R.3 Dividing Polynomials

We will be doing a quick review of long division since we need to know this when working with rational functions. To divide using long division we do the same steps as if we are working with numbers.

EXAMPLE: Divide by using long division: $\left(2x^2+3x-35\right)\div(x+5)$

We will do this the same way we did the number example. First we need to set it up:

$$\begin{array}{r} 2x-7 \\ x+5\overline{)2x^2+3x-35} \\ \underline{2x^2+10x} \\ -7x-35 \\ \underline{-7x-35} \\ 0 \end{array}$$

First we ask ourselves how many times x goes into $2x^2$. We ignore the + 5 at the moment. We know x goes into $2x^2$ an amount of 2x. We then multiply x + 5 by 2x to get $2x^2+10x$. We write this on the next line and then we subtract. Then we get -7x – 35. We now ask how many times does x go into -7x. The answer is -7. We then multiply x + 5 by -7 to get -7x – 35. We subtract and get a remainder of zero.

EXAMPLE: Divide by using long division: $\left(3x^3+x+5\right)\div(x+1)$

We notice that this one is missing a term. You want to make sure that each term is accounted for. We need to add in a $0x^2$ as a place keep.

$$\begin{array}{r} 3x^2-3x+4 \\ x+1\overline{)3x^3+0x^2+x+5} \\ \underline{3x^3+3x^2} \\ -3x^2+x \\ \underline{-3x^2-3x} \\ 4x+5 \\ \underline{4x+4} \\ 1 \end{array}$$

Remember you are always subtracting when doing long division.

This one has a remainder. Here is how you want to write your answer: $3x^2-3x+4+\dfrac{1}{x+1}$

EXAMPLE: Divide by using long division: $\left(x+3x^3-x^2-2\right)\div(x^2+2)$

First we need to make sure the terms are written in descending order. So we wan to write this as: $\left(3x^3-x^2+x-2\right)\div(x^2+2)$. The next thing we notice is that we are missing a term in x^2+2. The x term is missing so we must add a 0x to make it x^2+0x+2.

$$\begin{array}{r} 3x-1 \\ x^2+0x+2\overline{)3x^3-x^2+x-2} \\ \underline{3x^2+0x^2+6x} \\ -x^2-5x-2 \\ \underline{-x^2+0x-2} \\ -5x \end{array}$$

Remember you are always subtracting when doing long division.

This one also has a remainder. Here is how you want to write your answer: $3x-1+\dfrac{-5x}{x^2+2}$.

EXAMPLE: Divide by using long division: $\left(-3x^4-2x-1\right)\div(x-1)$

We notice that this one is missing terms. We need to put in a $0x^3$ and $0x^2$.

$$\begin{array}{r} -3x^3-3x^2-3x-5 \\ x-1\overline{)-3x^4+0x^3+0x^2-2x-1} \\ \underline{-3x^4+3x^3} \\ -3x^3+0x^2 \\ \underline{-3x^3+3x^2} \\ -3x^2-2x \\ \underline{-3x^2+3x} \\ -5x-1 \\ \underline{-5x+5} \\ -6 \end{array}$$

Remember you are always subtracting when doing long division.

We have a remainder once more. Here is how you want to write your answer: $-3x^3-3x^2-3x-5+\dfrac{-6}{x-1}$

Synthetic Division is alternative to long division. It is used when dividing by something in the form x – or x + something. In all the problems in this section pertaining to synthetic division there will be a one in front of the x.

EXAMPLE: Divide using synthetic division: $\left(x^3-7x^2-13x+15\right)\div\left(x+2\right)$

In order to do this we must first set it up. In the problem above we have x + 2. You want to reverse the sign of the number after the x and place it in a half box like shown below. In this case we have a negative 2 since we reversed the sign. After this number you want to write the coefficients of your equation afterwards. A coefficient is just the number that comes in front of the variable. In synthetic division there are no variables. You are only working with numbers. Under this line skip a space and then draw a line. Your set up should look like this:

$$\begin{array}{r|rrrr} -2 & 1 & -7 & -13 & 15 \\ & & & & \\ \hline \end{array}$$

Here is how you do synthetic division. You always drop down the number right after the box like the following:

```
-2| 1  -7  -13  15
    ___________
    1
```

Now we will multiply the 1 by -2 and we will write this result under the -7.

```
-2| 1  -7  -13  15
    ___-2________
    1
```

In synthetic division we always add. We will add -7 and -2 to get -9.

```
-2| 1  -7  -13  15
    ___-2________
    1  -9
```

Now multiply -9 by -2 to get 18. This number gets written under the -13.

```
-2| 1  -7  -13  15
    ___-2__18___
    1  -9
```

Now add -13 and 18 to get 5.

```
-2| 1  -7  -13  15
    ___-2__18___
    1  -9    5
```

Multiply 5 by -2 to get -10. Now write this under the 15.

```
-2| 1  -7  -13   15
    ___-2__18__-10_
    1  -9    5
```

Now add 15 and -10 to get 5.

```
-2| 1  -7  -13   15
    ___-2__18__-10_
    1  -9    5    5
```

Now we need to write our answer (see next page).

```
-2| 1  -7  -13   15
    ___-2__18__-10_
    1  -9    5    5
```

The last line above, which shows 1 -9 5 5 are the coefficients for our answer. The last 5 is our remainder, which we write over x + 2. We always start our answer with a power one less than what we began with. Since the original started with x^3 then our answer will start with x^2. Write our answer as: $x^2 - 9x + 5 + \frac{5}{x+2}$.

EXAMPLE: Divide using synthetic division: $\left(3x^2 - 2x^3 + x^4 - 4\right) \div (x-3)$

Before we set this problem up we need to make sure we have descending powers. We also need to make sure no powers are missing. If they are then we need to insert a zero as a place keeper. We will rewrite this as: $\left(x^4 - 2x^3 + 3x^2 + 0x - 4\right) \div (x-3)$. This one the x term was missing so this is a zero place keeper.

In the problem above we have x – 3. You want to reverse the sign of the number after the x and place it in a half box like shown below. In this case we have a positive 3 since we reversed the sign. After this number you want to write the coefficients of your equation afterwards. Under this line skip a space and then draw a line. Notice below we still wrote the 0 to hold the place. Your set up should look like this:

```
3| 1 -2 3 0 -4
   __________
```
Start by dropping the 1 below the line we just drew.

```
3| 1 -2 3 0 -4
   __________
    1
```
Now multiply 1 by 3 and write this result under -2.

```
3| 1 -2 3 0 -4
   ___3_______
    1
```
Now ADD the -2 and 3 to get 1. We always add with synthetic division.

```
3| 1 -2 3 0 -4
   ___3_______
    1 1
```
Now multiply the 1 by 3 and write this under the 3.

```
3| 1 -2 3 0 -4
   ___3_3_____
    1 1 6
```
Now multiply 6 by 3 to get 18. Write this under 0.

```
3| 1 -2 3 0 -4
   ___3_3_18_____
    1 1 6
```
Add and repeat the process to the end. You will get:

```
3| 1 -2 3 0   -4
   ___3_3_18_54___
    1 1 6 18 50
```
The last line shows the coefficients for our answer. We always start our answer with a power one less than what we began with. Since the original started with x^4 then our answer will start with x^3. The 50 above is our remainder, which we write over x – 2. Write our answer as: $x^3 + x^2 + 6x + 18 + \frac{50}{x-3}$.

EXAMPLE: Divide using synthetic division: $(2x^5 - 6x^3 - 14x) \div (x-2)$

We also need to make sure no powers are missing. If they are then we need to insert a zero as a place keeper. We will rewrite this as: $(2x^5 + 0x^4 - 6x^3 + 0x^2 - 14x + 0) \div (x-2)$. We had the x^4, x^2 and the constant term missing. You need all three of these zeros as place keepers. We are dividing by x – 2. You want to reverse the sign of the number after the x and place it in a half box like shown below. In this case we have a positive 2 since we reversed the sign. Your set up should look like this:

```
2| 2 0 -6 0 -14 0
   ______________
```
Start by dropping the 2 below the line we just drew.

```
2| 2 0 -6 0 -14 0
   ______________
   2
```
Multiply the 2 by 2 and write this under the 0.

```
2| 2 0 -6 0 -14 0
   _4_____________
   2 4
```
Add 0 and 4 to get 4. Now multiply the 4 by 2 to get 8. Write this under -6.

```
2| 2  0  -6  0  -14  0
      4   8
   2  4
```

Add -6 and 8 to get 2. Then multiply the 2 by 2 to get 4. Write this under the 0.

```
2| 2  0  -6  0  -14  0
      4   8  4
   2  4   2
```

Add 0 and 4 to get 4. Now multiply the 4 by 2 to get 8. Write this under -14.

```
2| 2  0  -6  0  -14  0
      4   8  4    8
   2  4   2  4
```

Add -14 and 8 to get -6. Now multiply the -6 by 2 to get -12. Write this under 0.

```
2| 2  0  -6  0  -14   0
      4   8  4    8  -12
   2  4   2  4   -6
```

Now add 0 to -12 to get -12. This is our remainder.

```
2| 2  0  -6  0  -14   0
      4   8  4    8  -12
   2  4   2  4   -6  -12
```

Now we need to write our answer. The last line shows the coefficients for our answer. We always start our answer with a power one less than what we began with. Since the original started with x^5 then our answer will start with x^4. The -12 above is our remainder, which we write over x – 2. Write our answer as:

$$2x^4 + 4x^3 + 2x^2 + 4x - 6 + \frac{-12}{x-2}.$$