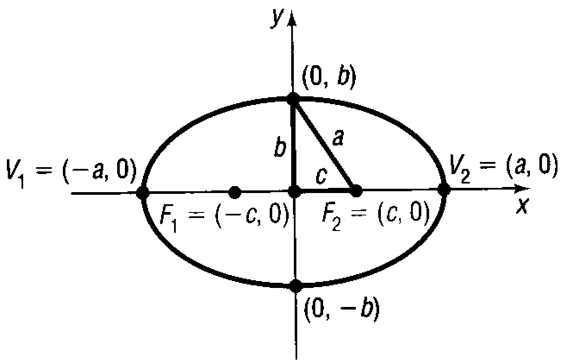
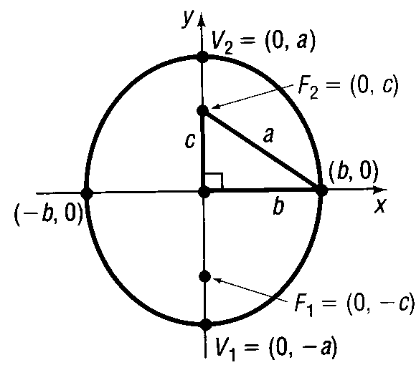
7.1 The Ellipse

In this section we will be looking at the ellipse which is basically an elongated circle. The pictures below show the two different ways an ellipse can be drawn that are both centered at the origin. The formulas below contain a and b. The a is the length of the major axis, or the longest axis. The b is the length of the shortest axis. It is very important to note that a is **ALWAYS** larger than b. If the larger number is under the x then the ellipse is drawn horizontally. If the larger number is under the y then the graph is drawn vertically.

Ellipses centered at (0, 0).



(a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



(b) $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

In both of these cases the length of the major axis is 2a. The length of the minor axis is 2b. To find the c value in any of these graphs, use the equation $c=\sqrt{a^2-b^2}$.

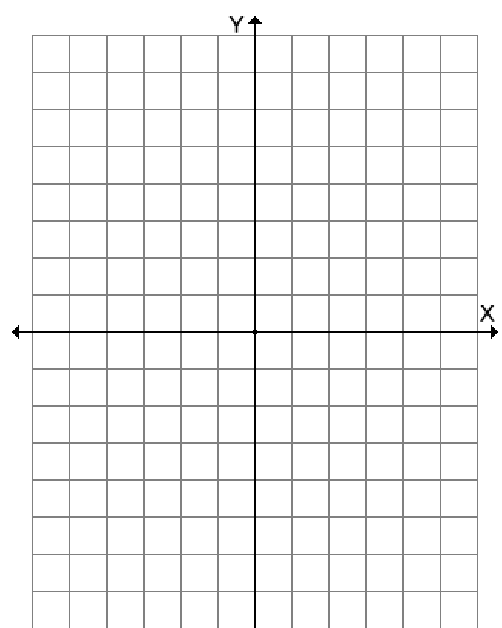
Eccentricity: a measure of how much the ellipse resembles a circle. The formula is $e=\frac{c}{a}$. If e = 0 then the ellipse is a circle. The larger the ellipse the skinnier it becomes.

EXAMPLE: Graph $\frac{x^2}{25}+\frac{y^2}{9}=1$ and identify the foci, eccentricity, center, length of the major axis, length of the minor axis, and the two vertices on the major axis.

Center:_______________ Vertices:___________________

Foci:_________________ Eccentricity:________________

Major:_______________ Minor:_____________________

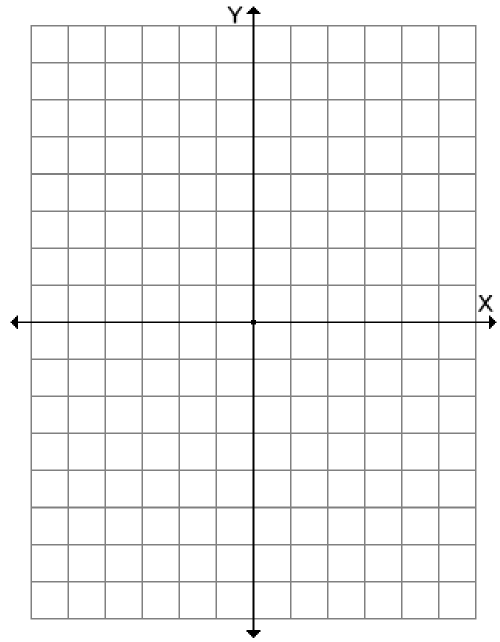


EXAMPLE: Graph $36x^2 + 4y^2 = 144$ and identify the foci, eccentricity, center, length of the major axis, length of the minor axis, and the two vertices on the major axis.

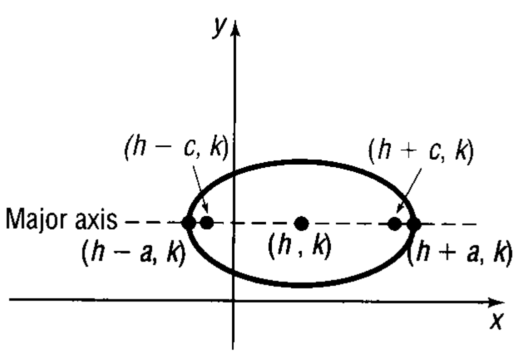
Center:______________ Vertices:__________________

Foci:________________ Eccentricity:_______________

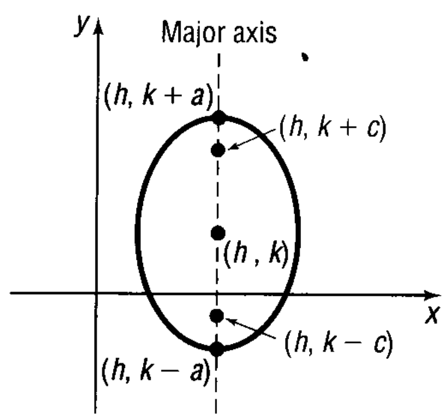
Major:______________ Minor:____________________



Ellipses centered at (h, k).



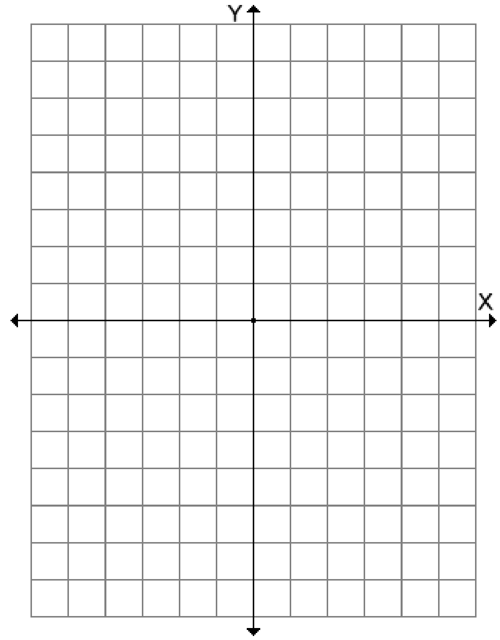
(a) $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$



(b) $\dfrac{(x-h)^2}{b^2} + \dfrac{(y-k)^2}{a^2} = 1$

Once again in both of these cases the length of the major axis is 2a. The length of the minor axis is 2b. To find the c value in any of these graphs, use the equation $c = \sqrt{a^2 - b^2}$. The eccentricity is still $e = \dfrac{c}{a}$.

EXAMPLE: Graph $\frac{(x-2)^2}{16}+\frac{(y-1)^2}{4}=1$ and identify the foci, eccentricity, center, length of the major axis, length of the minor axis, and the two vertices on the major axis.

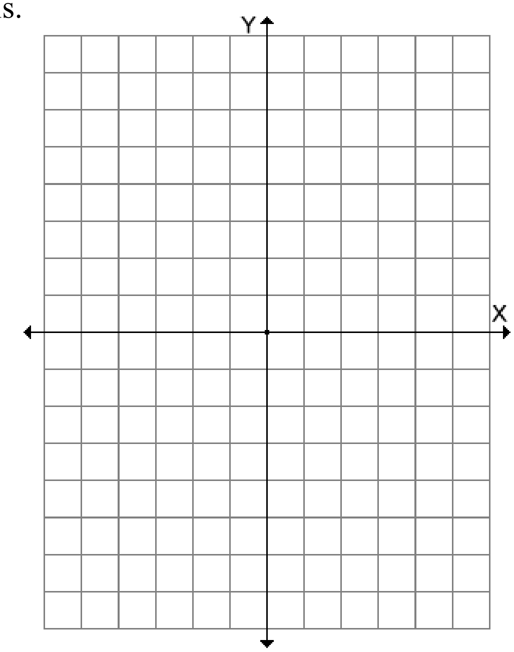


Center:_______________ Vertices:___________________

Foci:_________________ Eccentricity:________________

Major:_______________ Minor:_____________________

EXAMPLE: Graph $16x^2+9y^2+64x-54y+1=0$ and identify the foci, eccentricity, center, length of the major axis, length of the minor axis, and the two vertices on the major axis.



Center:_______________ Vertices:___________________

Foci:_________________ Eccentricity:________________

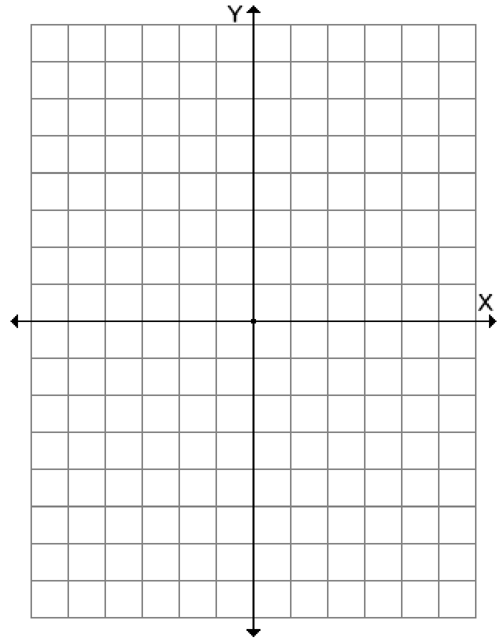
Major:_______________ Minor:_____________________

EXAMPLE: Graph $x^2 + 16y^2 - 160y + 384 = 0$ and identify the foci, eccentricity, center, length of the major axis, length of the minor axis, and the two vertices on the major axis.

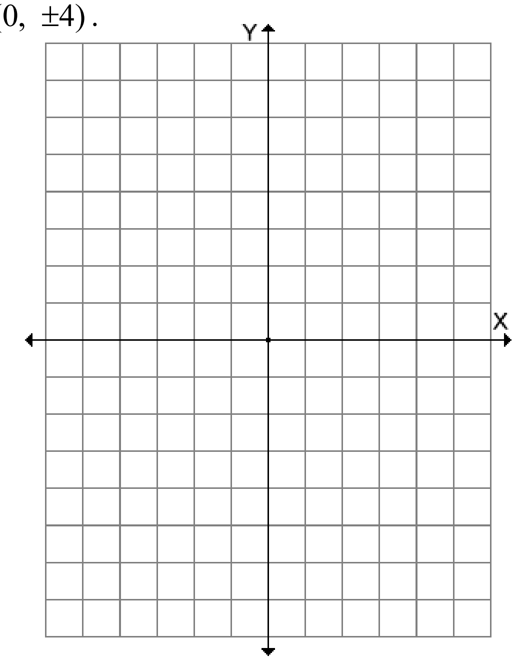
Center:_______________ Vertices:____________________

Foci:_________________ Eccentricity:________________

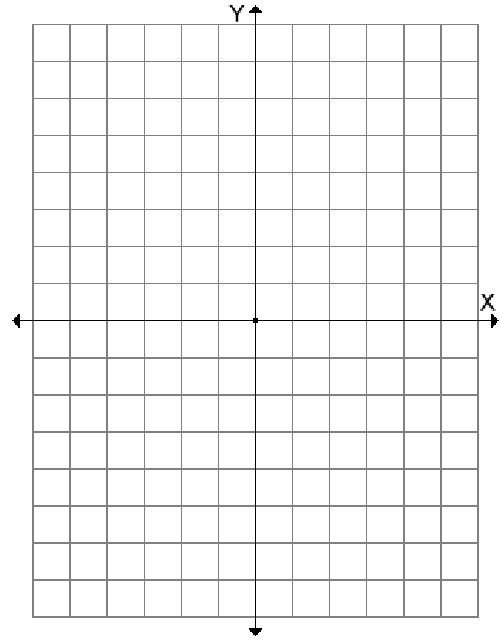
Major:_______________ Minor:______________________



EXAMPLE: Find the equation of an ellipse with foci $(0, \pm 1)$ and vertices $(0, \pm 4)$.



EXAMPLE: Find the equation of an ellipse centered at the origin with an eccentricity of $\frac{3}{5}$ and with one vertice at (-5, 0).



EXAMPLE: Find the equation of an ellipse with foci at (1, 2) and (-3, 2) and with one vertex at (-4, 2).