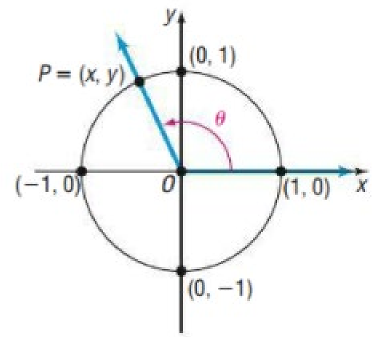
6.3 Properties of the Trigonometric Functions

In this section we will be by looking at the domain and range of the six trigonometric functions. We can get the domains from the unit circle. The ranges come from the graphs of these functions. We see that the x and y values are between -1 and 1. We can apply this to the individual trig functions. In the table below, *n* represents any integer.

Function	Symbol	Domain	Range
sine	$f(\theta)=\sin\theta$	$(-\infty,\infty)$	$[-1,1]$
cosine	$f(\theta)=\cos\theta$	$(-\infty,\infty)$	$[-1,1]$
tangent	$f(\theta)=\tan\theta$	$\left(-\infty,\frac{n\pi}{2}\right)\cup\left(\frac{n\pi}{2},\infty\right)$	$(-\infty,\infty)$
cosecant	$f(\theta)=\csc\theta$	$(-\infty,n\pi)\cup(n\pi,\infty)$	$(-\infty,-1]\cup[1,\infty)$
secant	$f(\theta)=\sec\theta$	$\left(-\infty,\frac{n\pi}{2}\right)\cup\left(\frac{n\pi}{2},\infty\right)$	$(-\infty,-1]\cup[1,\infty)$
cotangent	$f(\theta)=\cot\theta$	$(-\infty,n\pi)\cup(n\pi,\infty)$	$(-\infty,\infty)$



Even – Odd Properties

$\cos(-t)=\cos t$ $\qquad$ $\sec(-t)=\sec t$

$\sin(-t)=-\sin t$ $\qquad$ $\csc(-t)=-\csc t$

$\tan(-t)=-\tan t$ $\qquad$ $\cot(-t)=-\cot t$

EXAMPLE: Use the even-odd properties to find the exact value of $\cos(-30^\circ)$ without using a calculator.

EXAMPLE: Use the even-odd properties to find the exact value of $\csc\left(-\frac{\pi}{4}\right)$ without using a calculator.

Periodic Properties

If we start at an angle and go around one revolution (360° or 2π radians) we will end up at the same angle we started with. The k value is any integer, and represents how many revolutions are going around. If you want to use degrees, replace the $2\pi k$ in the equations below with $360k$. For tangent and cotangent you will end up at the same spot if you add 2π, however you will also get the same value if you just add π.

$\sin(t \pm 2\pi k) = \sin t$ $\quad\quad$ $\csc(t \pm 2\pi k) = \csc t$

$\cos(t \pm 2\pi k) = \cos t$ $\quad\quad$ $\sec(t \pm 2\pi k) = \sec t$

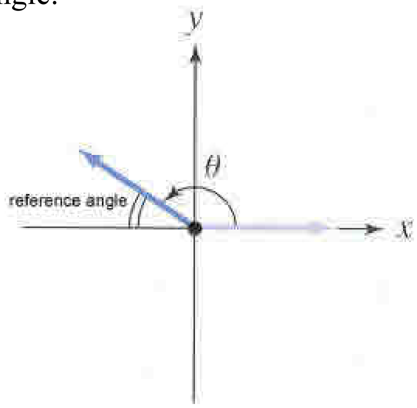
$\tan(t \pm \pi k) = \tan t$ $\quad\quad$ $\cot(t \pm \pi k) = \cot t$

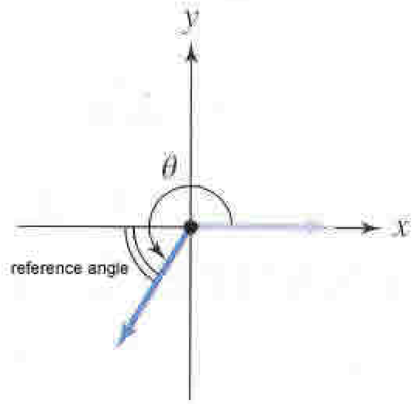
EXAMPLE: Find the EXACT value of $\sin\left(1485^\circ\right)$.

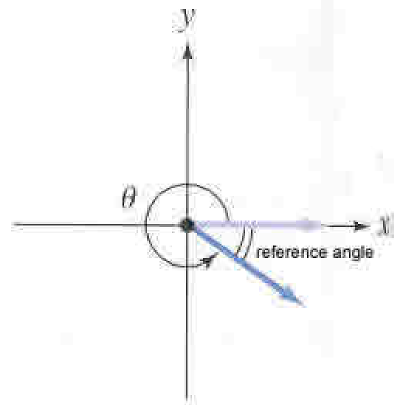
EXAMPLE: Find the EXACT value of $\tan\left(-\frac{17\pi}{4}\right)$.

EXAMPLE: Use the Even-Odd Properties and Periodic Properties to simplify:
$-2\sin(3t + 2\pi) - 3\sin(-3t) + \cos(-2t)$

Reference Angle – an angle between 0 and 90 that is formed by the terminal side of an angle and the x-axis. The reference angle is labeled below. It is indicated by the double curved lines. Notice that no matter where the angle is drawn it is measured from the x-axis. Under each drawing it tells you how to find the reference angle:







If $90^\circ < \theta < 180^\circ$ then
Ref. angle = $180^\circ - \theta$

If $\frac{\pi}{2} < \theta < \pi$ then
Ref. angle = $\pi - \theta$

If $180^\circ < \theta < 270^\circ$ then
Ref. angle = $\theta - 180^\circ$

If $\pi < \theta < \frac{3\pi}{2}$ then
Ref. angle = $\theta - \pi$

If $270^\circ < \theta < 360^\circ$ then
Ref. angle = $360^\circ - \theta$

If $\frac{3\pi}{2} < \theta < 2\pi$ then
Ref. angle = $2\pi - \theta$

EXAMPLE: Find the reference angle for 170°.

EXAMPLE: Find the reference angle for $\frac{9\pi}{5}$.

EXAMPLE: Draw 120° in standard position and then find its reference angle.

EXAMPLE: Draw $\frac{11\pi}{6}$ in standard position and then find its reference angle.

Sign values of sine, cosine, and tangent in each quadrant

$\sin\theta$ + $\cos\theta$ − $\tan\theta$ −	$\sin\theta$ + $\cos\theta$ + $\tan\theta$ +
$\sin\theta$ − $\cos\theta$ − $\tan\theta$ +	$\sin\theta$ − $\cos\theta$ + $\tan\theta$ −

Depending on which quadrant you are in the sine, cosine, and tangent functions will be either positive or negative.

The quadrants are number from 1 to 4 counterclockwise starting with the upper right quadrant. Each quadrant has a certain angle value: In quadrant 1: $0 < \theta < 90^\circ$, in quadrant 2: $90 < \theta < 180$, in quadrant 3: $180 < \theta < 270$, and in quadrant 4: $270 < \theta < 360$.

An easy way to remember the sign chart is the phrase 'All Students Take Calculus' The first letter of each word in the phrase tells you what is positive in each quadrant, starting in quad. 1 and going counterclockwise.

ALL Means all of them are positive in the first quadrant
S Means sine is the only one positive in quad 2.
T Means tangent is the only one positive in quad 3
C Means cosine is the only one positive in quad 4

EXAMPLE: Name the quadrant in which the angle θ lies given $\sin\theta < 0$ and $\cos\theta > 0$.

EXAMPLE: Name the quadrant in which the angle θ lies given $\tan\theta < 0$ and $\cos\theta < 0$.

How to find the trigonometric value for any angle:

1.) Find the reference angle.
2.) Apply the trig function to the reference angle
3.) Apply the appropriate sign.

EXAMPLE: Find the exact value of $\cos 135^\circ$ using reference angles. Draw the angle in standard position and indicate the reference angle.

EXAMPLE: Find the exact value of $\sin\frac{4\pi}{3}$ using reference angles. Draw the angle in standard position and indicate the reference angle.

EXAMPLE: Find the exact value of $\cos 330^{\circ}$ using reference angles. Draw the angle in standard position and indicate the reference angle.

EXAMPLE: Find the exact value of $\tan\frac{14\pi}{3}$ using reference angles. Draw the angle in standard position and indicate the reference angle.

EXAMPLE: Find the reference angle of $\tan(-225^{\circ})$ and find its EXACT value.

EXAMPLE: Find the reference angle of $\sec(-210^\circ)$ and find its EXACT value.

EXAMPLE: Find the reference angle of $\sin\left(-\frac{11\pi}{3}\right)$ and find its EXACT value.

EXAMPLE: Find the reference angle of $\csc\left(-\frac{19\pi}{4}\right)$ and find its EXACT value.

EXAMPLE: Given $\tan\theta = -\frac{3}{4}$ and $\sin\theta < 0$, find the exact value of the six trig functions.

EXAMPLE: Given $\cos\theta = -\frac{1}{4}$ and $180^\circ < \theta < 270^\circ$, find the exact value of the six trig functions.

EXAMPLE: Given $\csc\theta = 3$ and $\frac{\pi}{2} < \theta < \pi$, find the exact value of the six trig functions.