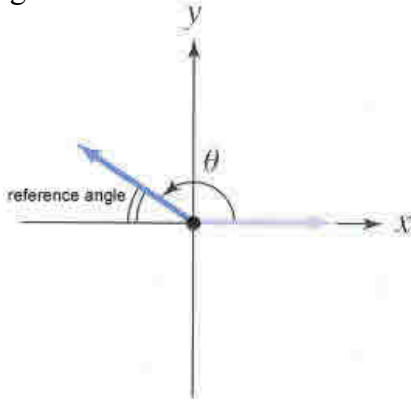


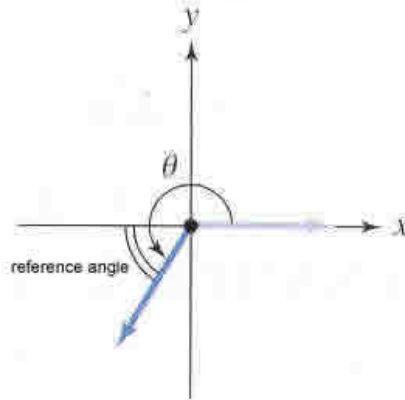
4.4 Reference Angles

Reference Angle – an angle between 0 and 90 that is formed by the terminal side of an angle and the x-axis. The reference angle is labeled below. It is indicated by the double curved lines. Notice that no matter where the angle is drawn it is measured from the x-axis. Under each drawing it tells you how to find the reference angle:



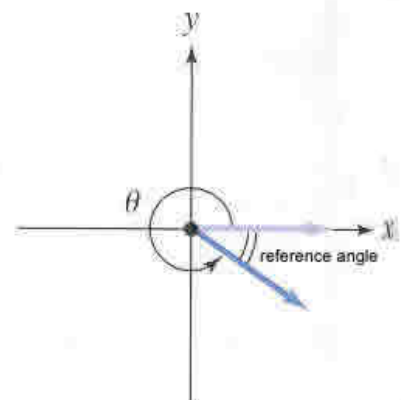
If $90^\circ < \theta < 180^\circ$ then
Ref. angle = $180^\circ - \theta$

If $\frac{\pi}{2} < \theta < \pi$ then
Ref. angle = $\pi - \theta$



If $180^\circ < \theta < 270^\circ$ then
Ref. angle = $\theta - 180^\circ$

If $\pi < \theta < \frac{3\pi}{2}$ then
Ref. angle = $\theta - \pi$



If $270^\circ < \theta < 360^\circ$ then
Ref. angle = $360^\circ - \theta$

If $\frac{3\pi}{2} < \theta < 2\pi$ then
Ref. angle = $2\pi - \theta$

EXAMPLE: Find the reference angle for 170° .

EXAMPLE: Find the reference angle for $\frac{9\pi}{5}$.

EXAMPLE: Draw 120° in standard position and then find its reference angle.

EXAMPLE: Draw $\frac{11\pi}{6}$ in standard position and then find its reference angle.

Sign values of sine, cosine, and tangent in each quadrant

$\sin \theta +$	$\sin \theta +$	Depending on which quadrant you are in the sine, cosine, and tangent functions will be either positive or negative. You will need this for
$\cos \theta -$	$\cos \theta +$	
$\tan \theta -$	$\tan \theta +$	
$\sin \theta -$	$\sin \theta -$	using reference angles to find trigonometric values.
$\cos \theta -$	$\cos \theta +$	
$\tan \theta +$	$\tan \theta -$	

The quadrants are numbered from 1 to 4 counterclockwise starting with the upper right quadrant. Each quadrant has a certain angle value: In quadrant 1: $0 < \theta < 90^\circ$, in quadrant 2: $90 < \theta < 180$, in quadrant 3: $180 < \theta < 270$, and in quadrant 4: $270 < \theta < 360$.

An easy way to remember the sign chart is the phrase 'All Students Take Calculus'. The first letter of each word in the phrase tells you what is positive in each quadrant, starting in quad. 1 and going counterclockwise.

- ALL Means all of them are positive in the first quadrant
- S Means sine is the only one positive in quad 2.
- T Means tangent is the only one positive in quad 3
- C Means cosine is the only one positive in quad 4

How to find the trigonometric value for any angle:

- 1.) Find the reference angle.
- 2.) Apply the trig function to the reference angle
- 3.) Apply the appropriate sign.

EXAMPLE: Find the exact value of $\cos 135^\circ$ using reference angles. Draw the angle in standard position and indicate the reference angle.

EXAMPLE: Find the exact value of $\sin \frac{4\pi}{3}$ using reference angles. Draw the angle in standard position and indicate the reference angle.

EXAMPLE: Find the exact value of $\cos 330^\circ$ using reference angles. Draw the angle in standard position and indicate the reference angle.

EXAMPLE: Find the exact value of $\tan \frac{14\pi}{3}$ using reference angles. Draw the angle in standard position and indicate the reference angle.

EXAMPLE: Find the reference angle of $\tan(-225^\circ)$ and find its EXACT value.

EXAMPLE: Find the reference angle of $\sec(-210^\circ)$ and find its EXACT value.

EXAMPLE: Find the reference angle of $\sin\left(-\frac{11\pi}{3}\right)$ and find its EXACT value.

EXAMPLE: Find the reference angle of $\csc\left(-\frac{19\pi}{4}\right)$ and find its EXACT value.

EXAMPLE: Given $\tan \theta = -\frac{3}{4}$ and $\sin \theta < 0$, find the exact value of the six trig functions.

EXAMPLE: Given $\cos \theta = -\frac{1}{4}$ and $180^\circ < \theta < 270^\circ$, find the exact value of the six trig functions.

EXAMPLE: Given $\csc \theta = 3$ and $\frac{\pi}{2} < \theta < \pi$, find the exact value of the six trig functions.