4.1 Polynomial Functions

Polynomial Function: $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_o$

The n in the formula above is called the **degree**, and this is the largest exponent of the polynomial. A polynomial can only have whole number exponents (no negatives, fractions or decimals). A polynomial must also be a smooth line with no breaks or corners.

The a_n is always in the term with the degree. In other words, it is the number in front of the x with the highest power.

EXAMPLE: Indicate whether the following are polynomials. If they are, indicate the degree and the a_n.

a.) $f(x) = 3x - 2x^3 + \frac{x^2}{3}$

b.) $f(x) = \sqrt{x} - 5$

c.) $f(x) = \frac{5}{x^2}$

d.) $f(x) = 6$

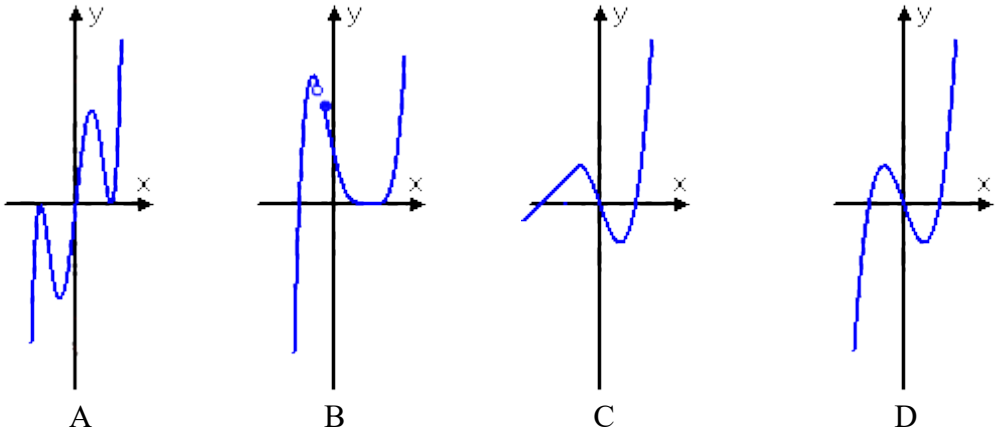
e.) $f(x) = (x-2)(x+5)$

Turning point – a point in which the graph changes directions. This happens at a peak or valley.

If n is the degree of a polynomial then the polynomial can have <u>at most</u> n – 1 turning points.

EXAMPLE: Up to how many turning points can $y = x^2 - x^5$ have?

EXAMPLE: Which of the following can be a degree 3 polynomial?



EXAMPLE: Form a degree 3 polynomial whose zeros are -2, 0, 2 passing through the point (–1,6).

EXAMPLE: Form a degree 3 polynomial whose zeros are -3, and 1 with a coefficient of 1.

The 2 that I added to either of those factors is called the **multiplicity**. *Multiplicity* is basically the power on each factored piece. Usually, you will indicate what the zero is and then classify its multiplicity.

At a zero with an even multiplicity, the graph ***touches*** the x-axis at that zero.
At a zero with an odd multiplicity, the graph ***crosses*** the x-axis at that zero.

EXAMPLE: Indicate the zeros and multiplicities of each zero from $y = 2x^3(x-1)^2(x+2)^4$. Indicate whether the graph crosses or touches the x-axis at each zero.

zero:_____ Multiplicity:_____ Crosses or touches?

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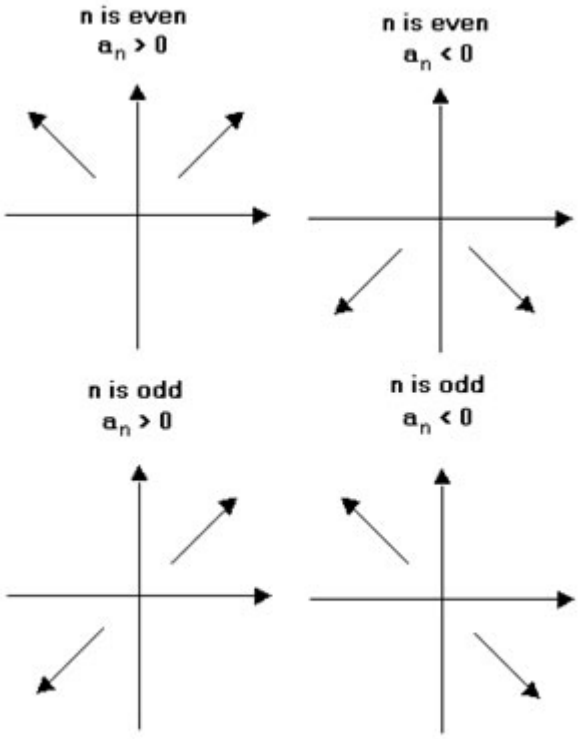
End behavior

This is what the graph will do when x is really big or really small. This is where you will need to know what the a_n is. Depending on what the degree is and what the a_n is the graph will do the following:



Some textbooks also refer to this end behavior as a "power function of degree *n*". This is a monomial function of the form $f(x) = ax^n$ where a is a real number, $a \neq 0$, and $n > 0$ is an integer. This is what equation the graph resembles for very large or very small values of |x|.

EXAMPLE: Construct a polynomial function for the graph below: