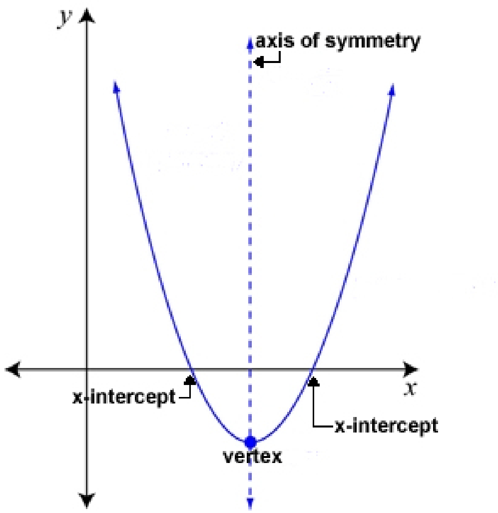
3.3 Quadratic Functions

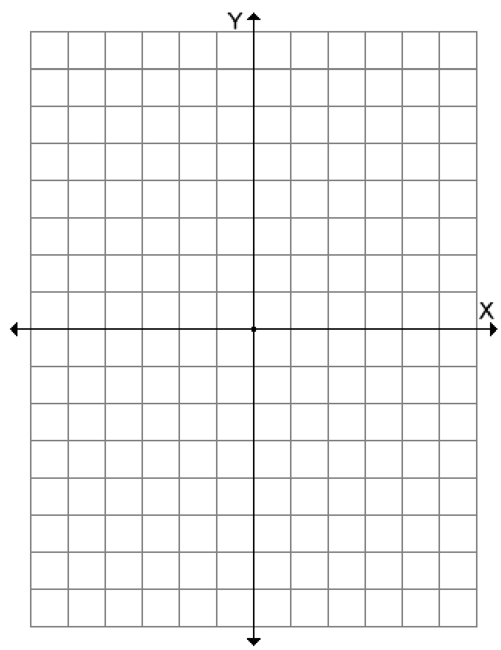
This section is all about quadratic functions, which give U shaped graphs called parabolas. First we need to define a couple of terms involving parabolas:



Vertex form: $y = a(x-h)^2 + k$.
Vertex: (h, k)

$\boldsymbol{a > 0}$ opens up, vertex is a minimum
$\boldsymbol{a < 0}$ opens down, vertex is a maximum

EXAMPLE: Find the vertex, axis of symmetry, intercepts and graph of $y = -2(x+3)^2 + 8$.



vertex:

axis of symmetry:

x-int: y=0

y-int: x=0

Vertex formula:

Given $y = ax^2 + bx + c$, then the x-coordinate of the vertex is $x = \frac{-b}{2a}$. To find the y-coordinate put the x back into the original equation.

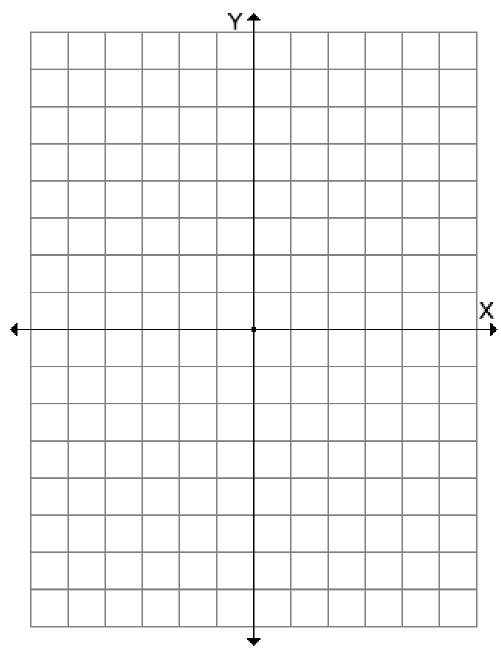
EXAMPLE: Find the vertex by using the formula: $y = 3x^2 + 6x + 1$.

EXAMPLE: Find the vertex by using the formula: $y = -2x^2 - 5x + 1$. Then find the axis of symmetry.

EXAMPLE: Determine, without graphing, whether the quadratic function $y = -2x^2 + 8x + 3$ has a maximum value or a minimum value and then find the value.

EXAMPLE: Find the intercepts, vertex, axis of symmetry, domain, range, and the graph of $y = x^2 - 6x + 5$.

vertex:



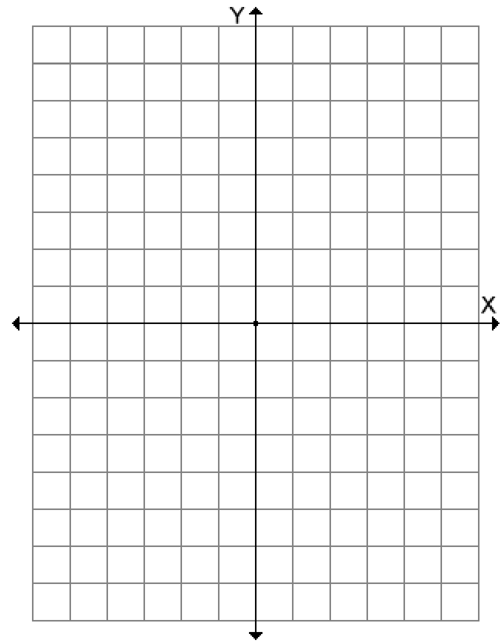
axis of symmetry:

x-int: y=0

y-int: x=0

EXAMPLE: Find the intercepts, vertex, axis of symmetry, domain, range, and the graph of $y = x^2 - 5$.

vertex:



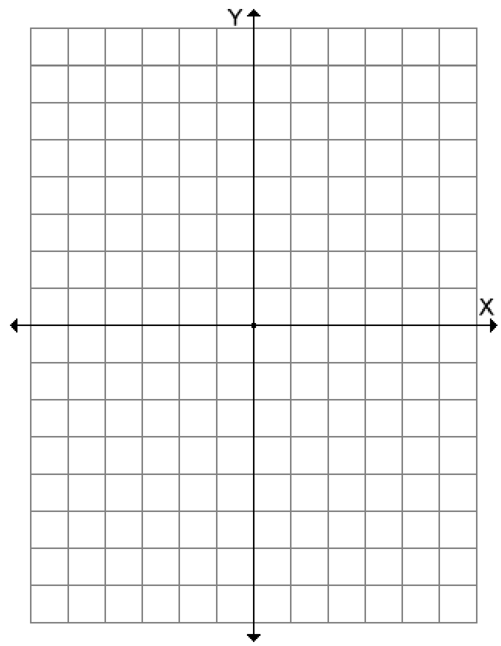
axis of symmetry:

x-int: y=0

y-int: x=0

EXAMPLE: Find the intercepts, vertex, axis of symmetry, domain, range, and the graph of $y = 6 - 4x + x^2$.

vertex:



axis of symmetry:

x-int: y=0

y-int: x=0

Modeling with Quadratic Functions

These problems involve finding maximum or minimum values. ALL of the problems in this section require you to find the vertex once you have your equation.

To find a maximum or minimum value you must find the vertex.

EXAMPLE: Suppose the height of an object shot straight up is given by $h = 512t - 16t^2$ where h is measured in feet and t is in seconds. Find the maximum height and the time at which the object hits the ground.

EXAMPLE: The daily revenue, R, achieved by selling x boxes of candy is figured to be $R(x) = 9.5x - 0.04x^2$. The daily cost, C, of selling x boxes of candy is $C(x) = 1.25x + 250$. How many boxes must be sold to maximize the profit? What will be the maximum profit?

EXAMPLE: Among all pairs of numbers whose difference is 24, find a pair whose product is as small as possible. What is the minimum product?

EXAMPLE: Suppose you have 900 ft of fencing and you want to enclose a rectangular field that borders a river, so only three sides require fencing. What is the largest area that can be enclosed?