

11.5 Partial Fraction Decomposition

Suppose we were asked to write the following as a single fraction: $\frac{4}{x+1} - \frac{3}{x-2}$.

Three Rules of How a Fraction Decomposes

Let $P(x)$ be a polynomial.

Rule 1:
$$\frac{P(x)}{(x-a_1)(x-a_2)\dots(x-a_n)} = \frac{A_1}{x-a_1} + \frac{A_2}{x-a_2} + \dots + \frac{A_n}{x-a_n}$$

Rule 2:
$$\frac{P(x)}{(x-a)^n} = \frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \frac{A_3}{(x-a)^3} + \dots + \frac{A_n}{(x-a)^n}$$

Rule 3:
$$\frac{P(x)}{(ax^2+bx+c)^n} = \frac{A_1x+B_1}{(ax^2+bx+c)} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \dots + \frac{A_nx+B_n}{(ax^2+bx+c)^n}$$

Where ax^2+bx+c is irreducible, or nonfactorable.

EXAMPLE: Set up the following for decomposition but DO NOT SOLVE: $\frac{3x-5}{x^2-6x+5}$.

EXAMPLE: Set up the following for decomposition but DO NOT SOLVE: $\frac{2x-4}{(x^2+x+1)^3}$.

EXAMPLE: Set up the following for decomposition but DO NOT SOLVE: $\frac{2x^2-1}{(3x-5)^4}$.

EXAMPLE: Set up the following for decomposition but DO NOT SOLVE: $\frac{4x^2-3x+1}{x(x-3)^2}$.

EXAMPLE: Determine the partial fraction decomposition: $\frac{5x + 27}{x^2 - 9}$.

EXAMPLE: Determine the partial fraction decomposition: $\frac{6-4x}{x^3-x^2-4x+4}$.

EXAMPLE: Determine the partial fraction decomposition: $\frac{7x}{(x-5)^2}$.

EXAMPLE: Determine the partial fraction decomposition: $\frac{x+2}{x^3-x^2}$.

EXAMPLE: Determine the partial fraction decomposition: $\frac{x^2 + 3}{x^3 + 6x^2 + 9x}$.

EXAMPLE: Determine the partial fraction decomposition: $\frac{x-7}{x^3+2x}$.