5.1 Evaluate Composite Functions

Composite Functions – a way of combining two functions

$(f \circ g)(x) = f(g(x))$ This is pronounced "f of g of x" DOES NOT MEAN F TIMES G!!!
$(g \circ f)(x) = g(f(x))$ This is pronounced "g of f of x" DOES NOT MEAN G TIMES F!!

These are not multiplications. The $(f \circ g)(x)$ means we place the g function into the f function.
The $(g \circ f)(x)$ means we place the f function into the g function.

EXAMPLE: Given: $f(x) = 5x - 4$ and $g(x) = 3x + 1$ find the following:
$(f \circ g)(2)$, $(g \circ f)(-1)$, $(f \circ g)(x)$, $(g \circ f)(x)$, $(f \circ f)(x)$, $(g \circ g)(x)$

a.) $(f \circ g)(2) = f(g(2))$ First we rewrite this using the definition, replacing x with 2.

Let's first do $g(2)$ since this is inside the parenthesis. To do this, replace the x with 2 in the g function:
$g(2) = 3(2) + 1 = 7$ Since now we know that $g(2) = 7$ we can now replace $g(2)$ with 7:
By replacing $g(2)$ with 7 now we need to find $f(7)$. Replace the x in the f function with 7:
$f(7) = 5(7) - 4 = 31$. So now conclude that $(f \circ g)(2) = 31$.

b.) $(g \circ f)(-1) = g(f(-1))$ First we rewrite this using the definition, replacing x with -1.

Let's first do $f(-1)$ since this is inside the parenthesis. To do this, replace the x with -1 in the f function:
$f(-1) = 5(-1) - 4 = -9$ Since now we know that $f(-1) = -9$ we can now replace $f(-1)$ with -9:
By replacing $f(-1)$ with -9 now we need to find $g(-9)$. Replace the x in the g function with -9:
$g(-9) = 3(-9) + 1 = -26$. So now conclude that $(g \circ f)(-1) = -26$.

c.) $(f \circ g)(x) = f(g(x))$ We use the definition.

Since we don't have a number for x this time there will not be a number answer for this one. What we can do is replace the $g(x)$ with the expression $3x + 1$. So now we will do $f(3x + 1)$. This means where ever there is an x in the f function we will replace it with 3x + 1:
$f(3x + 1) = 5(3x + 1) - 4$ Now simplify:
$f(3x + 1) = 15x + 5 - 4 = 15x + 1$ So we write our answer as: $(f \circ g)(x) = 15x + 1$
Notice what happens if we put a 2 in for x in our answer. We will get 31, which is the same as in part a.)

d.) $(g \circ f)(x) = g(f(x))$ We use the definition.

Since we don't have a number for x this time there will not be a number answer for this one. What we can do is replace the $f(x)$ with the expression $5x - 4$. So now we will do $g(5x - 4)$. This means where ever there is an x in the g function we will replace it with 5x - 4:
$g(5x - 4) = 3(5x - 4) + 1$ Now simplify:
$g(5x - 4) = 15x - 12 + 1 = 15x - 11$ So we write our answer as: $(g \circ f)(x) = 15x - 11$

Notice what happens if we put a -1 in for x in our answer. We will get -26, which is the same as in part b.)

e.) $(f \circ f)(x) = f(f(x))$

So for this one we are putting in *f* into itself. Replace $f(x)$ inside with $5x - 4$. You will have $f(5x-4)$. Now replace the x in $5x - 4$ with $5x - 4$: $5(5x-4)-4$. Now simplify: $25x-20-4=25x-24$. Therefore $(f \circ f)(x) = 25x-24$.

f.) $(g \circ g)(x) = g(g(x))$

So for this one we are putting in g into itself. Replace $g(x)$ inside with $3x + 1$. You will have $g(3x+1)$. Now replace the x in $3x + 1$ with $3x + 1$: $3(3x+1)+1$. Now simplify: $9x+3+1=9x+4$. Therefore $(g \circ g)(x) = 9x+4$.

EXAMPLE: Given: $f(x)=x+3$ and $g(x)=2x^2-1$ find the following:
$(f \circ g)(-1)$, $(g \circ f)(0)$, $(f \circ g)(x)$, $(g \circ f)(x)$, $(f \circ f)(x)$, $(g \circ g)(x)$

For this one the process is the same as I described about. I will only show the algebraic steps here.

a.) $(f \circ g)(-1) = f(g(-1))$
$g(-1) = 2(-1)^2 - 1$
$g(-1) = 1$
$f(g(-1)) = f(1) = 1+3 = 4$, so $(f \circ g)(-1) = 4$

b.) $(g \circ f)(0) = g(f(0))$
$f(0) = 0+3$
$f(0) = 3$
$g(f(0)) = g(3) = 2(3)^2 - 1 = 17$, so $(g \circ f)(0) = 17$

c.) $(f \circ g)(x) = f(g(x))$
$f(2x^2-1) = (2x^2-1)+3$, so $(f \circ g)(x) = 2x^2+2$

d.) $(g \circ f)(x) = g(f(x))$
$g(x+3) = 2(x+3)^2 - 1 = 2(x^2+6x+9) - 1 = 2x^2+12x+18-1$, so $(g \circ f)(x) = 2x^2+12x+17$

e.) $(f \circ f)(x) = f(f(x))$
$f(x+3) = (x+3)+3 = x+6$, so $(f \circ f)(x) = x+6$

f.) $(g \circ g)(x) = g(g(x))$
$g(2x^2-1) = 2(2x^2-1)^2 - 1 = 2(4x^4-4x^2+1) - 1 = 8x^4-8x^2+2-1$
$(g \circ g)(x) = 8x^4-8x^2+1$

EXAMPLE: Given: $f(x) = 4 - x^2$ and $g(x) = \dfrac{-1}{x}$ find the following: $(f \circ g)(0)$, $(f \circ g)(x)$, $(g \circ f)(x)$, $(f \circ f)(x)$, $(g \circ g)(x)$. Express all as single fractions in factored form if possible.

a.) $(f \circ g)(0) = f(g(0))$
$g(0) = undefined$
Since this is undefined, that means that $(f \circ g)(0)$ is also undefined.

b.) $(f \circ g)(x) = f(g(x))$

$f\left(\dfrac{-1}{x}\right) = 4 - \left(\dfrac{-1}{x}\right)^2 = 4 - \dfrac{1}{x^2}$ We now want to express this as a single fraction. Use common denominators.

$= 4\left(\dfrac{x^2}{x^2}\right) - \dfrac{1}{x^2} = \dfrac{4x^2}{x^2} - \dfrac{1}{x^2} = \dfrac{4x^2 - 1}{x^2}$ Now we just need to factor the numerator.

$(f \circ g)(x) = \dfrac{(2x-1)(2x+1)}{x^2}$

What is the domain of this? It is $(-\infty, 0) \cup (0, \infty)$.

c.) $(g \circ f)(x) = g(f(x))$

$g(4 - x^2) = \dfrac{-1}{4 - x^2}$ Since we already have a single fraction we now just need to factor the denominator.

$(g \circ f)(x) = \dfrac{-1}{(2-x)(2+x)}$

d.) $(f \circ f)(x)$

$f(f(x)) = f(4 - x^2) = 4 - (4 - x^2))^2 = 4 - (x^4 - 8x^2 + 16) = -x^4 + 8x^2 - 12$

e.) $(g \circ g)(x)$

$g(g(x)) = g\left(-\dfrac{1}{x}\right) = \dfrac{-1}{\left(-\dfrac{1}{x}\right)} = x$

Suppose they asked you to find the domain on this one. The answer to part *e* was just *x*. We know we can put in any number for *x*, however if you look at the algebra step right before, we see that *x* cannot be 0 otherwise we will have something that is undefined. So the domain for this problem is $(-\infty, 0) \cup (0, \infty)$. Therefore, when finding domains on these fraction problems, be careful. Make sure you consider all the algebraic steps.

EXAMPLE: If $f(x)=\frac{5}{x-3}$ and $g(x)=\frac{1}{x}$, find the domain of $(f\circ g)(x)$ in set builder notation.

First we will find $(f\circ g)(x)=f(g(x))=f\left(\frac{1}{x}\right)$. You will get $\frac{5}{\frac{1}{x}-3}$. Before we simplify this one, let's first analyze what we have. First of all we notice that if we put a 0 into this expression, it will be undefined. Therefore we know right away that 0 is not in the domain. Now let's keep going and simplify this as far as we can go: $(f\circ g)(x)=\frac{5}{\frac{1}{x}-3}=\frac{5}{\frac{1-3x}{x}}=\frac{5x}{1-3x}$. So now let's find the domain of $(f\circ g)(x)=\frac{5x}{1-3x}$. This means the bottom cannot equal zero. Setting the bottom equal to zero we get that x cannot be 1/3. So the domain is $\left\{x\mid x\neq 0,\ x\neq\frac{1}{3}\right\}$

EXAMPLE: Find functions f and g so that $(f\circ g)(x)=H(x)$ given that $H(x)=\left(1+x^2\right)^3$

This problem is the reverse of what we were just doing. Now we have the finished product and we want to go back to the beginning. We want to find the two functions f and g so that when we find $(f\circ g)(x)$ we will get the expression $\left(1+x^2\right)^3$. First let's find the $g(x)$. This is usually an 'inside' function, one put into something else. In H we have the expression $1+x^2$. This is inside a set of parenthesis raised to the third power. So we will let $g(x)=1+x^2$. Now the 'outside' function is the $f(x)$. In order to find this function, replace the 'inside' function with x in H(x) and then replace H(x) with f(x). In this case we will replace $1+x^2$ with x. You will get $f(x)=x^3$. Now let's check to make sure we did it correctly. If we let $f(x)=1+x^2$ and $g(x)=x^3$ let's find $(f\circ g)(x)$. To do this, we place the g into the f. We will get $\left(1+x^2\right)^3$, which is equal to H. So the answers are: $f(x)=x^3$ and $g(x)=1+x^2$.

EXAMPLE: Find functions f and g so that $(f\circ g)(x)=H(x)$ given that $H(x)=\sqrt{2x-1}-4x+2$

First let's factor H: $H(x)=\sqrt{2x-1}-2(2x-1)$. Notice we have the expression 2x – 1 repeating. This is our 'inside' function, so $g(x)=2x-1$. To find $f(x)$, replace the 2x – 1 with x in the H(x) equation and then replace H(x) with *f*(x). You will get $f(x)=\sqrt{x}-2x$. So our answers are $g(x)=2x-1$ and $f(x)=\sqrt{x}-2x$.

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EXAMPLE: Use the given table to evaluate each composition.

x	-3	-2	-1	0	1	2	3
$f(x)$	11	9	7	5	3	1	-1
$g(x)$	-8	-3	0	1	0	-3	-8

a.) Find $(f \circ g)(1)$.

First we can rewrite this as $f(g(1))$. First we will find $g(1)$. Go to the x row and find 1. Then go down to the $g(x)$ row. Therefore $g(1)=0$. So now our problem becomes $f(0)$. To find this we will go to the x row and find 0. Then go down to the $f(x)$ row and you will find $f(0)=5$. Therefore, $(f \circ g)(1)=5$.

b.) Find $(f \circ g)(2)$.

First we can rewrite this as $f(g(2))$. First we will find $g(2)$. Go to the x row and find 2. Then go down to the $g(x)$ row. Therefore $g(2)=-3$. So now our problem becomes $f(-3)$. To find this we will go to the x row and find –3. Then go down to the $f(x)$ row and you will find $f(-3)=11$. Therefore, $(f \circ g)(2)=11$.

c.) Find $(g \circ f)(2)$.

First we can rewrite this as $g(f(2))$. First we will find $f(2)$. Go to the x row and find 2. Then go down to the $f(x)$ row. Therefore $f(2)=1$. So now our problem becomes $g(1)$. To find this we will go to the x row and find 1. Then go down to the $g(x)$ row and you will find $g(1)=0$. Therefore, $(g \circ f)(2)=0$.

d.) Find $(g \circ g)(1)$.

First we can rewrite this as $g(g(1))$. First we will find $g(1)$. Go to the x row and find 1. Then go down to the $g(x)$ row. Therefore $g(1)=0$. So now our problem becomes $g(0)$. To find this we will go to the x row and find 0. Then go down to the $g(x)$ row and you will find $g(0)=1$. Therefore, $(g \circ g)(1)=1$.

e.) Find $(f \circ f)(3)$.

First we can rewrite this as $f(f(3))$. First we will find $f(3)$. Go to the x row and find 3. Then go down to the $f(x)$ row. Therefore $f(3)=-1$. So now our problem becomes $f(-1)$. To find this we will go to the x row and find –1. Then go down to the $f(x)$ row and you will find $f(-1)=7$. Therefore, $(f \circ f)(3)=7$.