6.4 Graphs of the Sine and Cosine Functions; Phase Shift

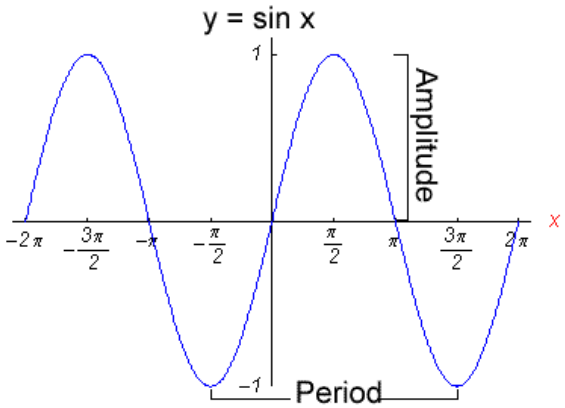
This section will introduce you to the sine and cosine functions. In order to generate each of these graphs, we can start with a table of values for the sine and cosine graph. Then we will plot the values below. We will look at specific angle since these provide easy to plot values. These specific angles are called **key points.** These values come directly from the unit circle.

$y = \sin\theta$

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = \sin\theta$					

$y = \cos\theta$

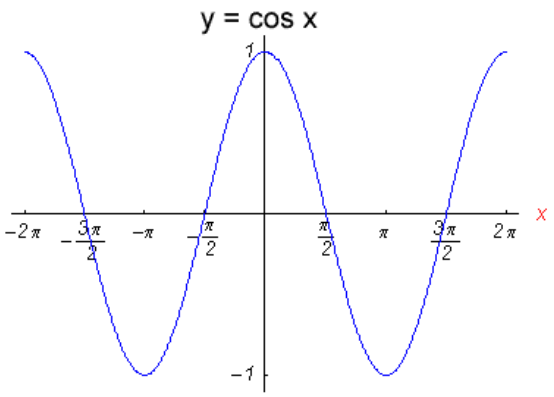
θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = \cos\theta$					



Period: How long it takes the graph to repeat itself
For sine graphs, the period is 2π .

Amplitude = $\frac{Highest\ \ value - Lowest\ \ value}{2}$

For the regular sine graph the amplitude is 1.



The period for cosine graphs is 2π

The amplitude for a regular cosine graph is 1.

General Form of a Sine or Cosine Equation:

$y = A\sin(Bx - C)$ or $y = A\cos(Bx - C)$

Amplitude = $|A|$, **Period** = $\frac{2\pi}{B}$, **Phase Shift** = $\frac{C}{B}$

The **phase shift** is a shift of the graph to the left or to the right. The direction depends on the sign of the phase shift:

If $\frac{C}{B} > 0$ the graph will shift to the right (this occurs when there is a minus sign between *Bx* and *C*).

If $\frac{C}{B} < 0$ the graph will shift to the left (this occurs when there is a plus sign between *Bx* and *C*).

The phase shift will always be one of the five key points. In the two regular graphs of sine and cosine, the phase shift is 0, That is why 0 is the starting key point of a cycle.

EXAMPLE: Indicate the amplitude, period, and phase shift without graphing: $y = -3.4\sin(5x - 7)$

EXAMPLE: Indicate the period, amplitude, and phase shift without graphing: $y = -\frac{1}{5}\sin\left(\frac{\pi}{2}x + \frac{2\pi}{3}\right) + 1$.

EXAMPLE: Graph over one period using transformations: $y = 4\sin x$

EXAMPLE: Graph over one period using transformations: $y = -2\cos x$

EXAMPLE: Graph over one period using transformations: $y = -3\sin x + 2$

EXAMPLE: Graph over one period using transformations: $y = 2\cos\left(\frac{1}{2}x\right)$

EXAMPLE: Graph over one period using transformations: $y = 3\sin(\pi x)$

EXAMPLE: Identify the amplitude, period, phase shift and graph of $y = 3\cos\left(3x - \frac{\pi}{2}\right)$. (Graph 1 period).

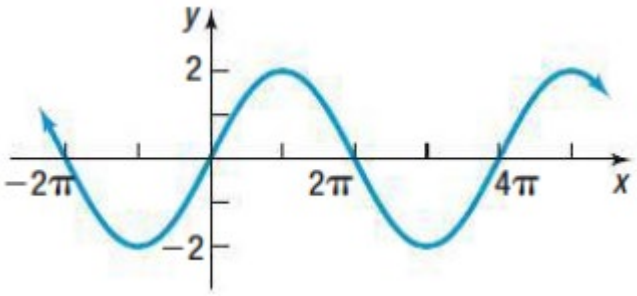
EXAMPLE: Identify the amplitude, period, phase shift and graph of $y = \frac{1}{2}\sin(\pi x + \pi)$. (Graph 1 period).

EXAMPLE: Identify the amplitude, period, phase shift and graph of $y = -2\cos\left(2x + \frac{\pi}{3}\right)$. (Graph 1 period).

EXAMPLE: Identify the amplitude, period, phase shift and graph of $y = -\sin\left(\frac{1}{3}x - \frac{\pi}{4}\right)$. (Graph 1 period).

EXAMPLE: Identify the amplitude, period, phase shift and graph of $-3\cos\left(-2x+\frac{\pi}{2}\right)$. (Graph 1 period).

EXAMPLE: Find the equation of the given graph in the form $y = A\cos(Bx)$ or $y = A\sin(Bx)$.



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