

5.3 Double Angle, Half Angle, and Reduction Formulas

If we have either a double angle 2θ or a half angle $\frac{\theta}{2}$ then these have special formulas:

Double Angle Formulas

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$\cos(2\theta) = 2 \cos^2 \theta - 1 \quad \text{There are three formulas for } \cos(2\theta)$$

$$\cos(2\theta) = 1 - 2 \sin^2 \theta$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

The above double angle formulas can be manipulated to derive power reducing formulas. These formulas will be useful primarily in Calculus 2:

Power Reducing Formulas

$$\sin^2 \theta = \frac{1 - \cos(2\theta)}{2} \quad \cos^2 \theta = \frac{1 + \cos(2\theta)}{2} \quad \tan^2 \theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

Besides double angle formulas there are also half angle formulas:

Half Angle Formulas

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}} \quad \text{You will choose plus or minus depending on what quadrant } \frac{\theta}{2} \text{ is.}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}} \quad \text{You will choose plus or minus depending on what quadrant } \frac{\theta}{2} \text{ is.}$$

$$\tan \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} \quad \text{You will choose plus or minus depending on what quadrant } \frac{\theta}{2} \text{ is.}$$

There are better formulas for $\tan \frac{\theta}{2}$ that don't involve a plus or minus:

$$\tan \frac{\theta}{2} = \frac{\sin \theta}{1 + \cos \theta}, \quad \tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta}$$

EXAMPLE: Compute $\sin \theta$, $\tan \theta$, $\csc \theta$, $\sec \theta$, $\cot \theta$, $\sin(2\theta)$, $\cos(2\theta)$, $\tan(2\theta)$, $\sin \frac{\theta}{2}$, $\cos \frac{\theta}{2}$, and $\tan \frac{\theta}{2}$ if you are given $\cos \theta = -0.8$ and $90^\circ \leq \theta \leq 180^\circ$. Round decimals to two decimal places.

EXAMPLE: Compute $\sin \theta$, $\tan \theta$, $\csc \theta$, $\sec \theta$, $\cot \theta$, $\sin(2\theta)$, $\cos(2\theta)$, $\tan(2\theta)$, $\sin \frac{\theta}{2}$, $\cos \frac{\theta}{2}$, and $\tan \frac{\theta}{2}$ if you are given $\cot \theta = \frac{1}{2}$ and $\sin \theta < 0$.

EXAMPLE: Compute $\sin(22.5^\circ)$ and $\tan(22.5^\circ)$ using a half-angle formula.

EXAMPLE: Rewrite $1 - 2\sin^2(\pi/12)$ using a double angle formula. Then find its exact value.

EXAMPLE: Establish the identity: $\frac{\cot \theta - \tan \theta}{\cot \theta + \tan \theta} = \cos 2\theta$

EXAMPLE: $(4 \sin \theta \cos \theta)(1 - 2 \sin^2 \theta) = \sin 4\theta$

EXAMPLE: Use power-reducing formulas to rewrite $\cos^4 \theta$ in terms of first powers of cosine.