4.7 Inverse Trigonometric Functions

From our tables in a previous section we know that $\sin 30^\circ = \frac{1}{2}$. We put in an angle and get a value as a result.

In inverse trig functions we put in the value and get an angle: $\sin^{-1}\frac{1}{2} = 30^\circ$. So here we put in the value of one half and got 30 degrees as a result. We are not allowed to put any number into our inverse trig functions. There are restrictions on the domain that are given in the following table:

	Domain	Range
$y = \sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

NOTE: in some textbooks, the inverse functions are written differently, for example instead of $y = \sin^{-1} x$, some textbooks may write this as $y = \arcsin x$. So instead of the $^{-1}$ symbol, it is replaced by the word *arc*. These two mean exactly the same thing. So $y = \arccos x$ would mean the same as $y = \cos^{-1} x$, etc.

EXAMPLE: Find the $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

EXAMPLE: Find the $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$.

EXAMPLE: Find the $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$.

EXAMPLE: Use a calculator to find $\cos^{-1} 0.7$, if possible, where $0 < \theta < 2\pi$. Round your answer to two decimal places.

EXAMPLE: Use a calculator to find $\sin^{-1}(-1.2)$, if possible, where $0 < \theta < 2\pi$. Round your answer to two decimal places.

Inverses and canceling

If we take $\cos^{-1}(\cos x)$ what will we get? Well, the inverse cosine and cosine will cancel and that will leave us with just x. However there are some restrictions on what x can be as listed below:

$$\cos^{-1}(\cos x) = x \quad if \quad 0 \le x \le \pi$$
$$\cos(\cos^{-1} x) = x \quad if \quad -1 \le x \le 1$$
$$\sin^{-1}(\sin x) = x \quad if \quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$
$$\sin(\sin^{-1} x) = x \quad if \quad -1 \le x \le 1$$
$$\tan^{-1}(\tan x) = x \quad if \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$$
$$\tan(\tan^{-1} x) = x \quad if \quad -\infty < x < \infty$$

EXAMPLE: Find the exact value if possible: $\tan(\tan^{-1} 5.3)$.

EXAMPLE: Find the exact value if possible: $\sin\left(\sin^{-1}\frac{98}{99}\right)$.

EXAMPLE: Find the exact value if possible: $\cos\left(\cos^{-1}\sqrt{2}\right)$.

EXAMPLE: Find the exact value: $\cos^{-1}\left(\cos\frac{\pi}{3}\right)$

EXAMPLE: Find the exact value: $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

EXAMPLE: Find the exact value: $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)$.

EXAMPLE: Use a sketch to find the exact value: $\sin\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$.

EXAMPLE: Use a sketch to find the exact value: $\cos\left(\sin^{-1}\left(-\frac{1}{\sqrt{6}}\right)\right)$.

EXAMPLE: Use a sketch to find the exact value: $\tan\left(\cos^{-1}\left(-\frac{3}{4}\right)\right)$.

EXAMPLE: Find the exact value: $\csc\left(\tan^{-1}\left(-\frac{1}{3}\right)\right)$.

EXAMPLE: Use right triangles to write in algebraic form: $\cos\left(\sin^{-1} u\right)$. Assume that *u* is positive and that the given inverse trigonometric function is defined for the expression in *u*.

EXAMPLE: Use right triangles to write in algebraic form: $\sec\left(\tan^{-1} 4u\right)$. Assume that *u* is positive and that the given inverse trigonometric function is defined for the expression in *u*.

EXAMPLE: Solve the triangle: