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MATH 181 TEST 2 SAMPLE TEST

NOTE: The actual exam will only have 13 questions. The different parts of each question (part A, B, etc.) are variations. Know how to do all the variations on this exam.

1A.) (4 pts) Find the derivative: $y=\frac{x^3}{3}+4\sqrt{x}-3e^{-x}+5\sin x-\frac{1}{\pi+1}$

1A. $y'=x^2+\frac{2}{\sqrt{x}}+\frac{3}{e^x}+5\cos x$

Section 3.3, 3.5

$$y'=x^2+4\cdot\frac{1}{2}x^{-\frac{1}{2}}-3\cdot\left(-e^{-x}\right)+5\cos x-0$$
$$y'=x^2+\frac{2}{\sqrt{x}}+\frac{3}{e^x}+5\cos x$$

1B.) (4 pts) Find the derivative: $y=\frac{x^4}{4}-6\cdot\sqrt[3]{x}+7e^x-3\cos x-\frac{2\sqrt{7}}{e^2}$

1B. $y'=x^3-\frac{2}{x^{\frac{2}{3}}}+7e^x+3\sin x$

Section 3.3, 3.5

$$y'=x^3-6\cdot\frac{1}{3}x^{-\frac{2}{3}}+7e^x-3(-\sin x)-0$$
$$y'=x^3-\frac{2}{x^{\frac{2}{3}}}+7e^x+3\sin x$$

2A.) (4 pts) Find the velocity and acceleration functions **Section 3.3, 3.4**

$v(t)=4-\frac{1}{t^{\frac{3}{2}}}$

given: $s(t)=\frac{20t^3-2t^2+10\sqrt{t^3}}{5t^2}$

$$s(t)=\frac{20t^3}{5t^2}-\frac{2t^2}{5t^2}+\frac{10t^{\frac{3}{2}}}{5t^2}$$

$a(t)=\frac{3}{2t^{\frac{5}{2}}}$

$$s(t)=4t-\frac{2}{5}+2t^{-\frac{1}{2}}$$
$$v(t)=4-0-t^{-\frac{3}{2}}$$
$$a(t)=-\frac{3}{2}t^{-\frac{5}{2}}$$

2B.) (4 pts) Find the velocity and acceleration functions

given: $s(t)=\dfrac{10t^2-3t+5\sqrt{t}}{\sqrt[5]{t}}$ **Section 3.3, 3.4**

$v(t)=18t^{\frac{4}{5}}-\dfrac{12}{5t^{\frac{1}{5}}}+\dfrac{3}{2t^{\frac{7}{10}}}$

$$s(t)=\frac{10t^2-3t+5\sqrt{t}}{\sqrt[5]{t}}$$

$$s(t)=\frac{10t^2}{t^{\frac{1}{5}}}-\frac{3t}{t^{\frac{1}{5}}}+\frac{5t^{\frac{1}{2}}}{t^{\frac{1}{5}}}$$

$$s(t)=10t^{\frac{9}{5}}-3t^{\frac{4}{5}}+5t^{\frac{3}{10}}$$

$$v(t)=18t^{\frac{4}{5}}-\frac{12}{5}t^{-\frac{1}{5}}+\frac{3}{2}t^{-\frac{7}{10}}$$

$$a(t)=\frac{72}{5}t^{-\frac{1}{5}}+\frac{12}{25}t^{-\frac{6}{5}}-\frac{21}{20}t^{-\frac{17}{10}}$$

$a(t)=\dfrac{72}{5t^{\frac{1}{5}}}+\dfrac{12}{25t^{\frac{6}{5}}}-\dfrac{21}{20t^{\frac{17}{10}}}$

3A.) (4 pts) Find the derivative of $f(x)=\dfrac{3x-1}{2-x^2}$ using the quotient rule.

3A. $f'(x)=\dfrac{3x^2-2x+6}{\left(2-x^2\right)^2}$

Section 3.3

$$f'(x)=\frac{\left(2-x^2\right)(3)-(3x-1)(-2x)}{\left(2-x^2\right)^2}$$

$$f'(x)=\frac{6-3x^2+6x^2-2x}{\left(2-x^2\right)^2}$$

$$f'(x)=\frac{3x^2-2x+6}{\left(2-x^2\right)^2}$$

3B.) (4 pts) Find the derivative of $f(x)=\dfrac{4x}{27-x^3}$ using the quotient rule.

3B. $f'(x)=\dfrac{8x^3+108}{\left(27-x^3\right)^2}$

Section 3.3

$$f'(x)=\frac{\left(27-x^3\right)(4)-(4x)\left(-3x^2\right)}{\left(27-x^3\right)^2}$$

$$f'(x)=\frac{108-4x^3+12x^3}{\left(27-x^3\right)^2}$$

$$f'(x)=\frac{8x^3+108}{\left(27-x^3\right)^2}$$

4A.) (5 pts) Find the derivative of $f(x)=\left(\dfrac{2}{x-1}\right)^3$.

4A. $f'(x)=-\dfrac{24}{(x-1)^4}$

Section 3.6

$$f(x)=\frac{8}{(x-1)^3}=8(x-1)^{-3}$$

$$f'(x)=-24(x-1)^{-4}(1)$$

$$f'(x)=-\frac{24}{(x-1)^4}$$

4B.) (5 pts) Find the derivative of $f(x)=\dfrac{2x}{\sqrt[4]{2x-5}}$.

4B. $f'(x)=\dfrac{2}{(2x-5)^{\frac{1}{4}}}-\dfrac{x}{(2x-5)^{\frac{5}{4}}}$

or $f'(x)=\dfrac{3x-10}{(2x-5)^{\frac{5}{4}}}$

Section 3.3, 3.6

$$f(x)=\frac{2x}{\sqrt[4]{2x-5}}=\frac{2x}{(2x-5)^{\frac{1}{4}}}$$

$$f'(x)=\frac{(2x-5)^{\frac{1}{4}}(2)-(2x)\cdot\frac{1}{4}(2x-5)^{-\frac{3}{4}}(2)}{\left((2x-5)^{\frac{1}{4}}\right)^2}$$

$$f'(x)=\left(2(2x-5)^{\frac{1}{4}}-\frac{x}{(2x-5)^{\frac{3}{4}}}\right)\cdot\frac{1}{(2x-5)^{\frac{1}{2}}}$$ Distribute.

$$f'(x)=\frac{2}{(2x-5)^{\frac{1}{4}}}-\frac{x}{(2x-5)^{\frac{5}{4}}}$$ This answer is okay. If you want to continue:

$$f'(x)=\frac{2}{(2x-5)^{\frac{1}{4}}}\cdot\left(\frac{2x-5}{2x-5}\right)-\frac{x}{(2x-5)^{\frac{5}{4}}}$$

$$f'(x)=\frac{4x-10-x}{(2x-5)^{\frac{5}{4}}}=\frac{3x-10}{(2x-5)^{\frac{5}{4}}}$$

5A.) (4 pts) Find y'' if $y=\tan x$.

5A. $y''=2\sec^2 x\tan x$

Section 3.3, 3.5, 3.6

$$y'=\sec^2 x=(\sec x)^2$$

$$y''=2\sec x\cdot\sec x\tan x$$

$$y''=2\sec^2 x\tan x$$

5B.) (4 pts) Find y'' if $y = -\frac{1}{2}\cos\left(x^2\right)$.

5B. $y'' = 2x^2\cos\left(x^2\right) + \sin\left(x^2\right)$

Section 3.3, 3.5, 3.6

$$y' = -\frac{1}{2}\left(-\sin\left(x^2\right)\right)(2x)$$
$$y' = x \cdot \sin\left(x^2\right)$$

$$y'' = x \cdot \cos\left(x^2\right)(2x) + \sin\left(x^2\right)(1)$$
$$y'' = 2x^2\cos\left(x^2\right) + \sin\left(x^2\right)$$

6A.) (5 pts) Find the derivative of $f(\theta) = 4\sqrt{\sin\left(\frac{\theta}{2}\right)}$.

6A. $f'(\theta) = \dfrac{\cos\left(\frac{\theta}{2}\right)}{\sqrt{\sin\left(\frac{\theta}{2}\right)}}$

Section 3.5, 3.6

$$f(\theta) = 4\left(\sin\left(\frac{1}{2}\theta\right)\right)^{\frac{1}{2}}$$

$$f'(\theta) = 2\left(\sin\left(\frac{1}{2}\theta\right)\right)^{-\frac{1}{2}} \cdot \cos\left(\frac{1}{2}\theta\right) \cdot \frac{1}{2}$$

$$f'(\theta) = \frac{\cos\left(\frac{1}{2}\theta\right)}{\sqrt{\sin\left(\frac{1}{2}\theta\right)}}$$

6B.) (5 pts) Find the derivative of $f(x) = \cos^4\left(x^2 - 3\right)$.

6B. $f'(x) = -8x\left(\sin\left(x^2 - 3\right)\right)\cos^3\left(x^2 - 3\right)$

Section 3.5, 3.6

$$f(x) = \left(\cos\left(x^2 - 3\right)\right)^4$$

$$f'(x) = 4\left(\cos\left(x^2 - 3\right)\right)^3 \cdot \left(-\sin\left(x^2 - 3\right)\right) \cdot (2x)$$

$$f'(x) = -8x\left(\sin\left(x^2 - 3\right)\right)\left(\cos\left(x^2 - 3\right)\right)^3$$

7A.) (6 pts) Find the derivative of $y = 3x^7\sqrt{x^2-3x}$ using product rules and chain rules.

7A. $y' = \dfrac{3x^7(2x-3)}{2\sqrt{x^2-3x}} + 21x^6\sqrt{x^2-3x}$

Section 3.3, 3.6

$$y' = 3x^7 \cdot \frac{1}{2}\left(x^2-3x\right)^{-\frac{1}{2}}(2x-3) + \sqrt{x^2-3x}\left(21x^6\right)$$

$$y' = \frac{3x^7(2x-3)}{2\sqrt{x^2-3x}} + 21x^6\sqrt{x^2-3x}$$

7B.) (6 pts) Find the derivative of $f(x) = \left(5x^6-2x\right)\cot^3 x$ using product rules and chain rules.

Section 3.3, 3.6

$$f'(x) = \left(5x^6-2x\right)\cdot 3\cot^2 x\left(-\csc^2 x\right) + \cot^3 x \cdot \left(30x^5-2\right)$$

$$f'(x) = -3\left(5x^6-2x\right)\cdot \cot^2 x \cdot \csc^2 x + \left(30x^5-2\right)\cot^3 x$$ ←ANSWER

8A.) (5 points) Use $4x^2 + y^2 - 2xy - 7 = 0$ for the following questions:

Section 3.7

i.) Find $\dfrac{dy}{dx}$ by implicit differentiation.

i. $\dfrac{dy}{dx} = \dfrac{y-4x}{y-x}$

$$8x + 2y\frac{dy}{dx} - \left(2x\frac{dy}{dx} + y(2)\right) = 0$$

$$4x + y\frac{dy}{dx} - x\frac{dy}{dx} - y = 0$$

$$y\frac{dy}{dx} - x\frac{dy}{dx} = y - 4x$$

$$\frac{dy}{dx}\left(y-x\right) = y - 4x$$

$$\frac{dy}{dx} = \frac{y-4x}{y-x}$$

ii.) Use your answer in part a to find the **equation** of the tangent line at the point (1, 3) on the curve.

ii. $y = -\frac{1}{2}x + \frac{7}{2}$

$$\frac{dy}{dx} = \frac{y-4x}{y-x} = \frac{3-4(1)}{3-1} = \frac{-1}{2} = m$$

$3 = -\frac{1}{2}(1) + b$, so $\frac{7}{2} = b$ Then $y = -\frac{1}{2}x + \frac{7}{2}$

8B.) (5 points) Use $\sin xy = x + y$ for the following questions:

Section 3.7

i.) Find $\frac{dy}{dx}$ by implicit differentiation.

i. $\frac{dy}{dx} = \frac{1 - y\cos(xy)}{x\cos(xy) - 1}$

$$\cos(xy) \cdot \left(x\frac{dy}{dx} + y(1) \right) = 1 + \frac{dy}{dx}$$

$$x\cos(xy)\frac{dy}{dx} + y\cos(xy) = 1 + \frac{dy}{dx}$$

$$x\cos(xy)\frac{dy}{dx} - \frac{dy}{dx} = 1 - y\cos(xy)$$

$$\frac{dy}{dx}(x\cos(xy) - 1) = 1 - y\cos(xy)$$

$$\frac{dy}{dx} = \frac{1 - y\cos(xy)}{x\cos(xy) - 1}$$

ii.) Use your answer in part a to find the **equation** of the tangent line at the point (0, 0) on the curve.

ii. $y = -x$

$$\frac{dy}{dx} = \frac{1 - y\cos(xy)}{x\cos(xy) - 1} = \frac{1 - 0\cos(0 \cdot 0)}{0\cos(0 \cdot 0) - 1} = -1 = m$$

$0 = -1(0) + b$, so $0 = b$ Then $y = -x$

9A.) (4 pts) Find the derivative of $f(x)=\ln\left(3x-4e^{x^{-2}}\right)$.

9A. $f'(x)=\dfrac{3x^3+8e^{\frac{1}{x^2}}}{x^3\left(3x-4e^{\frac{1}{x^2}}\right)}$

Section 3.8

$$f'(x)=\frac{3-4e^{x^{-2}}(-2x^{-3})}{3x-4e^{x^{-2}}}$$

$$f'(x)=\frac{3+\dfrac{8e^{\frac{1}{x^2}}}{x^3}}{3x-4e^{\frac{1}{x^2}}}$$

$$f'(x)=\frac{\dfrac{3x^3+8e^{\frac{1}{x^2}}}{x^3}}{3x-4e^{\frac{1}{x^2}}}=\frac{3x^3+8e^{\frac{1}{x^2}}}{x^3\left(3x-4e^{\frac{1}{x^2}}\right)}$$

9B.) (4 pts) Find the derivative of $f(x)=\ln\left(\sec x-2e^{3x}\right)$.

9B. $f'(x)=\dfrac{\sec x\tan x-6e^{3x}}{\sec x-2e^{3x}}$

Section 3.8

$$f'(x)=\frac{\sec x\tan x-2e^{3x}(3)}{\sec x-2e^{3x}}$$

$$f'(x)=\frac{\sec x\tan x-6e^{3x}}{\sec x-2e^{3x}}$$

10A.) (5 pts) Find the derivative of $y=\ln\sqrt{\dfrac{2x-3}{x-4}}$.

10A. $y'=\dfrac{1}{2x-3}-\dfrac{1}{2(x-4)}$

Section 3.8

$$y=\ln\left(\frac{2x-3}{x-4}\right)^{\frac{1}{2}}=\frac{1}{2}\ln\left(\frac{2x-3}{x-4}\right)=\frac{1}{2}\left[\ln(2x-3)-\ln(x-4)\right]=\frac{1}{2}\ln(2x-3)-\frac{1}{2}\ln(x-4)$$

$$y=\frac{1}{2}\ln(2x-3)-\frac{1}{2}\ln(x-4)$$

$$y'=\frac{1}{2}\cdot\frac{2}{2x-3}-\frac{1}{2}\cdot\frac{1}{x-4}$$

$$y'=\frac{1}{2x-3}-\frac{1}{2(x-4)}$$

You do not need to get common denominators.

10B.) (5 pts) Find the derivative of $y = \ln \frac{(x^3-1)^4 \cdot \sqrt{3x-1}}{x^2+4}$.

10B. $y' = \frac{12x^2}{x^3-1} + \frac{3}{2(3x-1)} - \frac{2x}{x^2-4}$

Section 3.8

$$y = \ln \frac{(x^3-1)^4 \cdot (3x-1)^{\frac{1}{2}}}{x^2+4} = \ln(x^3-1)^4 + \ln(3x-1)^{\frac{1}{2}} - \ln(x^2+4)$$

$$y = 4\ln(x^3-1) + \frac{1}{2}\ln(3x-1) - \ln(x^2+4)$$

$$y' = 4 \cdot \frac{3x^2}{x^3-1} + \frac{1}{2} \cdot \frac{3}{3x-1} - \frac{2x}{x^2-4}$$

$y' = \frac{12x^2}{x^3-1} + \frac{3}{2(3x-1)} - \frac{2x}{x^2-4}$ You do not need to get common denominators.

11A.) (4 pts) Find the derivative of $y = 3x \cdot 2^{-5x}$.

11A. $y' = 3 \cdot 2^{-5x}[1 - 5x\ln 2]$ or

$y' = \frac{3 \cdot (1-5x\ln 2)}{2^{5x}}$

$$y' = 3x \cdot 2^{-5x}(-5)\ln 2 + 2^{-5x}(3)$$

$$y' = 3 \cdot 2^{-5x}[-5x\ln 2 + 1]$$

Section 3.8

or $y' = \frac{3 \cdot (1-5x\ln 2)}{2^{5x}}$

11B.) (4 pts) Find the derivative of $y = \frac{x^3}{3^x}$.

11B. $y' = \frac{x^2 3^x (3 - x\ln 3)}{3^{2x}}$ or

$y' = \frac{x^2(3 - x\ln 3)}{3^x}$

$$y' = \frac{3^x(3x^2) - x^3(3^x \ln 3)}{(3^x)^2}$$

$y' = \frac{x^2 3^x (3 - x\ln 3)}{3^{2x}}$ or

Section 3.8

$$y' = \frac{x^2(3 - x\ln 3)}{3^x}$$

12A.) (6 pts) Use logarithmic differentiation to find the derivative of: $y=\left(\frac{4\theta\cdot\sin\theta}{1-\cos\theta}\right)^{\frac{1}{4}}$

$$\ln y=\ln\left(\frac{4\theta\cdot\sin\theta}{1-\cos\theta}\right)^{\frac{1}{4}}$$

$$\ln y=\frac{1}{4}\left[\ln 4\theta+\ln\sin\theta-\ln(1-\cos\theta)\right]$$

$$\ln y=\frac{1}{4}\ln 4\theta+\frac{1}{4}\ln\sin\theta-\frac{1}{4}\ln(1-\cos\theta)$$

$$\frac{y'}{y}=\frac{1}{4}\cdot\frac{4}{4\theta}+\frac{1}{4}\cdot\frac{\cos\theta}{\sin\theta}-\frac{1}{4}\cdot\frac{\sin\theta}{1-\cos\theta}$$

$$y'=y\left(\frac{1}{4\theta}+\frac{1}{4}\cot\theta-\frac{\sin\theta}{4(1-\cos\theta)}\right)$$

$$y'=\left(\frac{4\theta\cdot\sin\theta}{1-\cos\theta}\right)^{\frac{1}{4}}\left(\frac{1}{4\theta}+\frac{1}{4}\cot\theta-\frac{\sin\theta}{4(1-\cos\theta)}\right)$$

12A. $y'=\left(\frac{4\theta\cdot\sin\theta}{1-\cos\theta}\right)^{\frac{1}{4}}\left(\frac{1}{4\theta}+\frac{1}{4}\cot\theta-\frac{\sin\theta}{4(1-\cos\theta)}\right)$

Section 3.8

12B.) (6 pts) Use logarithmic differentiation to find the derivative of: $y=\theta^{\csc\theta}$

$$\ln y=\ln\theta^{\csc\theta}$$

$$\ln y=\csc\theta\cdot\ln\theta$$

$$\frac{y'}{y}=\csc\theta\cdot\frac{1}{\theta}+\ln\theta(-\csc\theta\cot\theta)$$

$$y'=y\left(\frac{\csc\theta}{\theta}-\ln\theta\csc\theta\cot\theta\right)$$

$$y'=\theta^{\csc\theta}\left(\frac{\csc\theta}{\theta}-\ln\theta\csc\theta\cot\theta\right)$$

12B. $y'=\theta^{\csc\theta}\left(\frac{\csc\theta}{\theta}-\ln\theta\csc\theta\cot\theta\right)$

Section 3.8

13A.) (4 pts) Find the derivative of $y = \ln 3 \cdot \log_3(\tan x)$ and simplify.

13A. $y' = \dfrac{\sec^2 x}{\tan x}$ or $y' = \dfrac{1}{\sin x \cos x}$

Section 3.8

$$y' = \ln 3 \cdot \frac{\sec^2 x}{\tan x(\ln 3)}$$

$$y' = \frac{\sec^2 x}{\tan x} \quad \text{or} \quad y' = \frac{\frac{1}{\cos^2 x}}{\frac{\sin x}{\cos}} = \frac{1}{\sin x \cos x}$$

13B.) (4 pts) Find the derivative of $y = \log_4\left(\dfrac{3x-5}{2x+7}\right)$ and simplify.

13B. $y' = \dfrac{3}{(3x-5)\ln 4} - \dfrac{2}{(2x+7)\ln 4}$

or $y' = \dfrac{3}{2(3x-5)\ln 2} - \dfrac{1}{(2x+7)\ln 2}$

Section 3.8

$$y' = \log_4(3x-5) - \log_4(2x+7)$$

$$y' = \frac{3}{(3x-5)\ln 4} - \frac{2}{(2x+7)\ln 4}$$

$$\text{or} \quad y' = \frac{3}{(3x-5)\ln 2^2} - \frac{2}{(2x+7)\ln 2^2} = \frac{3}{2(3x-5)\ln 2} - \frac{1}{(2x+7)\ln 2}$$