5.3 Exponential Functions

In this section we will be working with exponents, so here is a quick review of the exponent rules:

Laws of Exponents

$a^s \cdot a^t = a^{s+t}$ Example:

$\frac{a^s}{a^t} = a^{s-t}$ Example:

$(a^s)^t = a^{s \cdot t}$ Example:

$(a \cdot b)^s = a^s \cdot b^s$ Example:

$1^s = 1$ Example:

$a^0 = 1$ Example:

$a^{-s} = \frac{1}{a^s}$ Example:

$\left(\frac{a}{b}\right)^{-s} = \left(\frac{b}{a}\right)^s$ Example:

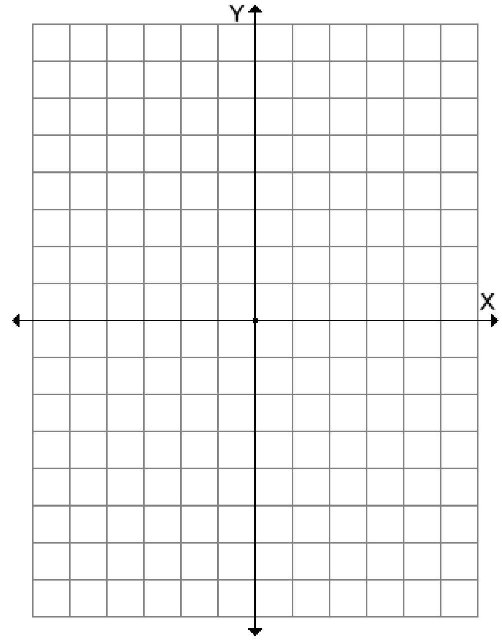
EXAMPLE: Approximate $5^{\sqrt{3}}$ using a calculator. Round your answer to three decimal places.

EXAMPLE: Approximate $e^{1.6}$ using a calculator. Round your answer to two decimal places.

Exponential function: $y = b^x$

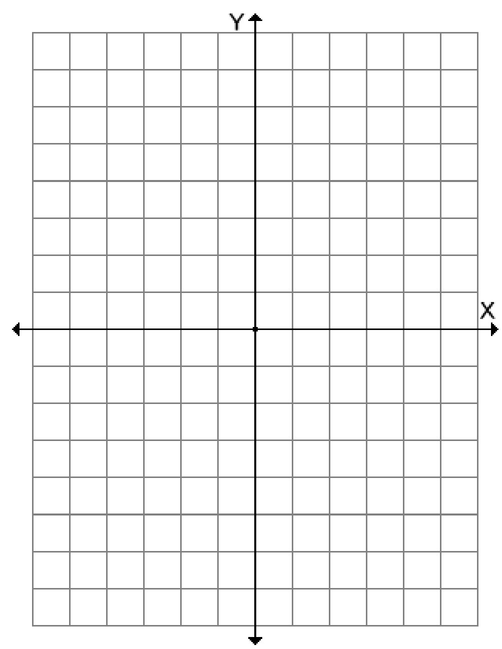
Graph of $y = 2^x$

x	$y = 2^x$	(x, y)
-2		
-1		
0		
1		
2		



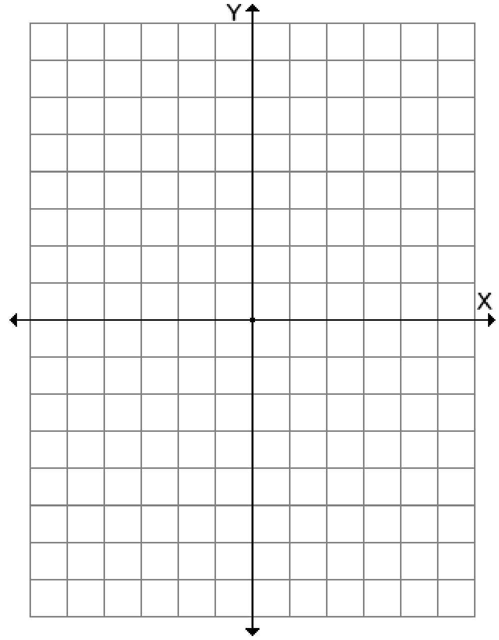
Graph of $y = e^x$

x	$y = e^x$	(x, y)
-2		
-1		
0		
1		
2		

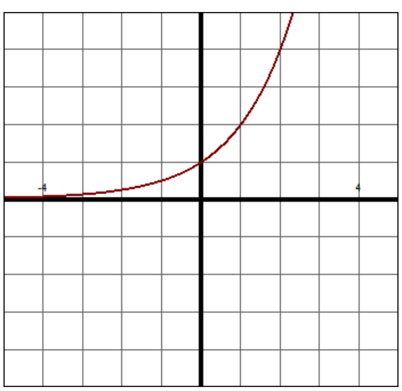


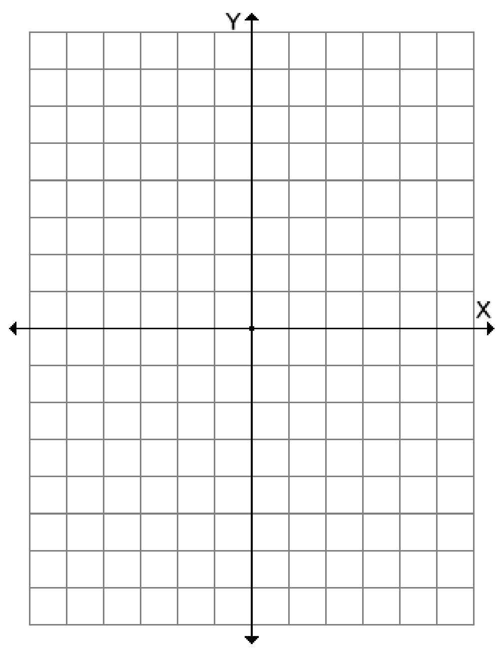
Graph of $y = \left(\frac{1}{2}\right)^x$

x	$y = \left(\frac{1}{2}\right)^x$	(x, y)
-2		
-1		
0		
1		
2		



EXAMPLE: Graph using transformations: $y = 2^x - 4$
Indicate the domain and range. State the horizontal asymptote.



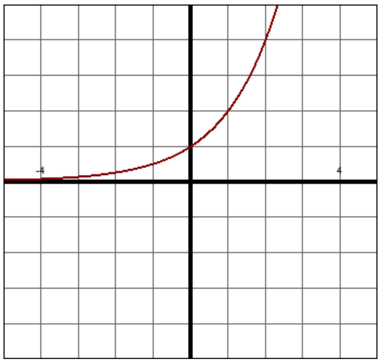


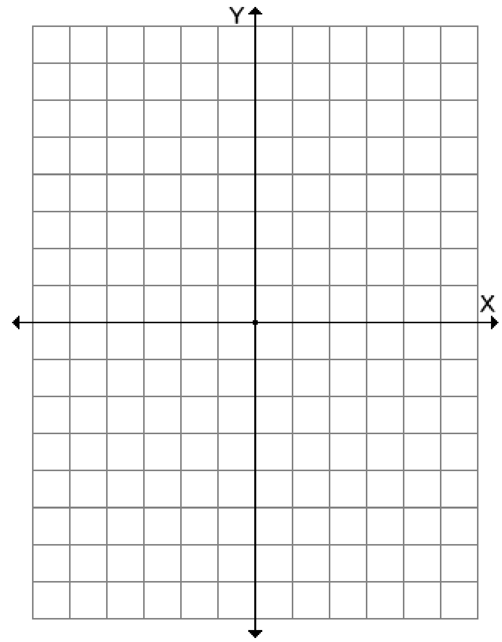
Domain:

H.A.:

Range:

EXAMPLE: Graph using transformations: $y = -2^x$.
Indicate the domain and range. State the horizontal asymptote.

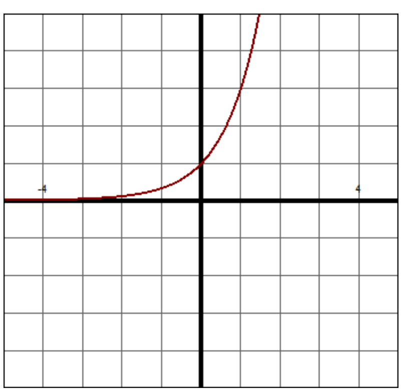


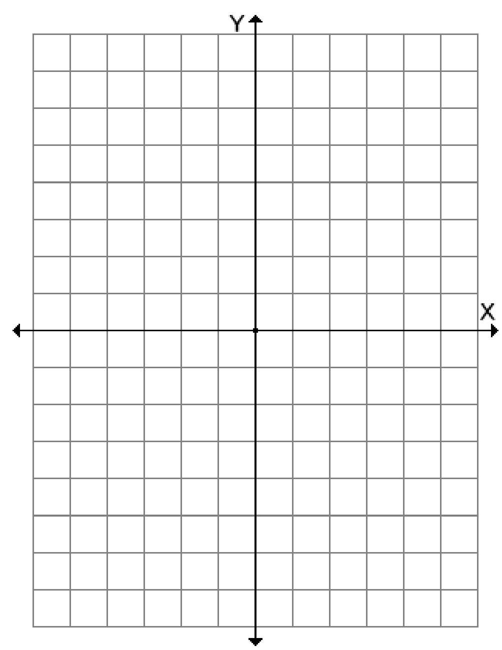


Domain: H.A.

Range:

EXAMPLE: Graph using transformations: $y = 3^{x+2} - 2$.
Indicate the domain and range. State the horizontal asymptote.



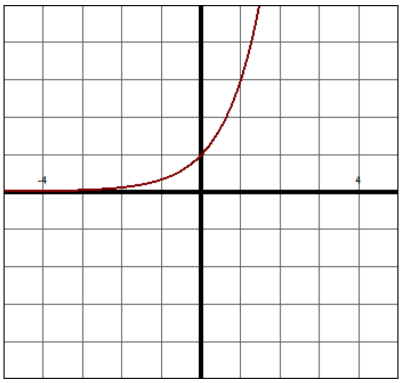


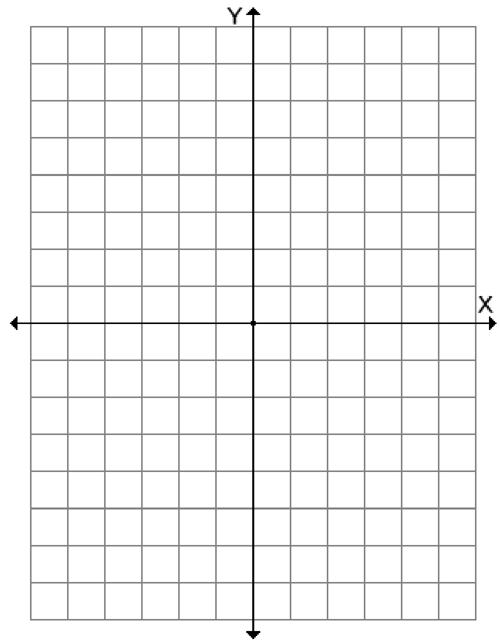
Domain: H.A.

Range:

EXAMPLE: Graph using transformations: $y = \left(\frac{1}{3}\right)^{x-3} + 1$.

Indicate the domain and range. State the horizontal asymptote.



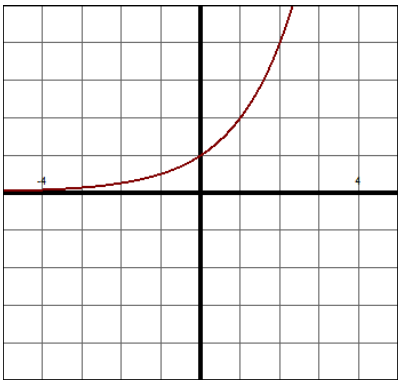


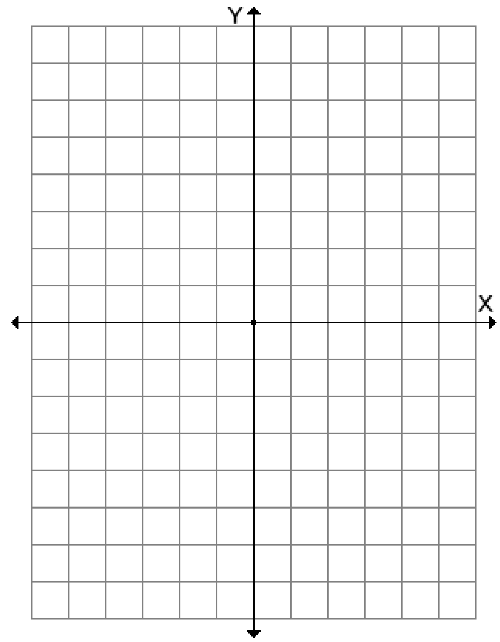
Domain:

Range:

H.A.

EXAMPLE: Graph using transformations: $y = -\left(\frac{1}{2}\right)^x + 3$.





Domain:

Range:

H.A.

Compound Interest

The amount *A* after *t* years due to a principal *P* invested at an annual interest rate *r* (expressed as a decimal) compounded n times per year is $A = P \cdot \left(1 + \frac{r}{n}\right)^{nt}$. The compound interest formula actually starts with the simple interest formula, $I = Prt$. Interest is calculated once, and then this interest is added to the principal, and the formula is repeated again. This process continues until it is compounded *n* times. So instead of that long process, we have the compound interest formula.

In working with these kinds of problems, they will give you different payment periods as listed below, which also tell you what to enter for *n*:

Annually: Once a year, $n = 1$
Semiannually: Twice a year, $n = 2$
Quarterly: Four times a year, $n = 4$
Monthly: 12 times a year, $n = 12$
Weekly: 52 times a year, $n = 52$
Daily: 365 days per year, $n = 365$

EXAMPLE: \$300 is invested at 12% compounded monthly for $1\frac{1}{2}$ years. Find the amount that results from this investment.

Continuous Compounding

If you take the same compound interest formula as above and have n go to infinity, you will get $A = P \cdot e^{rt}$, which is the formula for compounding continuously.

EXAMPLE: \$300 is invested at 12% compounded continuously for $1\frac{1}{2}$ years. Find the amount that results from this investment.

EXAMPLE: Determine the rate that represents the better deal:

9% compounded quarterly or $9\frac{1}{4}\%$ compounded annually

EXAMPLE: What will a $90,000 house cost 10 years from now if the price appreciation for homes over that period averages 4% compounded annually?