4.1 Angles

This section will cover how angles are drawn and also arc length and rotations.

Angles are measured a couple of different ways. The first unit of measurement is a **degree** in which 360° (degrees) is equal to one revolution. Most likely the reason why we use 360 is from the Babylonians, whose year is based on 360 days.

Another unit of measurement for angles is **radians**. In radians, 2π is equal to one revolution. So a conversion between radians and degrees is $2\pi = 360^\circ$, or $\pi = 180^\circ$.

When converting from degrees to radians:

Multiply your degrees by $\frac{\pi}{180}$

When converting from radians to degrees:

Multiply your radians by $\frac{180}{\pi}$

EXAMPLE: Convert 60° to radians.

EXAMPLE: Convert -405° to radians.

EXAMPLE: Convert $\frac{4\pi}{3}$ into degrees.

EXAMPLE: Convert $-\frac{3\pi}{2}$ into degrees.

Degrees, Minutes, Seconds

A degree can be divided into 60 equal parts called **minutes** $(\min')$ and each minute is divided into 60 equal parts called **seconds** $(\sec'')$. Below are the conversions:

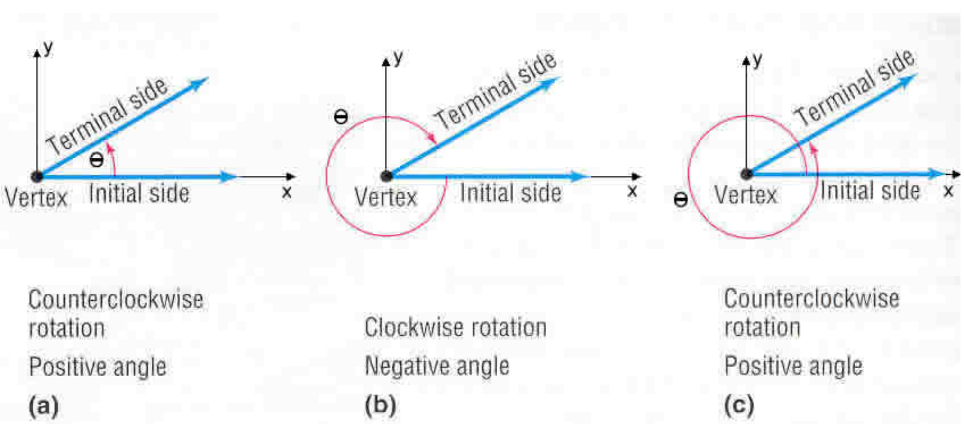
$$1 \text{ min} = \left(\frac{1}{60}\right)^{\circ} \text{ or } 1' = \left(\frac{1}{60}\right)^{\circ}$$

$$1 \text{ sec} = \left(\frac{1}{60}\right)' = \left(\frac{1}{3600}\right)^{\circ} \text{ or } 1'' = \left(\frac{1}{60}\right)' = \left(\frac{1}{3600}\right)^{\circ}$$

EXAMPLE: Write 83 degrees, 24 minutes, 13 seconds in the correct notation. Then convert to decimal degrees. Round to 4 decimal places.

EXAMPLE: Convert 163.36° to degree, minute, second form.

We will use θ (theta) to represent an angle's measurement. In the figure below it describes how you know if an angle is positive or negative. The vertex of the angle is at the origin of a rectangular coordinate system. The positive x axis is always where an angle is measure from, and this is called the initial side. An angle drawn this way is said to be in **standard form**. An angle that goes counterclockwise is always positive, and clockwise angles are negative.



EXAMPLE: Draw each angle in standard position. Indicate which quadrant the angle lies.

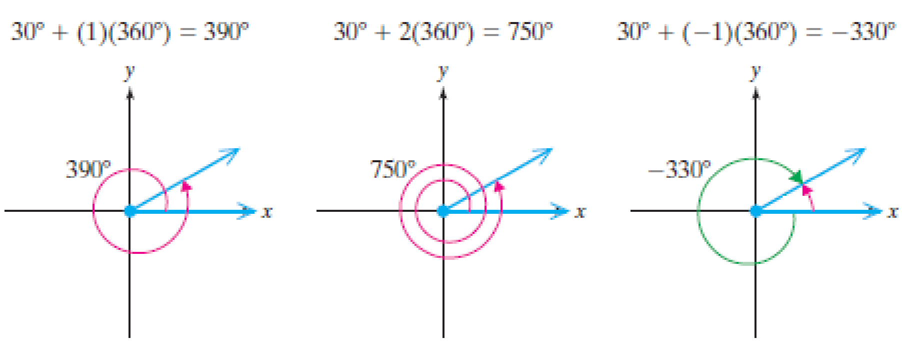
a.) $\frac{5\pi}{6}$

b.) -135°

c.) $\frac{8\pi}{3}$

Coterminal Angles

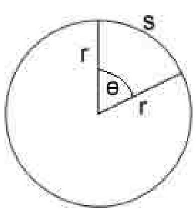
Two angles in standard position with the same initial side and same terminal side are called coterminal angles. The figure below shows three angles in standard position that are coterminal to 30 degrees. Notice that each angle is 30 degrees plus or minus some number of full revolutions clockwise or counterclockwise.



EXAMPLE: Find an angle coterminal to 960° between 0 and 360 degrees. Draw the original angle and the new angle in standard position.

EXAMPLE: Find an angle coterminal to -225° between 0 and 360 degrees. Draw the original angle and the new angle in standard position.

Arc length – this is the length of the arc between the two lines shown with θ.

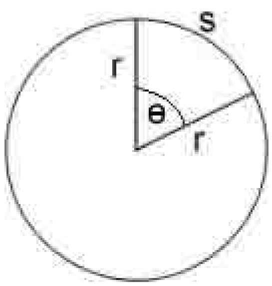


The equation is $S = r\theta$, where S is the arc length, r is the radius, and θ MUST be measured in RADIANS! The θ is also called the **central angle**.

EXAMPLE: Find the arc length of a sector whose radius is 3 inches and whose central angle is $\frac{\pi}{3}$.

EXAMPLE: Find the arc length of a sector whose radius is 7 inches and whose central angle is 45°.

Area of a Sector – this is the area of the piece of pie shown below.

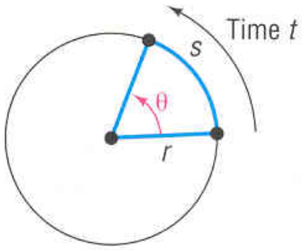


The equation is $A = \frac{1}{2}r^2\theta$, where A is the area of the sector, r is the radius, and again θ MUST be measured in RADIANS!

EXAMPLE: Find the area of a sector with a radius of 4 inches with a central angle of $\frac{4\pi}{3}$.

EXAMPLE: Find the area of a sector with a radius of 6 inches with a central angle of 150°.

Angular Speed (ω)



Angular speed is the angle θ that can be swept out in time t. The formula is:

$\omega = \dfrac{\theta}{t}$ where θ is in radians.

Linear Speed (*v*)

This is the speed at which a point on the circle is moving. It is measured by what arc length is traveled in time t. The formula is:

$v = r\omega$ where ω is measured in radians per unit of time.

EXAMPLE: A circular gear rotates at a rate of 75 rpm (revolutions per minute). What is the angular speed (in radians per minute)? What is the linear speed of a point on the gear 3mm from the center?

EXAMPLE: A wheel rotates at a rate of 2160° per second. What is the angular speed (in radians per minute)? What is the linear speed of a point on the wheel 30 cm from the center?

EXAMPLE: To approximate the speed of the current of a river, a circular paddle wheel with a radius of 4 feet is lowered into the water. If the current causes the wheel to rotate at a speed of 10 rpm, what is the speed of the current in miles per hour (1 mile = 5280 feet).