

5.1 Evaluate Composite Functions

Sum, Difference, Product, and Quotient of Functions

Given functions f and g , the functions $f + g$, $f - g$, $f \cdot g$, and $\frac{f}{g}$ are defined by

$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \text{ provided that } g(x) \neq 0$$

EXAMPLE: Let $f(x) = x^2 - 9x$, $g(x) = -6x$.

a.) Find $(f + g)(x)$ and $(f + g)(-2)$

b.) Find $(f - g)(x)$ and $(f - g)(1)$

c.) Find $(f \cdot g)(x)$ and $(f \cdot g)(3)$

d.) Find $\left(\frac{f}{g}\right)(x)$ and $\left(\frac{f}{g}\right)(15)$

Composite Functions – a way of combining two functions

$(f \circ g)(x) = f(g(x))$ This is pronounced “f of g of x” DOES NOT MEAN F TIMES G!!!

$(g \circ f)(x) = g(f(x))$ This is pronounced “g of f of x” DOES NOT MEAN G TIMES F!!

These are not multiplications. The $(f \circ g)(x)$ means we place the g function into the f function.

The $(g \circ f)(x)$ means we place the f function into the g function.

EXAMPLE: Given: $f(x) = 5x - 4$ and $g(x) = 3x + 1$ find the following:

$$(f \circ g)(2), (g \circ f)(-1), (f \circ g)(x), (g \circ f)(x), (f \circ f)(x), (g \circ g)(x)$$

a.) $(f \circ g)(2)$

b.) $(g \circ f)(-1)$

c.) $(f \circ g)(x)$

d.) $(g \circ f)(x)$

e.) $(f \circ f)(x)$

f.) $(g \circ g)(x)$

EXAMPLE: Given: $f(x) = x + 3$ and $g(x) = 2x^2 - 1$ find the following:

$$(f \circ g)(-1), (g \circ f)(0), (f \circ g)(x), (g \circ f)(x), (f \circ f)(x), (g \circ g)(x)$$

a.) $(f \circ g)(-1)$

b.) $(g \circ f)(0)$

c.) $(f \circ g)(x)$

d.) $(g \circ f)(x)$

e.) $(f \circ f)(x)$

f.) $(g \circ g)(x)$

EXAMPLE: Given: $f(x) = 4 - x^2$ and $g(x) = \frac{-1}{x}$ find the following: $(f \circ g)(0)$, $(f \circ g)(x)$, $(g \circ f)(x)$, $(f \circ f)(x)$, $(g \circ g)(x)$. Find the domain of $(f \circ g)(x)$, $(g \circ f)(x)$, $(f \circ f)(x)$, $(g \circ g)(x)$ in interval notation.

a.) $(f \circ g)(0)$

b.) $(f \circ g)(x)$

c.) $(g \circ f)(x)$

d.) $(f \circ f)(x)$

e.) $(g \circ g)(x)$

EXAMPLE: If $f(x) = \frac{5}{x-3}$ and $g(x) = \frac{1}{x}$, find the domain of $(f \circ g)(x)$ and $(g \circ f)(x)$ using interval notation.

EXAMPLE: Find functions f and g so that $(f \circ g)(x) = H(x)$ given that $H(x) = (1 + x^2)^3$

EXAMPLE: Find functions f and g so that $(f \circ g)(x) = H(x)$ given that $H(x) = \sqrt{2x-1} - 4x + 2$

EXAMPLE: Use the given table to evaluate each composition.

x	-3	-2	-1	0	1	2	3
$f(x)$	11	9	7	5	3	1	-1
$g(x)$	-8	-3	0	1	0	-3	-8

a.) Find $(f \circ g)(1)$.

b.) Find $(f \circ g)(2)$.

c.) Find $(g \circ f)(2)$.

d.) Find $(g \circ g)(1)$.

e.) Find $(f \circ f)(3)$.