.04)$

Section 1.5

$\left|\dfrac{x}{4} - 2\right| < 0.01$

$-0.01 < \dfrac{x}{4} - 2 < 0.01$

$1.99 < \dfrac{x}{4} < 2.01$

$7.96 < x < 8.04$

$(7.96,\ 8.04)$

ii.) Give a value for $\delta > 0$ such that for all x satisfying $0 < |x - x_0| < \delta$ the inequality $|f(x) - L| < \varepsilon$ holds:

6ii. $\delta = 0.04$

Section 1.5

$0 < |x - 8| < \delta$

$|x - 8| < \delta$

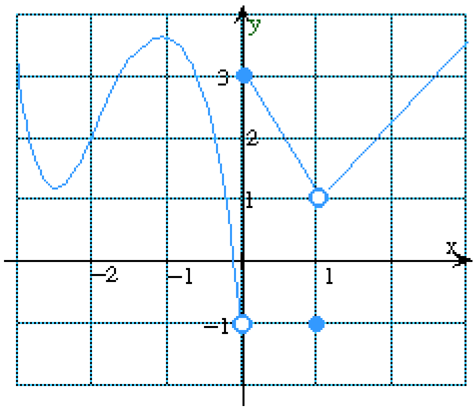
$-\delta < x - 8 < \delta$

$8 - \delta < x < 8 + \delta$

$8 - \delta = 7.96$ and $8 + \delta = 8.04$

$\delta = 0.04$ and $\delta = 0.04$

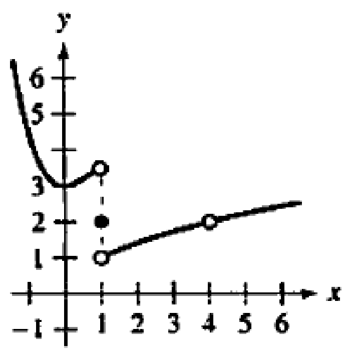
7A.) (5 pts.) Find the following by using the graph of f(x) below. If it doesn't exist, write d.n.e.



Section 1.6

$f(1)$:	-1	$f(0)$:	3
$\lim\limits_{x\to 0^-} f(x)$:	-1	$\lim\limits_{x\to 0^+} f(x)$:	3
$\lim\limits_{x\to 0} f(x)$:	DNE	$\lim\limits_{x\to 1^+} f(x)$:	1
$\lim\limits_{x\to 1^-} f(x)$:	1	$\lim\limits_{x\to 1} f(x)$:	1
$\lim\limits_{x\to -2^-} f(x)$:	2	$\lim\limits_{x\to -2} f(x)$:	2

7B.) (5 pts.) Find the following by using the graph of f(x) below. If it doesn't exist, write d.n.e.



Section 1.6

$f(1)$:	2	$f(4)$:	DNE
$\lim\limits_{x\to 1^-} f(x)$:	3.5	$\lim\limits_{x\to 1^+} f(x)$:	1
$\lim\limits_{x\to 1} f(x)$:	DNE	$\lim\limits_{x\to 4^+} f(x)$:	2
$\lim\limits_{x\to 4^-} f(x)$:	2	$\lim\limits_{x\to 4} f(x)$:	2
$\lim\limits_{x\to 0^-} f(x)$:	3	$\lim\limits_{x\to 0} f(x)$:	3

8A.) (4 pts) Find the exact value: $\lim\limits_{x\to 0}\left(\frac{x^2 - x + \sin 8x}{2x}\right)$

8A. $\frac{7}{2}$

Section 1.6

$$\lim_{x\to 0} \frac{x^2}{2x} - \frac{x}{2x} + \frac{\sin 8x}{2x} = \lim_{x\to 0} \frac{x}{2} - \frac{1}{2} + \frac{\sin 8x}{2x}\left(\frac{\frac{1}{8x}}{\frac{1}{8x}}\right) = \lim_{x\to 0} \frac{x}{2} - \frac{1}{2} + \frac{\frac{\sin 8x}{8x}}{\frac{1}{4}} = \lim_{x\to 0} \frac{x}{2} - \frac{1}{2} + 4\cdot\frac{\sin 8x}{8x}$$

$$= \frac{0}{2} - \frac{1}{2} + 4\cdot(1) = \frac{7}{2}$$

8B.) (4 pts) Find the exact value: $\lim_{x\to 0}\left(\frac{\sin\frac{x}{3}}{6x}\right)$

8B. $\frac{1}{18}$

Section 1.6

$$\lim_{x\to 0}\left(\frac{\sin\frac{x}{3}}{6x}\right) = \lim_{x\to 0}\frac{\sin\frac{x}{3}}{6x}\left(\frac{\frac{1}{18}}{\frac{1}{18}}\right) = \frac{1}{18}\lim_{x\to 0}\frac{\sin\frac{x}{3}}{\frac{x}{3}} = \frac{1}{18}\cdot 1 = \frac{1}{18}$$

8C.) (4 pts) Find the exact value: $\lim_{x\to 0}\frac{\sin x(1-\cos x)}{x^2}$

8C. 0

Section 1.6

$$\lim_{x\to 0}\frac{\sin x(1-\cos x)}{x^2} = \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{1-\cos x}{x} = 1\cdot 0 = 0$$

For problem 9, on the first blank, indicate the x-values (if any) at which f is not continuous. On the second blank, indicate which discontinuity is removable (if any), and on the third blank, indicate which discontinuity is non-removable (if any). If *f* is continuous, just write "none" in the first blank and don't write anything in the other 2 blanks. **Section 1.7**

9A.) (3 pts) $f(x) = 2\tan\theta\cos\theta$ on $[0,\pi]$

9A. $\frac{\pi}{2}$

$f(x) = 2\tan\theta\cos\theta$

$\frac{\pi}{2}$

$f(x) = 2\tan\theta\cos\theta$

none

$f(x) = \frac{2}{1}\cdot\frac{\sin\theta}{\cos\theta}\cdot\frac{\cos\theta}{1}$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$ this is the discontinuity. Since the cosines cancel, there is a hole at $\theta = \frac{\pi}{2}$ so it is a removable discontinuity.

9B.) (3 pts) $f(x)=\dfrac{x+5}{x^3+x^2-20x}$

9B. $0,\ 4,\ -5$

-5

$0, 4$

$$f(x)=\frac{x+5}{x^3+x^2-20x}$$
$$f(x)=\frac{x+5}{x\left(x^2+x-20\right)}$$
$$f(x)=\frac{x+5}{x(x-4)(x+5)}$$

$x(x-4)(x+5)=0$

$x=0,\ 4,\ -5$ These are the discontinuities. –5 is removable and 0, 4 are nonremovable.

10A.) (4 pts) Find the limit (if possible): $\lim\limits_{x\to -5^+}\dfrac{x-3}{x^2+2x-15}$

10A. ∞

Section 1.8

$$\lim_{x\to -5^+}\frac{x-3}{x^2+2x-15} = \lim_{x\to -5^+}\frac{x-3}{(x-3)(x+5)} = \lim_{x\to -5^+}\frac{1}{x+5}$$

No matter what, we will get division by zero. So test a number slight larger than –5, like –4.999. Plug this into $\dfrac{1}{x+5}$. You will get $\dfrac{1}{-4.999+5}=1000$. So the closer you get to –5 (from the positive side) the larger the numbers get. Therefore $\lim\limits_{x\to -5^+}\dfrac{x-3}{x^2+2x-15}=\infty$.

10B.) (4 pts) Find the limit (if possible): $\lim\limits_{x\to -10^-}-\dfrac{2x^2+13x+15}{2x^2+23x+30}$

10B. $-\infty$

Section 1.8

$$\lim_{x\to -10^-}-\frac{2x^2+13x+15}{2x^2+23x+30} = \lim_{x\to -10^-}-\frac{(2x+3)(x+5)}{(2x+3)(x+10)} = \lim_{x\to -10^-}-\frac{x+5}{x+10}$$

No matter what, we will get division by zero. So test a number slight smaller than –10, like –10.999. Plug this into $\dfrac{x+5}{x+10}$. You will get $-\dfrac{-10.001+5}{-10.001+10}=-\dfrac{-5.001}{-0.001}=-5001$. So the closer you get to –10 (from the negative side) the larger negative numbers you get. Therefore $\lim\limits_{x\to -10^-}-\dfrac{2x^2+13x+15}{2x^2+23x+30}=-\infty$.

11A.) (4 pts) Find the limit (if possible): $\lim_{y\to 4^-} \frac{y+4}{y^2-10y-24}$ 11A. $-\frac{1}{6}$

Section 1.8

For this one, plugging in 4 does not result in division by zero. Therefore, we can immediately evaluate the limit: $\lim_{y\to 4^-} \frac{y+4}{y^2-10y-24} = \frac{4+4}{4^2-10(4)-24} = \frac{8}{-48} = -\frac{1}{6}$

11B.) (4 pts) Find the limit (if possible): $\lim_{\theta\to 0^+} \frac{6\sin\theta - 1}{\cos\theta + 1}$ 11B. $-\frac{1}{2}$

Section 1.8

For this one, plugging in 0 does not result in division by zero. Therefore, we can immediately evaluate the limit: $\lim_{\theta\to 0^+} \frac{6\sin\theta - 1}{\cos\theta + 1} = \frac{6\sin(0)-1}{\cos(0)+1} = \frac{0-1}{1+1} = -\frac{1}{2}$

12A.) (4 pts) Find the exact value: $\lim_{\theta\to\infty} \cos\left(\frac{\sin\theta}{\theta}\right)$ 12A. 1

Section 1.8

$$\lim_{\theta\to\infty} \cos\left(\frac{\sin\theta}{\theta}\right) = \lim_{\theta\to\infty} \cos\left(\lim_{\theta\to\infty} \frac{\sin\theta}{\theta}\right) = \lim_{\theta\to\infty} \cos(0) = 1$$

12B.) (4 pts) Find the exact value: $\lim_{x\to\infty} \frac{x^4-3x^5}{7x^5-3x+\sin(x^5)}$ 12B. $-\frac{3}{7}$

Section 1.8

$$\lim_{x\to\infty} \frac{x^4-3x^5}{7x^5-3x+\sin(x^5)} = \lim_{x\to\infty} \frac{\frac{x^4}{x^5}-\frac{3x^5}{x^5}}{\frac{7x^5}{x^5}-\frac{3x}{x^5}+\frac{\sin(x^5)}{x^5}} = \lim_{x\to\infty} \frac{\frac{1}{x}-3}{7-\frac{3}{x^4}+\frac{\sin(x^5)}{x^5}} = \lim_{x\to\infty} \frac{0-3}{7-0+0} = -\frac{3}{7}$$

13A.) (4 pts) Find the following infinite limits (if possible): $\lim_{w\to\infty}\frac{\sqrt{5+36w^2}}{3w-4}$ 13A. 2

Section 1.8

If $w\to\infty$ then let $w=\sqrt{w^2}$. Divide the top by $\sqrt{w^2}$ and the bottom by w.

$$\lim_{w\to\infty}\frac{\frac{\sqrt{5+36w^2}}{w^2}}{\frac{3w-4}{w}} = \lim_{w\to\infty}\frac{\sqrt{\frac{5+36w^2}{w^2}}}{\frac{3w}{w}-\frac{4}{w}} = \lim_{w\to\infty}\frac{\sqrt{\frac{5}{w^2}+\frac{36w^2}{w^2}}}{3-\frac{4}{w}} = \lim_{w\to\infty}\frac{\sqrt{\frac{5}{w^2}+36}}{3-\frac{4}{w}} = \frac{\sqrt{0+36}}{3-0} = \frac{\sqrt{36}}{3}=2$$

13B.) (4 pts) Find the following infinite limits (if possible): $\lim_{x\to-\infty}\frac{10x-1}{\sqrt{4x+5x^2}}$ 13B. $-2\sqrt{5}$

Section 1.8

If $x\to-\infty$ then let $x=-\sqrt{x^2}$. Divide the top by $-\sqrt{x^2}$ and the bottom by x.

$$\lim_{x\to-\infty}\frac{\frac{10x-1}{x}}{\frac{\sqrt{4x+5x^2}}{-\sqrt{x^2}}} = \lim_{x\to-\infty}\frac{\frac{10x}{x}-\frac{1}{x}}{-\sqrt{\frac{4x+5x^2}{x^2}}} = \lim_{x\to-\infty}\frac{10-\frac{1}{x}}{-\sqrt{\frac{4x}{x^2}+\frac{5x^2}{x^2}}} = \lim_{x\to-\infty}\frac{10-\frac{1}{x}}{-\sqrt{\frac{4}{x}+5}} = \frac{10-0}{-\sqrt{0+5}} = \frac{10}{-\sqrt{5}}=-2\sqrt{5}$$

14A.) (5 pts) Find $\frac{dy}{dx}$ if $y=\frac{1}{\sqrt{x-1}}$. Use $\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$. 14A. $\frac{-1}{2(x-1)^{\frac{3}{2}}}$

Section 2.2

$$\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$

$$\lim_{h\to0}\frac{\frac{1}{\sqrt{x+h-1}}-\frac{1}{\sqrt{x-1}}}{h}$$

$$\lim_{h\to0}\frac{\frac{1}{\sqrt{x+h-1}}\cdot\left(\frac{\sqrt{x-1}}{\sqrt{x-1}}\right)-\frac{1}{\sqrt{x-1}}\cdot\left(\frac{\sqrt{x+h-1}}{\sqrt{x+h-1}}\right)}{h}$$

$$\lim_{h\to0}\frac{\sqrt{x-1}-\sqrt{x+h-1}}{\sqrt{x-1}\sqrt{x+h-1}}\cdot\left(\frac{1}{h}\right)$$

$$\lim_{h\to0}\frac{\sqrt{x-1}-\sqrt{x+h-1}}{h\sqrt{x-1}\sqrt{x+h-1}}\cdot\left(\frac{\sqrt{x-1}+\sqrt{x+h-1}}{\sqrt{x-1}+\sqrt{x+h-1}}\right)$$

$$\lim_{h\to0}\frac{x-1-(x+h-1)}{h\sqrt{x-1}\sqrt{x+h-1}\left(\sqrt{x-1}+\sqrt{x+h-1}\right)}$$

$$\lim_{h\to0}\frac{x-1-x-h+1}{h\sqrt{x-1}\sqrt{x+h-1}\left(\sqrt{x-1}+\sqrt{x+h-1}\right)}$$

$$\lim_{h\to0}\frac{-h}{h\sqrt{x-1}\sqrt{x+h-1}\left(\sqrt{x-1}+\sqrt{x+h-1}\right)}$$

$$\lim_{h\to0}\frac{-1}{\sqrt{x-1}\sqrt{x+h-1}\left(\sqrt{x-1}+\sqrt{x+h-1}\right)}$$

$$\frac{-1}{\sqrt{x-1}\sqrt{x+0-1}\left(\sqrt{x-1}+\sqrt{x+0-1}\right)}$$

$$\frac{-1}{(x-1)\left(2\sqrt{x-1}\right)} \quad \text{or} \quad \frac{-1}{2(x-1)^{\frac{3}{2}}}$$

14B.) (5 pts) Find $\frac{dy}{dx}$ if $y=3-4x^2$. Use $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$.

14B. $\frac{dy}{dx}=-8x$

Section 2.2

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\lim_{h\to 0}\frac{3-4(x+h)^2-(3-4x^2)}{h}$$

$$\lim_{h\to 0}\frac{3-4(x^2+2xh+h^2)-3+4x^2}{h}$$

$$\lim_{h\to 0}\frac{3-4x^2-8xh-4h^2-3+4x^2}{h}$$

$$\lim_{h\to 0}\frac{-8xh-4h^2}{h}$$

$$\lim_{h\to 0}\frac{h(-8x-4h)}{h}$$

$$\lim_{h\to 0}(-8x-4h)=-8x-4(0)=-8x$$

$$\frac{dy}{dx}=-8x$$