1.4 Circles

In this section we will be analyzing graphs and equations of circles. First let us derive the equation of a circle. It follows from what we learned in the last section.

To derive the equation of a circle, we can use the distance formula with the points (h, k) and (x, y) and the distance r.

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ First we start with the distance formula.

$r=\sqrt{(x-h)^2+(y-k)^2}$ Next we substitute the given points.

$r^2=(x-h)^2+(y-k)^2$ Square both sides. This is the circle formula.

Standard Form

$(x-h)^2+(y-k)^2=r^2$ where (h, k) is the center and r is the radius

EXAMPLE1: Write the standard form of a circle with a center of (-2, 3) and a radius of 4 and graph.

We will use the standard form formula. Here, h = -2, k = 3, and r = 4. Plug them into the formula.

Equation: $(x+2)^2+(y-3)^2=4^2$ To graph, plot the center. The go vertically up 4 and make a point. Then from the center go down 4 and make a point. Then to the left and to the right. See all graphs on last page.

EXAMPLE2: Write the standard form of a circle with a center of (5, 1) and a radius of 3 and graph.

We will use the standard form formula. Here, h = 5, k = 1, and r = 3. Plug them into the formula.

Equation: $(x-5)^2+(y-1)^2=3^2$ To graph, plot the center. The go vertically up 3 and make a point. Then from the center go down 3 and make a point. Then to the left and to the right. See all graphs on next page.

EXAMPLE3: Find the center and radius and graph: $(x+1)^2+(y-3)^2=25$

We will use the standard form formula. To find the center take the opposite sign of each number inside the parenthesis. In the first parenthesis there is a +1 so the x coordinate of the center will be -1. In the second parenthesis there is a -3, so the y coordinate of the center will be 3. So the center is (-1, 3). The radius is always the square root of the number after the equal sign, so $\sqrt{25}=5$. See all graphs on next page.

EXAMPLE4: Find the center and radius and graph: $(x-3)^2+y^2=13$

We will put this into the standard form formula. We can rewrite the second term as: $(y-0)^2$. Now our equation becomes: $(x-3)^2+(y-0)^2=13$. To find the center take the opposite sign of each number inside the parenthesis. In the first parenthesis there is a -3 so the x coordinate of the center will be 3. In the second parenthesis there is a 0, so the y coordinate of the center will be 0. So the center is (3, 0). The radius is always the square root of the number after the equal sign, so $\sqrt{13} \approx 3.6$. We can't simplify this anymore. After plotting the center, go up, down, left, and right 3.6.

EXAMPLE5: Find the center and radius and graph: $x^2 + y^2 + 10x - 8y + 16 = 0$

This is not in standard form, like the above example so we need to first get it into the proper form. We want to do this by by completing the square.

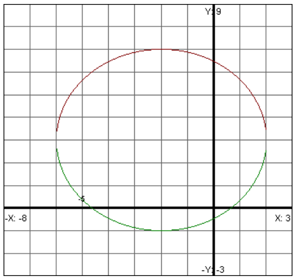
$x^2 + 10x + y^2 - 8y = -16$ First put like terms together and move the 16 to the other side.
Now we need to complete the square. Take the number in front of the variable that is not being squared and divide it by two. Then square this result. You will add this result to both sides of the equation. For example, there is a 10x, so first divide 10 by 2 and you get 5. Then square this number and you will get 25. Do the same for the 8y term. Divide -8 by 2 to get -4. Then square the -4 to get 16. We always divide by 2 and then square it. $x^2 + 10x + 25 + y^2 - 8y + 16 = -16 + 25 + 16$ Notice we added 25 to both sides and 16 to both sides.
$(x+5)^2 + (y-4)^2 = 25$ Now factor and simplify the right hand side.

Now this is in standard form, so we know the center is (-5, 4) and the radius is 5. See graphs below.

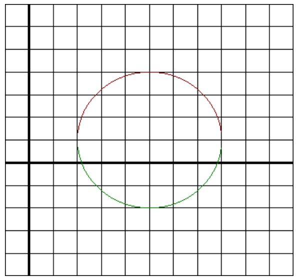
EXAMPLE6: Find the center and radius and graph: $2x^2 + 2y^2 + 8x + 7 = 0$

In order for complete the square to work, there must be a 1 in front of the squared terms, so I need to divide both sides of the equation by 2 to get: $x^2 + y^2 + 4x + \frac{7}{2} = 0$
Now we will put like terms together and moved the fraction to the other side to get
$x^2 + 4x + y^2 = -\frac{7}{2}$ The next step is to complete the square. We can only complete the square with the x since this has a non-squared term. We take the number 4 in front of the x and divide it by 2 and we get 2. Then we square this and get 4. We will add this to both sides.
$x^2 + 4x + 4 + y^2 = -\frac{7}{2} + 4$ Now we need to factor the left side and simplify the right side.
$(x+2)^2 + (y+0)^2 = \frac{1}{2}$ Notice that I rewrote y^2. It didn't change the problem, just put in standard form.
Center: (-2, 0) Radius: $\sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ The radius is always the square root of the number after the = sign.

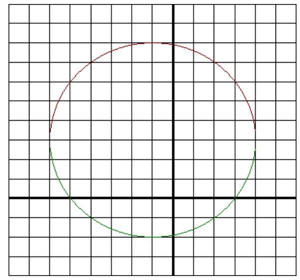
Example 1 Graph



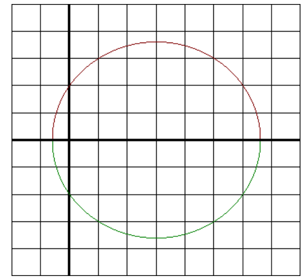
Example 2 Graph



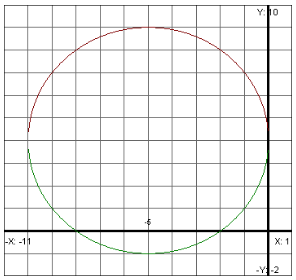
Example 3 Graph



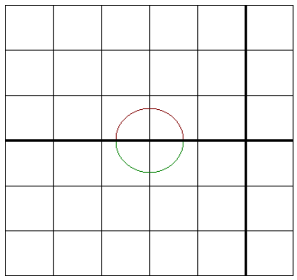
Example 4 Graph



Example 5 Graph



Example 6 Graph



EXAMPLE: The endpoints of the diameter of a circle is (2, 6) and (-4, -2). Determine the coordinates for the center of the circle, and find the length of the radius. Then write the standard form of the circle.

This is an application of the two formulas we have learned in this section. To find the center of the circle we need to find the center, since the center is the midpoint of the radius. The radius is defined as half the length of the diameter or the distance from the center to a point on the circle. So we have two ways of doing it.

Center: $M = \left(\frac{2+-4}{2}, \frac{6+-2}{2}\right)$ = (-1, 2) We have used the midpoint formula here.

Radius: $r = \sqrt{(2--1)^2 + (6-2)^2} = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$ Distance formula with points (-1, 2) and (2,6). You could have also used (-1, 2) and (-4, -2). You could also use (2, 6) and (-4, -2) and take half this distance.

Now we put this information into the standard form of a circle formula: $(x+1)^2 + (y-2)^2 = 25$