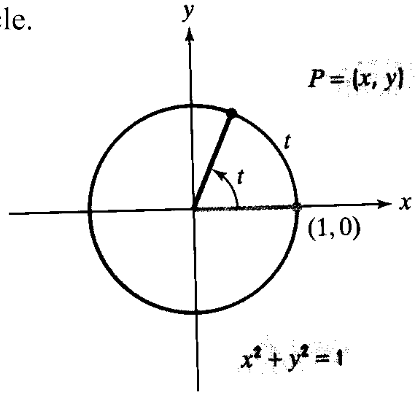
# 4.2 Trigonometric Functions: Unit Circle Approach

A **unit circle** is a circle centered at the origin with a radius of 1. Its equation is $x^2 + y^2 = 1$ as shown in the drawing below. Here the letter t represents an angle measure. The point P=(x, y) represents a point on the unit circle.




The following definitions are given based on this picture.

$\sin t = y \qquad \csc t = \dfrac{1}{y}$

$\cos t = x \qquad \sec t = \dfrac{1}{x}$

$\tan t = \dfrac{y}{x} \qquad \cot t = \dfrac{x}{y}$

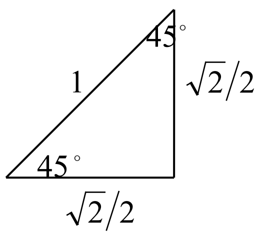
If we start with $x^2 + y^2 = 1$ and put in our definitions above we will have $\cos^2\theta + \sin^2\theta = 1$, an identity we will come back to.

EXAMPLE: Suppose a point on the unit circle is $\left(-\dfrac{5}{13}, -\dfrac{12}{13}\right)$. Find all six trigonometric values.

EXAMPLE: Use a calculator to find the approximate value of $\cos 14^\circ$ rounded to the two decimal places.

EXAMPLE: Use a calculator to find the approximate value of $\sin\frac{\pi}{8}$ rounded to the two decimal places.

**45 – 45 – 90   Triangle**




We can use our definitions of sine, cosine, and tangent to find exact values:

$\sin 45 =$    $\cos 45 =$    $\tan 45 =$

From our above definitions we can also find the following:

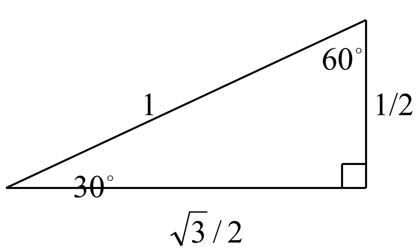
$\csc 45^\circ =$

$\sec 45^\circ =$

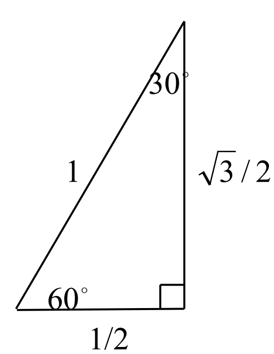
$\cot 45^\circ =$

Now let's look at two other special angles on the unit circle.  We will consider $t = 30^\circ$ and $t = 60^\circ$

**30 – 60 – 90  Triangle** ($t = 30^\circ$)




**30 – 60 – 90  Triangle** ($t = 60^\circ$)



We can use our definitions of sine, cosine, and tangent again to find exact values with 30 and 60 degrees.

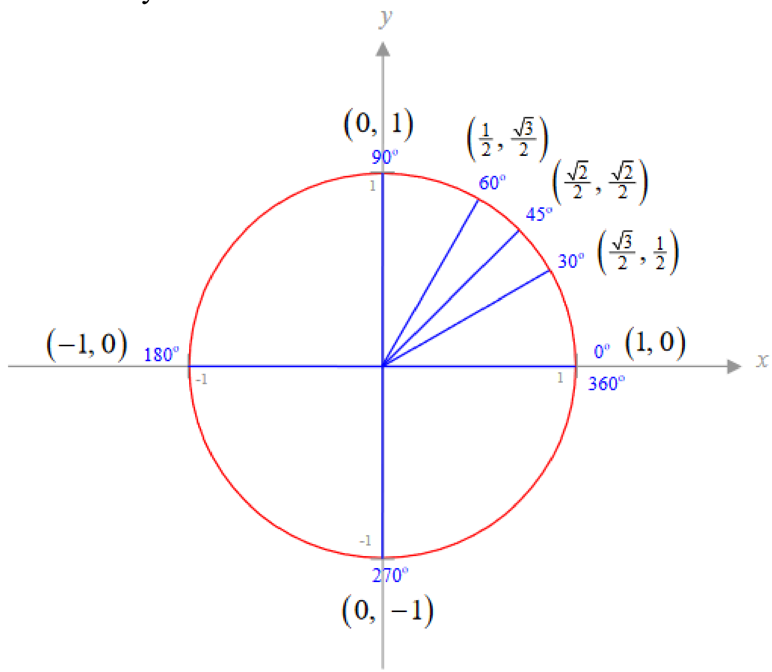
$\sin 30^{\circ} =$ $\cos 30^{\circ} =$ $\tan 30^{\circ} =$

$\sin 60^{\circ} =$ $\cos 60^{\circ} =$ $\tan 60^{\circ} =$

**Table of Trigonometric Values and Unit Circle**

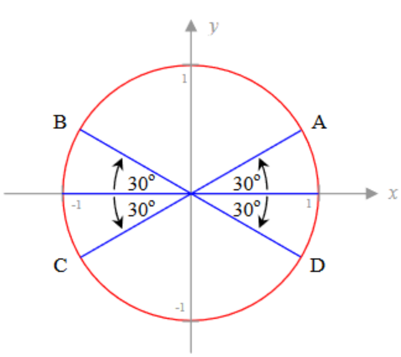
We can display this information in two different ways. The first way shown below is a table. We can also fill in the first quadrant of the unit circle as shown below.

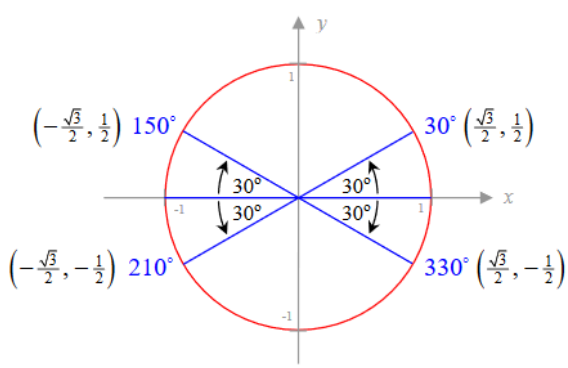
| $\theta$ (degrees) | $\theta$ (radians) | $\sin\theta$ | $\cos\theta$ | $\tan\theta$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 |
| 30 | $\frac{\pi}{6}$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{3}$ |
| 45 | $\frac{\pi}{4}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ | 1 |
| 60 | $\frac{\pi}{3}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\sqrt{3}$ |
| 90 | $\frac{\pi}{2}$ | 1 | 0 | undefined |



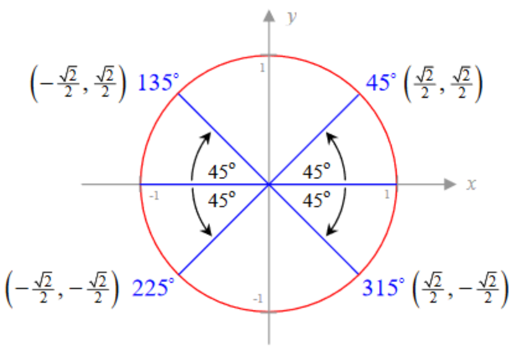

**Using Symmetry with the Unit Circle**

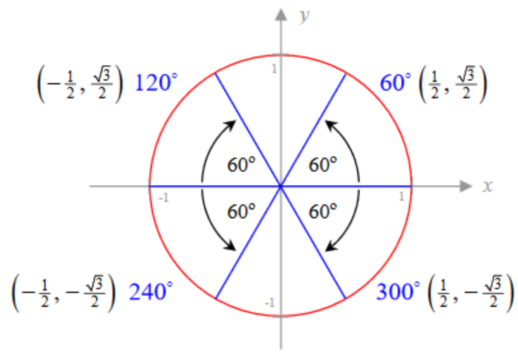
We have the first quadrant of the unit circle filled out, however it would be nice to fill out the rest of the circle. We can do this by symmetry. Below is a diagram that shows where there are other multiples of 30 degrees. As you can see, we can use symmetry to write the coordinates in other quadrants. For example, if we look at 30 degrees and 150 degrees, we see that these two points on the unit circle would have the same y value, however the x value would have opposite signs.



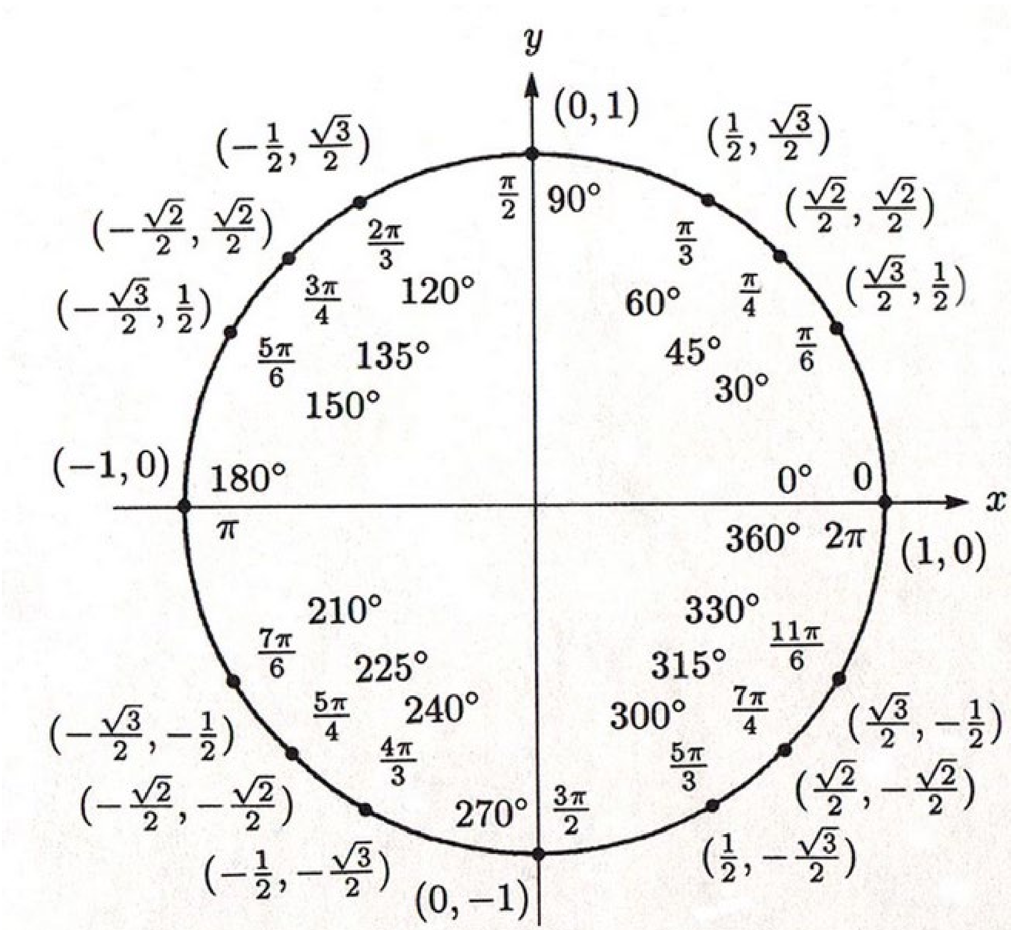



Now we can use this same logic to look at other multiples of 45 degrees and 60 degrees:




Now we can finally put all of this together into to get our unit circle with all special angles labeled:




EXAMPLE: Use the unit circle to evaluate the six trigonometric functions of the real number $t = \frac{2\pi}{3}$.

$\cos t =$ $\sin t =$ $\csc t =$

$\sec t =$ $\tan t =$ $\cot t =$

**Even – Odd Properties**

$\cos(-t) = \cos t$ $\qquad$ $\sec(-t) = \sec t$

$\sin(-t) = -\sin t$ $\qquad$ $\csc(-t) = -\csc t$

$\tan(-t) = -\tan t$ $\qquad$ $\cot(-t) = -\cot t$

**Periodic Properties**

If we start at an angle and go around one revolution ($360^\circ$ or $2\pi$ radians) we will end up at the same angle we started with. The *k* value is any integer, and represents how many revolutions are going around. If you want to use degrees, replace the $2\pi k$ in the equations below with 360*k*. For tangent and cotangent you will end up at the same spot if you add $2\pi$, however you will also get the same value if you just add $\pi$.

$\sin(t \pm 2\pi k) = \sin t$ $\qquad$ $\csc(t \pm 2\pi k) = \csc t$

$\cos(t \pm 2\pi k) = \cos t$ $\qquad$ $\sec(t \pm 2\pi k) = \sec t$

$\tan(t \pm \pi k) = \tan t$ $\qquad$ $\cot(t \pm \pi k) = \cot t$

EXAMPLE: Given that $\sec\left(\frac{11\pi}{12}\right) = \sqrt{2} - \sqrt{6}$, determine the value of $\sec\left(-\frac{13\pi}{12}\right)$.

EXAMPLE: Use the Even-Odd Properties and Periodic Properties to simplify:
$-2\sin(3t + 2\pi) - 3\sin(-3t) + \cos(-2t)$