NAME:___________KEY______________________

MATH 181 TEST 4 SAMPLE

NOTE: The actual exam will only have 11 questions. The different parts of each question (part A, B, etc.) are variations. Know how to do all the variations on this exam.

1A.) (6 pts) Find the indefinite integral: $\int\left(\frac{9+4y^3}{\sqrt[3]{y^2}}\right)dy$

1A. $27y^{\frac{1}{3}}+\frac{6}{5}y^{\frac{10}{3}}+C$

$$\int\frac{9}{y^{\frac{2}{3}}}+\frac{4y^3}{y^{\frac{2}{3}}}dy \quad\Rightarrow\quad \int 9y^{-\frac{2}{3}}+4y^{\frac{7}{3}}\ dy \quad\Rightarrow\quad 9\frac{y^{\frac{1}{3}}}{\frac{1}{3}}+4\frac{y^{\frac{10}{3}}}{\frac{10}{3}}+C \quad\Rightarrow\quad 27y^{\frac{1}{3}}+\frac{6}{5}y^{\frac{10}{3}}+C$$

1B.) (6 pts) Find the indefinite integral: $\int\left(\frac{1-5\sqrt{x}+6x^{-\frac{1}{2}}}{\sqrt{x}}\right)dx$

1B. $2\sqrt{x}-5x+6\ln x+C$

$$\int\frac{1}{x^{\frac{1}{2}}}-\frac{5x^{\frac{1}{2}}}{x^{\frac{1}{2}}}+\frac{6x^{-\frac{1}{2}}}{x^{\frac{1}{2}}}dx \quad\Rightarrow\quad \int x^{-\frac{1}{2}}-5+\frac{6}{x}\ dx \quad\Rightarrow\quad \frac{x^{\frac{1}{2}}}{\frac{1}{2}}-5x+6\ln x+C \quad\Rightarrow\quad 2x^{\frac{1}{2}}-5x+6\ln x+C$$

2A.) (6 pts) Use the **limit process** to find the area between the graph $y = 9 - x^2$ and the x-axis over [0, 3]. 2A. 18

$$\Delta x = \frac{3-0}{n} = \frac{3}{n} \qquad c_i = 0 + \frac{3}{n}i = \frac{3i}{n}$$

$$\lim_{x\to\infty}\sum_{i=1}^{n} f\left(\frac{3i}{n}\right)\cdot\frac{3}{n} \quad\Rightarrow\quad \lim_{x\to\infty}\sum_{i=1}^{n}\left(9-\left(\frac{3i}{n}\right)^2\right)\cdot\frac{3}{n} \quad\Rightarrow\quad \lim_{x\to\infty}\sum_{i=1}^{n}\left(9-\frac{9i^2}{n^2}\right)\cdot\frac{3}{n}$$

$$\Rightarrow\quad \lim_{x\to\infty}\sum_{i=1}^{n}\left(\frac{27}{n}-\frac{27i^2}{n^3}\right) \quad\Rightarrow\quad \lim_{x\to\infty}\left(\sum_{i=1}^{n}\frac{27}{n}-\sum_{i=1}^{n}\frac{27i^2}{n^3}\right) \quad\Rightarrow\quad \lim_{x\to\infty}\left(\frac{27}{n}\sum_{i=1}^{n}1-\frac{27}{n^3}\sum_{i=1}^{n}i^2\right)$$

$$\Rightarrow\quad \lim_{x\to\infty}\left(\frac{27}{n}\cdot n-\frac{27}{n^3}\cdot\frac{2n^3+3n^2+n}{6}\right) \quad\Rightarrow\quad \lim_{x\to\infty}\left(27-\frac{9}{n^3}\cdot\frac{2n^3+3n^2+n}{2}\right)$$

$$\Rightarrow\quad \lim_{x\to\infty}\left(27-\frac{18n^3+27n^2+9n}{2n^3}\right) \quad\Rightarrow\quad \lim_{x\to\infty}\left(27-9+\frac{27}{2n}+\frac{9}{2n^2}\right) \quad\Rightarrow\quad 27-9+0+0=18$$

2B.) (6 pts) Use the **limit process** to find the area between the graph 2B. $-\frac{1}{4}$

$y = \frac{1}{2}x - 1$ and the x-axis over [1, 2].

$$\Delta x = \frac{2-1}{n} = \frac{1}{n} \qquad c_i = 1 + \frac{1}{n}i = 1 + \frac{i}{n}$$

$$\lim_{x\to\infty}\sum_{i=1}^{n} f\left(1+\frac{i}{n}\right)\cdot\frac{1}{n} \quad\Rightarrow\quad \lim_{x\to\infty}\sum_{i=1}^{n}\left(\frac{1}{2}\left(1+\frac{i}{n}\right)-1\right)\cdot\frac{1}{n} \quad\Rightarrow\quad \lim_{x\to\infty}\sum_{i=1}^{n}\left(\frac{1}{2}+\frac{i}{2n}-1\right)\cdot\frac{1}{n}$$

$$\Rightarrow\quad \lim_{x\to\infty}\sum_{i=1}^{n}\left(\frac{i}{2n}-\frac{1}{2}\right)\cdot\frac{1}{n} \quad\Rightarrow\quad \lim_{x\to\infty}\sum_{i=1}^{n}\left(\frac{i}{2n^2}-\frac{1}{2n}\right) \quad\Rightarrow\quad \lim_{x\to\infty}\left(\frac{1}{2n^2}\sum_{i=1}^{n}i-\frac{1}{2n}\sum_{i=1}^{n}1\right)$$

$$\Rightarrow\quad \lim_{x\to\infty}\left(\frac{1}{2n^2}\cdot\frac{n^2+n}{2}-\frac{1}{2n}\cdot n\right) \quad\Rightarrow\quad \lim_{x\to\infty}\left(\frac{n^2+n}{4n^2}-\frac{1}{2}\right) \quad\Rightarrow\quad \lim_{x\to\infty}\left(\frac{n^2}{4n^2}+\frac{n}{4n^2}-\frac{1}{2}\right)$$

$$\Rightarrow\quad \lim_{x\to\infty}\left(\frac{1}{4}+\frac{1}{4n}-\frac{1}{2}\right) \quad\Rightarrow\quad \frac{1}{4}+0-\frac{1}{2}=-\frac{1}{4}$$

3A.) (6 pts) Find the indefinite integral:

$$\int \left(-4\csc^2(2\theta) + 14\cos(7\theta) - 2e^{5\theta}\right)d\theta$$

3A. $= 2\cot(2\theta) + 2\sin(7\theta) - \frac{2}{5}e^{5\theta} + C$

$$= 4\cdot\frac{1}{2}\cot(2\theta) + 14\cdot\frac{1}{7}\sin(7\theta) - 2\cdot\frac{1}{5}e^{5\theta} + C$$

$$= 2\cot(2\theta) + 2\sin(7\theta) - \frac{2}{5}e^{5\theta} + C$$

3B.) (6 pts) Find the indefinite integral:

$$\int \left(3\sec\left(\frac{\theta}{2}\right)\tan\left(\frac{\theta}{2}\right) - 8\sin(\pi\theta) + 2^{7\theta}\right)d\theta$$

3B. $6\sec\left(\frac{\theta}{2}\right) + \frac{8}{\pi}\cos(\pi\theta) + \frac{2^{7\theta}}{7\ln 2} + C$

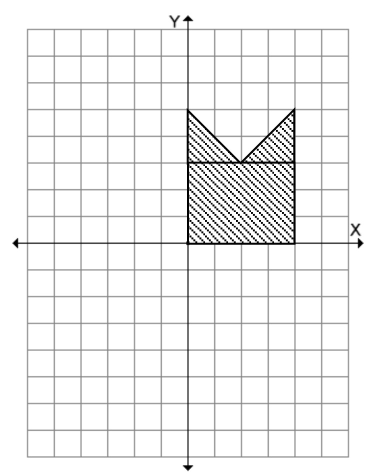
$$= 3\cdot\frac{1}{\frac{1}{2}}\sec\left(\frac{\theta}{2}\right) - 8\cdot -\frac{1}{\pi}\cos(\pi\theta) + \frac{1}{7\ln 2}2^{7\theta} + C$$

$$= 6\sec\left(\frac{\theta}{2}\right) + \frac{8}{\pi}\cos(\pi\theta) + \frac{2^{7\theta}}{7\ln 2} + C$$

4A.) (4 pts) Sketch the region whose area is $\int_0^4 \left(|x-2|+3\right)\,dx$. Then use **geometric formulas** to evaluate the integral.

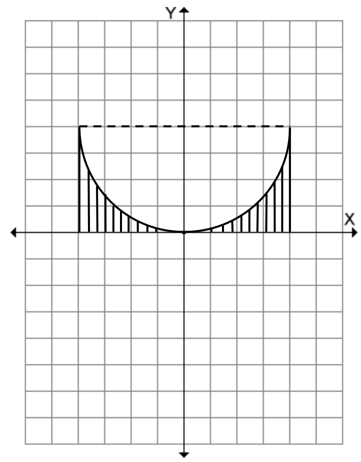
4A. 16

$$\textit{Total Area} = \frac{1}{2}(2)(2) + \frac{1}{2}(2)(2) + 4(3) = 16$$



4B.) (4 pts) Sketch the region whose area is $\int_{-4}^{4}\left(4-\sqrt{16-x^2}\right)dx$. 4B. $32-8\pi$

Then use **geometric formulas** to evaluate the integral.



$$Total\ \ Area = 8(4)-\frac{1}{2}\pi(4)^2 = 32-8\pi$$

5A.) (4 pts) Suppose *f* and *g* are integrable and that: $\int_0^3 f(x)dx = 4$, 5A. 19

$\int_0^3 g(x)dx = 9$. Use integral rules to find the following:

$\int_3^0 [2f(x)-3g(x)]dx + \int_3^3 f(x)\ dx$

$$2\int_3^0 f(x)\ dx - 3\int_3^0 g(x)\ dx + 0 \quad \Rightarrow \quad -2\int_0^3 f(x)\ dx + 3\int_0^3 g(x)\ dx \quad \Rightarrow \quad -2(4)+3(9)=19$$

5B.) (4 pts) Suppose *f* and *g* are integrable and that: $\int_{-1}^{1} f(x)dx = \frac{1}{2}$, 5B. -7

$\int_0^1 f(x)dx = -3$. Use integral rules to find $\int_0^{-1} 2f(x)dx$.

$$\int_{-1}^{0} f(x)dx + \int_0^1 f(x)dx = \int_{-1}^{1} f(x)dx \quad \Rightarrow \quad \int_{-1}^{0} f(x)dx + (-3) = \frac{1}{2} \quad \Rightarrow \quad \int_{-1}^{0} f(x)dx = \frac{7}{2}$$

$$\int_0^{-1} 2f(x)dx \quad \Rightarrow \quad 2\int_0^{-1} f(x)dx \quad \Rightarrow \quad -2\int_{-1}^{0} f(x)dx \quad \Rightarrow \quad -2\left(\frac{7}{2}\right) = -7$$

6A.) (4 pts) Find the **total area** between $y = 2x^3 - 2x^2 - 4x$ and the *x*-axis on [–1, 2]. 6A. $\frac{37}{6}$

First we need to set *y* equal to 0 to find out how we will break up the integral:

$$0 = 2x^3 - 2x^2 - 4x \quad \Rightarrow \quad 0 = 2x\left(x^2 - x - 2\right) \quad \Rightarrow \quad 0 = 2x(x+1)(x-2), \text{ so } x = 0, -1, 2.$$

So the intervals are [–1, 0] and [0, 2]. Next we want to do each integral separately:

$$\int_{-1}^{0} 2x^3 - 2x^2 - 4x \, dx$$
$$= \frac{1}{2}x^4 - \frac{2}{3}x^3 - 2x^2 \Big|_{-1}^{0}$$
$$= 0 - \left(\frac{1}{2}(-1)^4 - \frac{2}{3}(-1)^3 - 2(-1)^2\right)$$
$$= -\left(\frac{1}{2} + \frac{2}{3} - 2\right) = \quad = -\left(-\frac{5}{6}\right) = \frac{5}{6}$$

$$\int_{0}^{2} 2x^3 - 2x^2 - 4x \, dx$$
$$= \frac{1}{2}x^4 - \frac{2}{3}x^3 - 2x^2 \Big|_{0}^{2}$$
$$= \left(\frac{1}{2}(2)^4 - \frac{2}{3}(2)^3 - 2(2)^2\right) - 0$$
$$= \left(8 - \frac{16}{3} - 8\right) = -\frac{16}{3}$$

Finally we add the absolute values of these answers: $\left|\frac{5}{6}\right| + \left|-\frac{16}{3}\right| = \frac{5}{6} + \frac{16}{3} = \frac{37}{6}$

6B.) (4 pts) Find the **total area** between $y = x^3 - 9x$ and the *x*-axis on [–3, 3]. 6B. $\frac{81}{2}$

First we need to set *y* equal to 0 to find out how we will break up the integral:

$$0 = x^3 - 9x \quad \Rightarrow \quad 0 = x\left(x^2 - 9\right) \quad \Rightarrow \quad 0 = x(x+3)(x-3), \quad \text{so } x = 0, -3, 3.$$

So the intervals are [–3, 0] and [0, 3]. Next we want to do each integral separately:

$$\int_{-3}^{0} x^3 - 9x \, dx$$
$$= \frac{x^4}{4} - \frac{9x^2}{2} \Big|_{-3}^{0}$$
$$= 0 - \left(\frac{(-3)^4}{4} - \frac{9}{2}(-3)^2\right)$$
$$= -\left(\frac{81}{4} - \frac{81}{2}\right) = -\left(-\frac{81}{4}\right) = \frac{81}{4}$$

$$\int_{0}^{2} x^3 - 9x \, dx$$
$$= \frac{x^4}{4} - \frac{9x^2}{2} \Big|_{0}^{3}$$
$$= \left(\frac{(3)^4}{4} - \frac{9}{2}(3)^2\right) - 0$$
$$= \left(\frac{81}{4} - \frac{81}{2}\right) = -\frac{81}{4}$$

Finally we add the absolute values of these answers: $\left|\frac{81}{4}\right| + \left|-\frac{81}{4}\right| = \frac{81}{4} + \frac{81}{4} = \frac{81}{2}$

7A.) (6 pts) Find the indefinite integral: $\int \frac{y\sin(2y^2)}{1+\cos(2y^2)}\ dy$

7A. $-\frac{1}{4}\ln\left|1+\cos\left(2y^2\right)\right|+C$

Let $u = 1+\cos\left(2y^2\right)$

$du = -\sin\left(2y^2\right)\cdot 4y\ dy$

$\frac{du}{-4y\sin\left(2y^2\right)} = dy$

$\int \frac{y\sin(2y^2)}{u}\cdot\frac{du}{-4y\sin\left(2y^2\right)}$

$-\frac{1}{4}\int\frac{1}{u}\ du \quad\Rightarrow\quad -\frac{1}{4}\ln\left|1+\cos\left(2y^2\right)\right|+C$

7B.) (6 pts) Find the indefinite integral: $\int \frac{(x-2)(x+5)}{2x^3+9x^2-60x}\ dx$

7B. $\frac{1}{6}\ln\left|2x^3+9x^2-60x\right|+C$

Let $u = 2x^3+9x^2-60x$

$du = 6x^2+18x-60\ dx$

$\frac{du}{6(x-2)(x+5)} = dx$

$\int \frac{(x-2)(x+5)}{u}\cdot\frac{du}{6(x-2)(x+5)}$

$\frac{1}{6}\int\frac{1}{u}\ du \quad\Rightarrow\quad \frac{1}{6}\ln\left|2x^3+9x^2-60x\right|+C$

8A.) (6 pts) Find the indefinite integral: $\int (x+1)\sqrt{2-x}\ dx$

8A. $-2(2-x)^{\frac{3}{2}}+\frac{2}{5}(2-x)^{\frac{5}{2}}+C$

Let $u = 2-x \quad\Rightarrow\quad x = 2-u$

$du = -1\ dx$

$-du = dx$

$\int (x+1)\sqrt{u}\cdot -du \quad\Rightarrow\quad -\int((2-u)+1)\sqrt{u}\ du$

$-\int(3-u)u^{\frac{1}{2}}\ du \quad\Rightarrow\quad -\int 3u^{\frac{1}{2}}-u^{\frac{3}{2}}\ du$

$\Rightarrow\quad -3\frac{u^{\frac{3}{2}}}{\frac{3}{2}}+\frac{u^{\frac{5}{2}}}{\frac{5}{2}}+C \quad\Rightarrow\quad -2u^{\frac{3}{2}}+\frac{2}{5}u^{\frac{5}{2}}+C$

$\Rightarrow\quad -2(2-x)^{\frac{3}{2}}+\frac{2}{5}(2-x)^{\frac{5}{2}}+C$

8B.) (6 pts) Find the indefinite integral: $\int x^5\left(x^3-5\right)^4 \, dx$

8B. $\dfrac{\left(x^3-5\right)^6}{18}+\dfrac{\left(x^3-5\right)^5}{3}+C$

Let $u = x^3 - 5 \Rightarrow x^3 = u + 5$

$du = 3x^2 \, dx$

$\dfrac{du}{3x^2} = dx$

$$\int x^5 u^4 \cdot \frac{du}{3x^2} \Rightarrow \frac{1}{3}\int x^3 u^4 \, du \Rightarrow$$

$$\Rightarrow \frac{1}{3}\int (u+5)u^4 \, du \Rightarrow \frac{1}{3}\int u^5 + 5u^4 \, du$$

$$\Rightarrow \frac{1}{3}\cdot\frac{u^6}{6}+\frac{5}{3}\cdot\frac{u^5}{5}+C \Rightarrow \frac{u^6}{18}+\frac{u^5}{3}+C$$

$$\Rightarrow \frac{\left(x^3-5\right)^6}{18}+\frac{\left(x^3-5\right)^5}{3}+C$$

9A.) (6 pts) Find the indefinite integral: $\int \dfrac{3dx}{\sqrt{9-(2x-1)^2}}$

9A. $\dfrac{3}{2}\sin^{-1}\left(\dfrac{2x-1}{3}\right)+C$

Let $u = 2x - 1$

$du = 2 \, dx$

$\dfrac{du}{2} = dx$

$$3\int \frac{1}{\sqrt{3^2-u^2}}\cdot\frac{du}{2} \Rightarrow \frac{3}{2}\int \frac{du}{\sqrt{3^2-u^2}}$$

$$\Rightarrow \frac{3}{2}\sin^{-1}\left(\frac{u}{3}\right)+C \Rightarrow \frac{3}{2}\sin^{-1}\left(\frac{2x-1}{3}\right)+C$$

9B.) (6 pts) Find the indefinite integral: $\int \dfrac{dx}{x\ln x\sqrt{(\ln x)^2-4}}$

9B. $\dfrac{1}{2}\sec^{-1}\left(\dfrac{|\ln x|}{2}\right)+C$

Let $u = \ln x$

$du = \dfrac{1}{x} \, dx$

$x \, du = dx$

$$\int \frac{1}{x\cdot u\sqrt{u^2-2^2}}\cdot x \, du \Rightarrow \int \frac{du}{u\sqrt{u^2-2^2}}$$

$$\Rightarrow \frac{1}{2}\sec^{-1}\left(\frac{|u|}{2}\right)+C \Rightarrow \frac{1}{2}\sec^{-1}\left(\frac{|\ln x|}{2}\right)+C$$

10A.) (6 pts) Find the indefinite integral: $\int \sin(2x)\cdot\cot(\cos(2x))\,dx$ 10A. $-\frac{1}{2}\ln\left|\sin(\cos(2x))\right|+C$

Let $u=\cos(2x)$

$du=-2\sin(2x)\ dx$

$\frac{du}{-2\sin(2x)}=dx$

$$\int \sin(2x)\cdot\cot(u)\cdot\frac{du}{-2\sin(2x)} \Rightarrow -\frac{1}{2}\int \cot(u)\ du$$

$$\Rightarrow -\frac{1}{2}\ln\left|\sin u\right|+C \Rightarrow -\frac{1}{2}\ln\left|\sin(\cos(2x))\right|+C$$

10B.) (6 pts) Find the indefinite integral: $\int \frac{3}{\sqrt{2x-7}}\cdot\sec\left(\sqrt{2x-7}\right)dx$ Answer:

$3\ln\left|\sec\sqrt{2x-7}+\tan\sqrt{2x-7}\right|+C$

Let $u=\sqrt{2x-7}$

$du=\frac{1}{2}(2x-7)^{-\frac{1}{2}}(2)\ dx$

$du=\frac{1}{\sqrt{2x-7}}dx$

$\sqrt{2x-7}\cdot du=dx$

$$\int \frac{3}{\sqrt{2x-7}}\cdot\sec(u)\cdot\sqrt{2x-7}\cdot du \Rightarrow 3\int \sec(u)\ du$$

$$\Rightarrow 3\ln\left|\sec u+\tan u\right|+C \Rightarrow$$

$$3\ln\left|\sec\sqrt{2x-7}+\tan\sqrt{2x-7}\right|+C$$

11A.) (6 pts) Evaluate the definite integral $\int_0^3 \left(3x^2-10x+3\right)^3(24x-40)dx$ 11A. -81

Let $u=3x^2-10x+3$

$du=6x-10\ dx$

$\frac{du}{2(3x-5)}=dx$

$$\int u^3\cdot 8(3x-5)\cdot\frac{du}{2(3x-5)} \Rightarrow 4\int u^3\ du$$

$$\Rightarrow 4\frac{u^4}{4} \Rightarrow \left(3x^2-10x+3\right)^4\Big|_0^3$$

$$\Rightarrow \left(3(3)^2-10(3)+3\right)^4-\left(3(0)^2-10(0)+3\right)^4$$

$$= (0)^4-(3)^4 = -81$$

11B.) (6 pts) Evaluate the definite integral $\int_0^{\frac{\pi}{2}} (\sin x) e^{\cos x}\ dx$ 11A. $e-1$

Let $u = \cos x$

$du = -\sin x\ dx$

$\frac{du}{-\sin x} = dx$

$\int \sin x \cdot e^u \cdot \frac{du}{-\sin x} \Rightarrow -\int e^u\ du$

$\Rightarrow -e^{\cos x} \Rightarrow -e^{\cos x}\Big|_0^{\frac{\pi}{2}}$

$\Rightarrow -e^{\cos\frac{\pi}{2}} + e^{\cos 0} = -e^0 + e^1$

$= -1 + e$

11C.) (6 pts) Evaluate the definite integral $\int_1^e \frac{\ln\left(\sqrt{x}\right)}{x}\ dx$ 11B. $\frac{1}{4}$

Let $u = \ln\left(x^{\frac{1}{2}}\right) = \frac{1}{2}\ln x$

$du = \frac{1}{2x}\ dx$

$2x\ du = dx$

$\int \frac{u}{x} \cdot 2x\ du \Rightarrow 2\int u\ du \Rightarrow 2 \cdot \frac{u^2}{2}$

$\Rightarrow \left(\frac{1}{2}\ln x\right)^2\Big|_1^e \Rightarrow \left(\frac{1}{2}\ln e\right)^2 - \left(\frac{1}{2}\ln 1\right)^2$

$\Rightarrow \left(\frac{1}{2}(1)\right)^2 - \left(\frac{1}{2}(0)\right)^2 = \frac{1}{4} - 0 = \frac{1}{4}$