1.2 Graphs of Equations in Two Variables

Finding x and y intercepts from an equation

To find the x-intercept, put in a zero for y and solve for x.
To find the y-intercept, put in a zero for x and solve for y.

EXAMPLE: Find the intercepts given $4x + y^2 = 4$.

x-int: $4x + (0)^2 = 4$ Put a zero in for y and solve for x.
$4x = 4$
x = 1, and the point is (1, 0)

y-int: $4(0) + y^2 = 4$ Put a zero in for x and solve for y.
$y^2 = 4$
$y = \pm 2$ and the point is $(0, \pm 2)$

EXAMPLE: Find the intercepts given $y = |x - 4| - 5$.

x-int: $0 = |x - 4| - 5$ Put a zero in for y and solve for x.
$|x - 4| = 5$ Isolate the absolute value
$x - 4 = 5$ and $x - 4 = -5$ Remember absolute values turn into two equations.
$x = 9$ and $x = -1$. As coordinates you can write (9,0) and (-1,0).

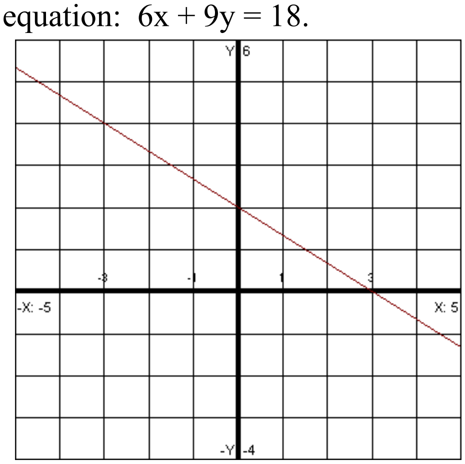
y-int: $y = |0 - 4| - 5$ Put a zero in for x and solve for y.
$y = |-4| - 5$
$y = 4 - 5$ Remember the absolute value is always positive.
$y = -1$ As a coordinate, you can write (0,-1).

EXAMPLE: Find the x and y intercepts and use them to graph the following equation: 6x + 9y = 18.

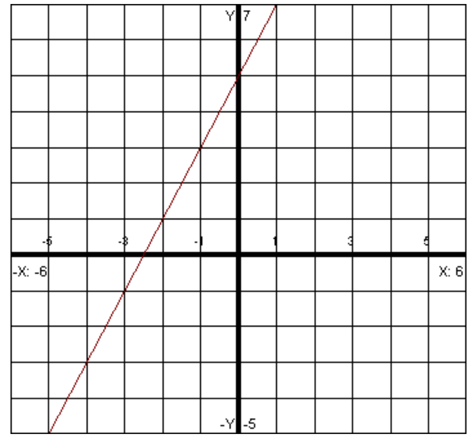
To find an x-intercept, put a zero in for y and solve: 6x + 9(0) = 18.
Simplifying will give us 6*x* = 18. Solve for x, so *x* = 3.
It is important to write our answer as a point. The *x*-intercept is (3, 0).

To find an y-intercept, put a zero in for x and solve: 6(0) + 9y = 18.
Simplifying will give us 9*y* = 18. Solve for y, so *y* = 2.
It is important to write our answer as a point. The *y*-intercept is (0, 2).

To graph, just plot each point and connect them with a line.



EXAMPLE: Find the x and y intercepts and use them to graph the following equation: $6x - 3y + 15 = 0$.



To find an x-intercept, put a zero in for y and solve: $6x - 3(0) + 15 = 0$.

Simplifying will give us $6x + 15 = 0$. Solve for x: $6x = -15$, so $x = -\frac{5}{2}$.

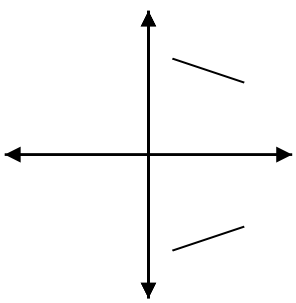
It is important to write our answer as a point. The x-intercept is $\left(-\frac{5}{2},\ 0\right)$.

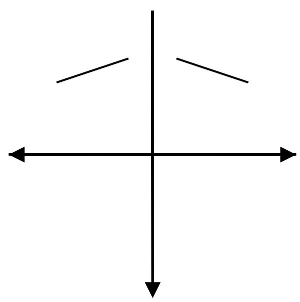
To find an y-intercept, put a zero in for x and solve: $6(0) - 3y + 15 = 0$.
Simplifying will give us $-3y + 15 = 0$. Solve for y: $-3y = -15$, so $y = 5$.
It is important to write our answer as a point. The y-intercept is (0, 5).

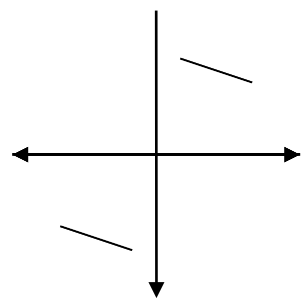
To graph, just plot each point. For fractions, you can change them into a decimal. Our x-intercept can be written as: (-2.5, 0).

Symmetry

Think of symmetry as a fold line. If a graph can be folded on top of itself and everything overlaps, then it has symmetry. The fold line that allows this to happen is called the line of symmetry. Below is the three types of symmetry that is possible.


x – axis symmetry

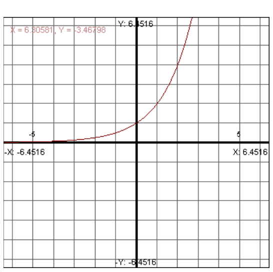

y-axis symmetry

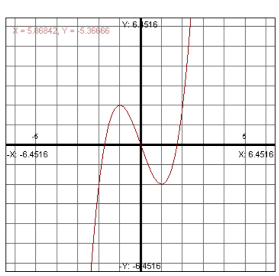

origin symmetry

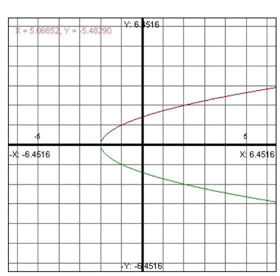
Looking at the above drawings we can come up with relationships. Let's begin with a point (x, y).

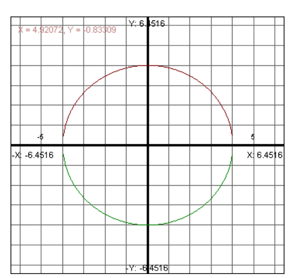
Two points symmetric about the x-axis would be (x, y) and (x, -y).
Two points symmetric about the y-axis would be (x, y) and (-x, y).
Two points symmetric about the origin would be (x, y) and (-x, -y).

EXAMPLE: Determine what kind of symmetry each graph has.


No symmetry


Origin symmetry


X-axis symmetry


X and Y axis, origin symmetries

How to test for symmetry without graphing

x-axis: Replace y with –y in the original equation. If it simplifies to the original equation, it has x-axis symmetry.

y-axis: Replace x with –x in the original equation. If it simplifies to the original equation, it has y-axis symmetry.

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origin: Replace x with –x and y by -y in the original equation. If it simplifies to the original equation, it has origin symmetry.

EXAMPLE: Test the following equation for symmetry: $4x + y^2 = 4$ and find the intercepts.

x-int: $4x + (0)^2 = 4$ Put a zero in for y and solve for x.
$4x = 4$
x = 1, and the point is (0, 1)

y-int: $4(0) + y^2 = 4$ Put a zero in for x and solve for y.
$y^2 = 4$
$y = \pm 2$ and the point is $(0, \pm 2)$

x-axis: $4x + (-y)^2 = 4$ Replace the y with –y in the original equation. Now simplify.
$4x + y^2 = 4$ This is the same equation we started with, so it has this symmetry.

y-axis: $4(-x) + y^2 = 4$ Replace x with –x and simplify.
$-4x + y^2 = 4$ No matter what this will never be the same as the original, so it does not have this sym.

Origin: $4(-x) + (-y)^2 = 4$ Replace x with –x and y with –y. Now simplify.
$-4x + y^2 = 4$ This is not the same as the original, so it does not have origin symmetry.

EXAMPLE: Test the following equation for symmetry: $y = x^2 - 5x$ and find the intercepts.

x-int: $0 = x^2 - 5x$ Put a zero in for y and solve for x.
$0 = x(x - 5)$
x = 0, and x = 5 and the points are (0, 0) and (5, 0)

y-int: $y = 0^2 - 5(0)$ Put a zero in for x and solve for y.
$y = 0$ and the point is (0, 0)

x-axis: $-y = x^2 - 5x$ Replace the y with –y in the original equation. This is not the same as what we started with, so it does not have x-axis symmetry.

y-axis: $y = (-x)^2 - 5(-x)$ Replace x with –x and simplify.
$y = x^2 + 5x$ No matter what this will never be the same as the original, so it does not have this sym.

Origin: $-y = (-x)^2 - 5(-x)$ Replace x with –x and y with –y. Now simplify.
$-y = x^2 + 5x$ This is not the same as the original, so it does not have origin symmetry.

So the above example does not have any axis symmetry, which is possible.

EXAMPLE: Test the following equation for symmetry: $y = x^3 - 4x$ and find the intercepts.

x-int: $0 = x^3 - 4x$ Put a zero in for y and solve for x.
$0 = x(x^2 - 4)$
$x = \pm 2$ and x = 0. The points are $(\pm 2, 0)$ and (0, 0).

y-int: $y = 0^3 - 4(0)$ Put a zero in for x and solve for y.
$y = 0$ and the point is (0, 0)

x-axis: $-y = x^3 - 4x$ Replace the y with –y in the original equation. This is not the same as what we started with, so it does not have x-axis symmetry.

y-axis: $y = (-x)^3 - 4(-x)$ Replace x with –x and simplify.
$y = -x^3 + 4x$ No matter what this will never be the same as the original, so it does not have this sym.

Origin: $-y = (-x)^3 - 4(-x)$ Replace x with –x and y with –y. Now simplify.
$-y = -x^3 + 4x$ You may think there is no symmetry because it is not the same as the original, but
$-(-y = -x^3 + 4x)$ let's multiply both sides by a negative one so we can solve for y.
$y = x^3 - 4x$ You do get the same as the original so it does have this symmetry.