1.7 Rates of Change and Behavior of Graphs

Average Rate of Change (A.R.C.)

The A.R.C. is an estimate of the slope between $x$ and $c$. Basically how much does something change between $x$ and $c$. The formula is as follows and is derived from the slope formula.

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

EXAMPLE: Find the A.R.C. for the $f(x) = -3x^2 + 3$ from $x_1 = 0$ to $x_2 = 2$.

EXAMPLE: Find the A.R.C. for the $f(x) = x^3 - x + 2$ from $x_1 = 1$ to $x_2 = 3$.

EXAMPLE: Find the A.R.C. for the $f(x) = \sqrt{x}$ from $x_1 = 9$ to $x_2 = 16$. 
If we wanted to find the slope of a curved line, the only way we can do this is by estimating it with a straight line. We will start with one point and then move over by a small amount $h$. Now we will use the slope formula. In the picture we have two points, A and B. The coordinates for these are: $(x, f(x))$ and $(x + h, f(x + h))$.

The slope, also called the **difference quotient** is: 

$$\frac{f(x + h) - f(x)}{h}$$

In calculus we will try to minimize $h$ so that it is so small that we end up at a point, which will be the exact slope of the curved line at $x$.

Now let’s look at some examples finding the difference quotient.

**EXAMPLE:** Let $f(x) = 2x - 3$. Find the difference quotient.
EXAMPLE: Let $f(x) = 3x^2 - x + 1$. Find the difference quotient.
EXAMPLE: $f(x) = \frac{3}{x-1}$. Find the difference quotient.
Symmetry

Think of symmetry as a fold line. If a graph can be folded on top of itself and everything overlaps, then it has symmetry. The fold line that allows this to happen is called the line of symmetry. Below is the three types of symmetry that is possible.

![x-axis symmetry](image1)
![y-axis symmetry](image2)
![origin symmetry](image3)

x-axis symmetry  y-axis symmetry  origin symmetry

Looking at the above drawings we can come up with relationships. Let's begin with a point (x, y).

Two points symmetric about the x-axis would be (x, y) and (x, -y).
Two points symmetric about the y-axis would be (x, y) and (-x, y).
Two points symmetric about the origin would be (x, y) and (-x, -y).

EXAMPLE: Determine what kind of symmetry each graph has.

![Graphs](image4)

How to test for symmetry without graphing

**x-axis:** Replace y with –y in the original equation. If it simplifies to the original equation, it has x-axis symmetry.

**y-axis:** Replace x with –x in the original equation. If it simplifies to the original equation, it has y-axis symmetry.

**origin:** Replace x with –x and y by -y in the original equation. If it simplifies to the original equation, it has origin symmetry.
EXAMPLE: Test the following equation for symmetry: \(4x + y^2 = 4\) and find the intercepts.

**x-int:** Put a zero in for \(y\) and solve for \(x\).

**y-int:** Put a zero in for \(x\) and solve for \(y\).

**x-axis:** Replace the \(y\) with \(-y\) in the original equation.

**y-axis:** Replace \(x\) with \(-x\) and simplify.

**Origin:** Replace \(x\) with \(-x\) and \(y\) with \(-y\).
EXAMPLE: Test the following equation for symmetry: \( y = x^2 - 5x \) and find the intercepts.

**x-int:** Put a zero in for \( y \) and solve for \( x \).

**y-int:** Put a zero in for \( x \) and solve for \( y \).

**x-axis:** Replace the \( y \) with \(-y\) in the original equation.

**y-axis:** Replace \( x \) with \(-x\) and simplify.

**Origin:** Replace \( x \) with \(-x\) and \( y \) with \(-y\).
EXAMPLE: Test the following equation for symmetry: $y = x^3 - 4x$ and find the intercepts.

x-int: Put a zero in for $y$ and solve for $x$.

y-int: Put a zero in for $x$ and solve for $y$.

x-axis: Replace the $y$ with $-y$ in the original equation.

y-axis: Replace $x$ with $-x$ and simplify.

Origin: Replace $x$ with $-x$ and $y$ with $-y$. 
Even and Odd functions

If $f(-x) = f(x)$ then the function is even, and symmetric to the y-axis.
If $f(-x) = -f(x)$ then the function is odd, and symmetric to the origin.

EXAMPLE: Determine whether the following are even, odd, or neither.

a.) $f(x) = x^4 + 7$

b.) $f(x) = 6x^5 - x^3$

c.) $f(x) = x^2 + x$

d.) $f(x) = \frac{|2x|}{x}$
Increasing, Decreasing, Constant Graphs; Relative Extrema

**Increasing**: as $x$ increases, $y$ increases (graph goes uphill as you move from left to right)

**Decreasing**: as $x$ increases, $y$ decreases (graph goes downhill as you move from left to right)

**Constant**: as $x$ increases, $y$ does not change (this part of the graph is horizontal)

**Relative maximum**: a point at which the graph increases and then decreases (peak)

**Relative minimum**: a point at which the graph decreases and then increases (valley)

e.) $f(x) = \frac{x^3}{x^2 - 9}$
EXAMPLE: Use the graph below to answer the following questions

a.) Indicate the interval(s) of which \( f \) is increasing

b.) Indicate the interval(s) of which \( f \) is decreasing.

c.) List the number where \( f \) has a relative maximum.

d.) What is the relative maximum?

e.) What is the relative minimum?
EXAMPLE: Use the graph below to answer the following questions

a.) Indicate the interval(s) of which $f$ is increasing

b.) Indicate the interval(s) of which $f$ is decreasing

c.) List the number(s) where $f$ has a relative minimum.

d.) What is the relative maximum(s)?

e.) What is the relative minimum(s)?

f.) What is the domain?

g.) What is the range?