

5.4 Sum-to-Product and Product-to-Sum Formulas

Product-to-Sum Formulas

$$\sin x \sin y = \frac{1}{2} [\cos(x - y) - \cos(x + y)]$$

$$\cos x \cos y = \frac{1}{2} [\cos(x - y) + \cos(x + y)]$$

$$\sin x \cos y = \frac{1}{2} [\sin(x - y) + \sin(x + y)]$$

$$\cos x \sin y = \frac{1}{2} [\sin(x + y) - \sin(x - y)]$$

EXAMPLE: Simplify: $\sin(6\theta)\sin(4\theta)$ using a product-to-sum formula.

EXAMPLE: Simplify: $\cos(3\theta)\cos(\theta)$ using a product-to-sum formula.

EXAMPLE: Simplify: $\sin(3\theta)\cos(5\theta)$ using a product-to-sum formula.

EXAMPLE: Find the exact value of $\cos\frac{5\pi}{12}\sin\frac{\pi}{12}$ using a product-to-sum formula.

Sum-to-Product Formulas

$$\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2 \sin\left(\frac{x-y}{2}\right) \cos\left(\frac{x+y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$

EXAMPLE: Simplify: $\sin(5\theta) - \sin(3\theta)$ using a sum-to-product formula.

EXAMPLE: Simplify: $\cos(3\theta) + \cos(2\theta)$ using a sum-to-product formula.

EXAMPLE: Simplify: $\cos(4\theta) - \cos(7\theta)$ using a sum-to-product formula.

EXAMPLE: Find the exact value of $\sin 15^\circ + \sin 75^\circ$ using a sum-to-product formula.