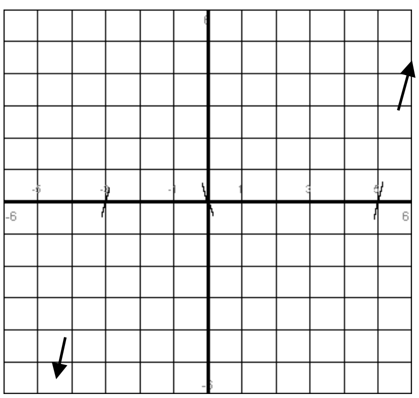
4.2 Graphing Polynomial Functions

In the last section we learned how to find polynomial characteristics. Now let's put this together to graph. On the test I will ask you to find the zeros, multiplicities, behavior at each zero, turning pts, y-intercept and the graph, and the correct numbers of blanks will be provided. Let's look at some examples.

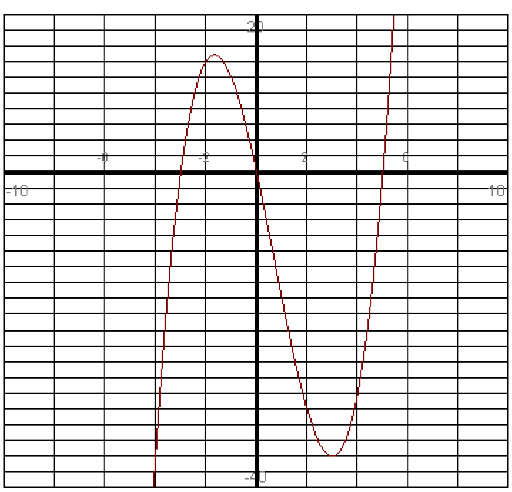
EXAMPLE: Find the zeros, multiplicities, turning pts, y-intercept and the graph of $f(x) = x(x-5)(x+3)$.

This was the same equation we used to find the behavior at each zero in the last section. The multiplicity on each zero is one in this case because 1 is the exponent on each factored piece. For the y-intercept, put in a zero for x. When you do you will get y = 0, so the y-intercept is (0, 0). What is our degree? We can multiply this out to get: $f(x) = x^3 - 2x^2 - 15x$ and we see that the degree is 3. This means it can have at most 3 – 1 or 2 turning points.

Now here is how we will graph this. First plot each of the zeros. Our equation is $f(x) = x^3 - 2x^2 - 15x$ which tells us our degree is odd and the $a_n > 0$. This means that from our end behavior models the graph will go down and to the left and will go up and to the right. Since we know this we can draw the arrows below. All of the multiplicities are odd, which means that the graph will cross at each zero. Let's put this information on the graph:



Since we know this we can connect these lines to get our graph (the graph is scaled by 2).



Your graph will be a sketch. You don't need to know exactly how high or how low the graph goes. I am only looking for the general shape.

What is the power function for this one? It will always be the term with the highest power. So here, $f(x) = x^3$. This means as x gets very large or very small, the graph will resemble $f(x) = x^3$.

EXAMPLE: Find the zeros, multiplicities, behavior at each zero, turning pts, y-intercept and the graph of $y = 2(x-3)^2(x+4)^2$.

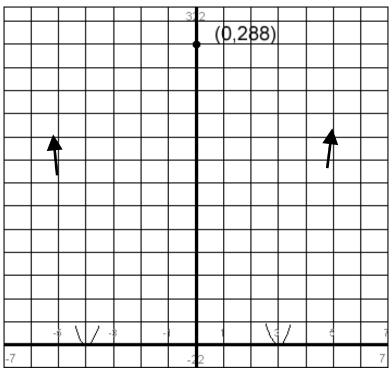
The quiz or test will have the correct number of blanks for the number of zeros. This one only has two zeros, so there are two blanks. Lets' first find the zeros and multiplicities. We will get 3 and -4 as our zeros. The exponent associated with each factor is 2. Therefore each multiplicity is 2. If we add the multiplicities this will give us the degree, which is 4. Once we know the degree we can subtract one and this will be the max turning points. We can also find out the y-intercept by putting in a 0 for x: $y = 2(0-3)^2(0+4)^2$, so y = 288. Our y-int is (0, 288). Since the multiplicities are both even, the graph will touch at each zero.

zero:___3___ Multiplicity:___2___ Touches at x = 3

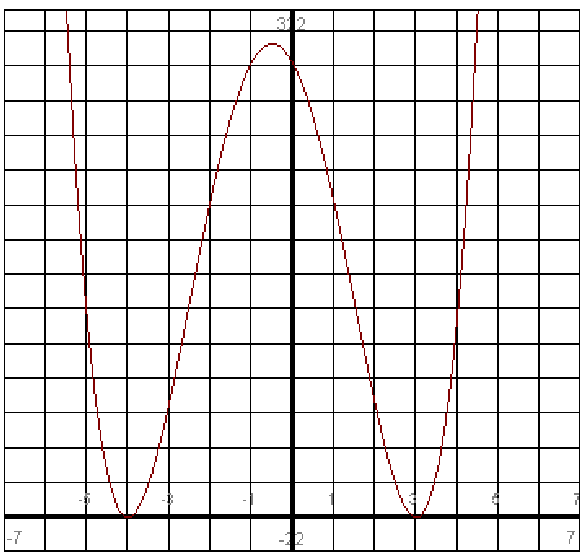
zero:___-4__ Multiplicity:___2___ Touches at x = -4

y-int:__(0, 288)_ Degree:______4_____ Max turning pts:___3____

To graph this we will first plot the x-intercepts. Then we can plot the y-intercept. Then we need to make a sketch of our behavior at zero equations. These will both be parabolas, or U shaped graphs. We found that our degree is even and the $a_n > 0$ because we just multiply our coefficients (number in front of each x). We have 2 on the outside and each x in the parenthesis has a 1 in front of it, so 2*1*1 = 2, which is greater than zero. Our end behavior models the graph will go up and to the left and will go up and to the right. We can put the arrows on our graph. We also know the graph touches at each zero, so we can also indicate that on the graph. This is why I have drawn parabola shapes at each zero. The graph will touch but not cross at these zeros.



Since we know this, we can connect these lines to get our graph (the graph is scaled by 20).



What is the power function on this one? Since the degree is 4 and we have a 2 in front of the equation, the power function will be $f(x) = 2x^4$.

EXAMPLE: Find the zeros, multiplicities, turning pts, y-intercept, and the graph of $y = -2x^3(x+2)$.

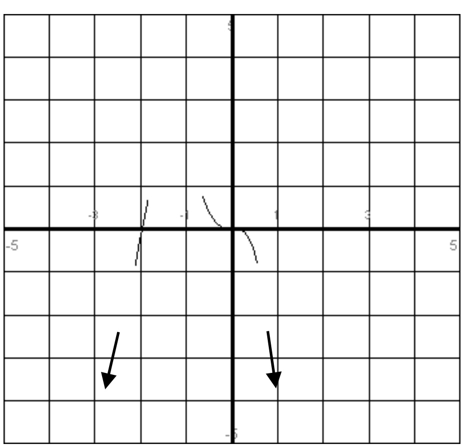
The quiz or test will have the correct number of blanks for the number of zeros. This one only has two zeros, so there are two blanks. Lets first find the zeros and multiplicities. We will get 0 and -2 as our zeros. The multiplicity of the zero, x = 0 is 3 and the multiplicity of the zero x = -2 is 1. Again, these are the exponents associated with each factor. If we add the multiplicities this will give us the degree, which is 4. Once we know the degree we can subtract one and this will be the max turning points. We can also find out the y-intercept by putting in a 0 for x: $y = -2(0)^3(0+2)$, so y = 0. Our y-intercept is (0, 0). The multiplicities are both odd which means the graph will cross the x-axis at each of these zeros.

zero:___0___ Multiplicity:___3___ Crosses at $x = 0$

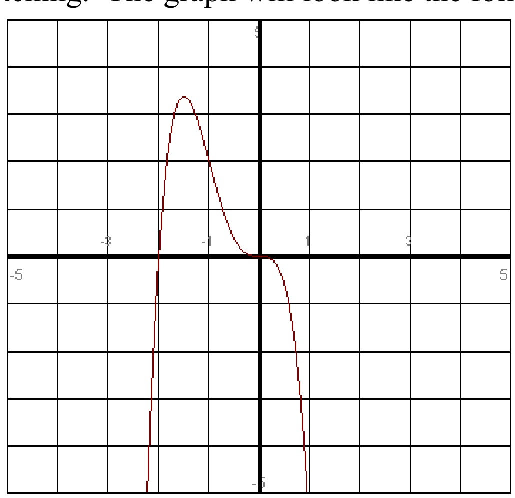
zero:___-2__ Multiplicity:___1___ Crosses at $x = -2$

y-int:__(0, 0)__ Degree:____4___ Max turning pts:___3____

To graph this we will first plot the x-intercepts. Then we can plot the y-intercept. We found that our degree is even and the $a_n < 0$ because we just multiply our coefficients (number in front of each x). We have -2 on the outside and the x in the parenthesis has a 1 in front of it, so -2*1 = -2, which is less than zero. Our end behavior models the graph will go down and to the left and will go down and to the right as indicated by the arrows below. We know the graph crosses at each zero, but how do we know what shape the graph will look like? This is where we use our multiplicities. At x = –2, the multiplicity is 1. This means the graph will resemble the graph of x, which is linear. At x = 0, the multiplicity is 3, so the graph will resemble an x^3 graph. This is why I drew this shape at x = 0 below.



Since we know this, we can connect these lines to get our graph. We do not know how high the graph goes, but that is okay. We are only sketching. The graph will look like the following:



What is the power function for this one? If we expand this one we will get $y = -2x^4 - 4x^3$. The power function is $y = -2x^4$.

EXAMPLE: Find the zeros, multiplicities, turning pts, y-intercept, and the graph of $y = x^3(x-1.5)^2(x+1.5)^2$.

We will have three zeros for this one. You will get 0, 1.5, and -1.5. The multiplicity of the zero, x = 0 is 3 and the multiplicity of the zero x = -1.5 and 1.5 are both 2. If we add the multiplicities this will give us the degree, which is 7. Once we know the degree we can subtract one and this will be the max turning points, which is 6. We can also find out the y-intercept by putting in a 0 for x: $y = 0^3(0-1.5)^2(0+1.5)^2$, so y = 0. So (0,0). The graph will cross at x = 0 since the multiplicity is odd. The graph will touch at both x = 1.5 and x = –1.5 since the associated multiplicities are even.

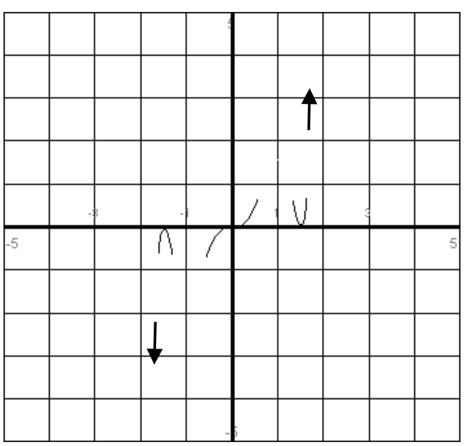
zero:___0___ Multiplicity:___3___ Crosses at $x = 0$

zero:___1.5__ Multiplicity:___2___ Touches at $x = 1.5$

zero:__-1.5___ Multiplicity:___2___ Touches at $x = -1.5$

y-int:__(0, 0)__ Degree:____7___ Max turning pts:___6____

To graph this we will first plot the x-intercepts. Then we can plot the y-intercept. We found that our degree is odd and the $a_n > 0$ because we just multiply our coefficients (number in front of each x). We have 1 on the outside and the x in the parenthesis has a 1 in front of each, so 1*1*1 = 1, which is greater than zero. Our end behavior models the graph will go down and to the left and will go up and to the right as indicated by the arrows below. The graph will touch at x = –1.5. Our end behavior indicates that the graph will be coming up from the bottom and will touch the x-axis and reverse directions. This is why I drew an upside down parabola at $x = -1.5$. The graph will also touch at x = 1.5. The end behavior says the graph will rise after this zero, which is why I drew a parabola opening up here. Now we need to figure out what the graph looks like when it crosses at x = 0. The multiplicity at x = 0 is 3, which means the graph will resemble x^3. I drew it in a way that it would connect with the other pieces we already drew.



Since we know this, we can connect these lines to get our graph.

The power function on this one is $y = x^7$.