5.2 Sum and Difference Identities

These formulas will allow use to find the exact value for other angles besides just 0, 30, 45, 60, and 90 degrees. The book uses the symbols α and β which are more confusing. Instead I will use x and y.

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$
$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$
$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \qquad \tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

EXAMPLE: Determine the exact value of $\sin 75^\circ$.

EXAMPLE: Determine the exact value of $\cos 165^\circ$.

EXAMPLE: Determine the exact value: $\tan 105^\circ$.

EXAMPLE: Write as a single trig value and then find the exact value: $\sin 40^\circ \cos 20^\circ + \cos 40^\circ \sin 20^\circ$.

EXAMPLE: Write as a single trig value and then find the exact value: $\cos\left(\frac{3\pi}{10}\right)\cos\left(\frac{\pi}{5}\right) - \sin\left(\frac{3\pi}{10}\right)\sin\left(\frac{\pi}{5}\right)$.

EXAMPLE: Write as a single trig value and then find the exact value: $\dfrac{\tan 50^\circ - \tan 20^\circ}{1 + \tan 50^\circ \tan 20^\circ}$.

EXAMPLE: Find the exact value of $\sin(\alpha + \beta)$ given $\sin(\alpha) = \dfrac{3}{8}$ for α in Quadrant II and $\cos(\beta) = \dfrac{12}{13}$ for β in Quadrant IV.

EXAMPLE: Establish the identity: $\sin\left(\frac{3\pi}{2} - \theta\right) = -\cos\theta$.

EXAMPLE: Establish the identity: $\frac{\cos(x + 3\pi/2)}{\sin(x - \pi/2)} = -\tan x$.

EXAMPLE: Establish the identity: $\dfrac{\sin(x+y)}{\cos x \cos y} = \tan x + \tan y$.

EXAMPLE: Establish the identity: $\dfrac{\cos(x+y)}{\cos(x-y)} = \dfrac{1-\tan x \tan y}{1+\tan x \tan y}$.