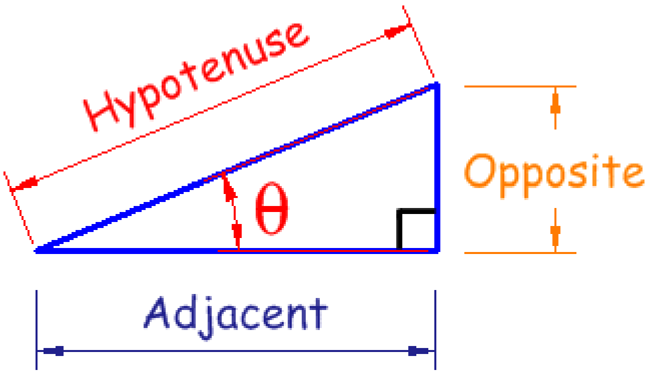
4.3 Right Triangle Trigonometry

This is a very important section since we are giving definitions for the six trigonometric functions you be using throughout the rest of this course and beyond. We need to first start with a drawing of a right triangle. The following definitions only apply to RIGHT TRIANGLES.

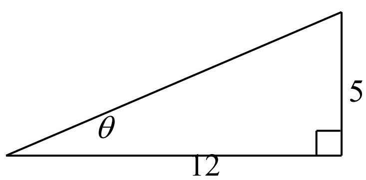


$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \csc\theta = \frac{hypotenuse}{opposite}$$

$$\cos\theta = \frac{adjacent}{hypotenuse} \qquad \sec\theta = \frac{hypotenuse}{adjacent}$$

$$\tan\theta = \frac{opposite}{adjacent} \qquad \cot\theta = \frac{adjacent}{opposite}$$

EXAMPLE: Find the exact value of the 6 trig functions using the following figure:

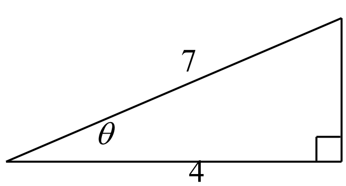


$\sin\theta =$ $\qquad$ $\csc\theta =$

$\cos\theta =$ $\qquad$ $\sec\theta =$

$\tan\theta =$ $\qquad$ $\cot\theta =$

EXAMPLE: Find the exact value of the 6 trig functions using the following figure:

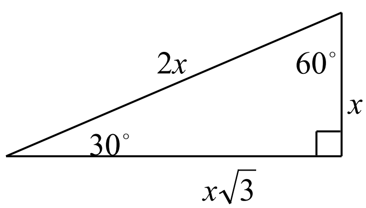


$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

30 – 60 – 90   Triangle



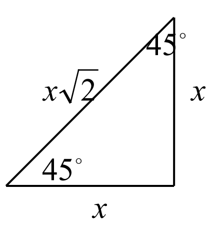
In this triangle the opposite side of 30 degrees is always half of the hypotenuse. The adjacent side is always $\sqrt{3}$ times the opposite. From this relationship we can get values for 30 and 60 degrees.

$\sin 30 =$ $\sin 60 =$

$\cos 30 =$ $\cos 60 =$

$\tan 30 =$ $\tan 60 =$

45 – 45 – 90   Triangle



In this triangle the opposite and adjacent sides are the same. The hypotenuse is always $\sqrt{2}$ times the opposite or adjacent. From this relationship we can get values for 45 degrees.

$\sin 45 =$

$\cos 45 =$

$\tan 45 =$

As with the unit circle, you can put all the information on table, but I will also add the other 3 trig functions:

Table of trigonometric values

θ (degrees)	θ (radians)	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

EXAMPLE: Find the exact value without using a calculator: $2\cos^2 30^\circ - \sin 30^\circ$

EXAMPLE: Find the exact value without using a calculator: $\dfrac{1-\cos 60^\circ}{\sin 60^\circ}$.

EXAMPLE: Find the exact value without using a calculator: $\tan\dfrac{\pi}{4} + \cot\dfrac{\pi}{4}$.

If you refer back to the previous table of trig values, you will notice that you see the same value repeated. This leads to the following identities:

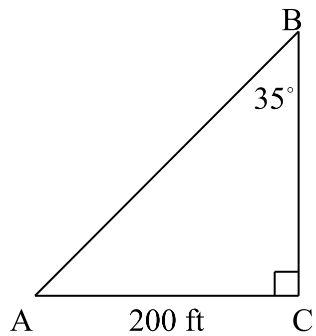
Cofunction Identities

$\sin\theta = \cos(90-\theta)$	$\csc\theta = \sec(90-\theta)$
$\cos\theta = \sin(90-\theta)$	$\sec\theta = \csc(90-\theta)$
$\tan\theta = \cot(90-\theta)$	$\cot\theta = \tan(90-\theta)$

EXAMPLE: Write the following as an equivalent cosine expression: $\sin 33$.

EXAMPLE: Given the function value, find a cofunction of another angle with the same value: $\csc\frac{\pi}{3} = \frac{2\sqrt{3}}{3}$.

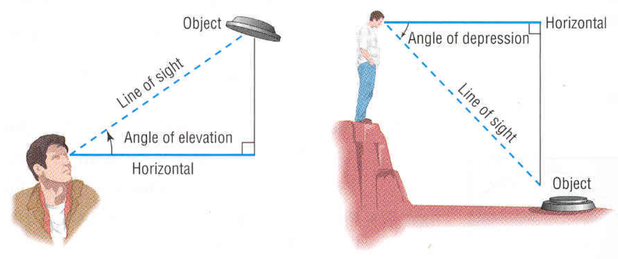
EXAMPLE: Solve the triangle:



EXAMPLE: A ladder is leaning against a building and forms an angle of 72 degrees with the ground. If the foot of the ladder is 6 feet from the base of the building how far up the building does the ladder reach? How long is the ladder?

Angle of elevation and depression

When you look up at something you have an angle of elevation, and when you look down on something you have an angle of depression.



EXAMPLE: While standing 4000 feet away from the base of the CN tower in Toronto, the angle of elevation was measured to be 24.4 degrees. Find the height of the tower.

EXAMPLE: An observer in a lighthouse is 66 feet above the surface of the water. The observer sees a ship and finds the angle of depression to be 0.7°. Estimate the distance from the ship to the base of the lighthouse in miles.

Bearing

Bearing is a way to measure direction. There are three parts to bearing. The first part is either a N or S. The second part is an acute angle that is ALWAYS measured from a vertical axis. The third part is an E or W. The picture to the right shows how North, South, East, and West is orientated. The next few examples show how to draw bearings.

EXAMPLE: Draw the following: $N40^\circ E$.

EXAMPLE: Draw the following: $N65^\circ W$.

EXAMPLE: Draw the following: $S70^\circ E$.

EXAMPLE: Draw the following: $S20^\circ W$

EXAMPLE: A cyclist rides west for 10 miles and then north 7 miles. What is the bearing from her starting point?

EXAMPLE: A jeep leaves its present location and travels along a bearing of $N58^{\circ}E$ for 31 miles. How far north and east of its original position is it? Round to two decimal places as needed.