

## 7.4 Verifying Trigonometric Identities

### Factoring Review

This section and a future section will require factoring to simplify expressions. Let's look at a few review problems from precalculus, and then we will apply this to trigonometric expressions.

EXAMPLE: Factor  $30x^3 + 42x^2$ .

EXAMPLE: Factor  $80\sin^4 x - 30\sin x$ .

EXAMPLE: Factor  $x^2 + 13x + 36$  completely, if possible.

EXAMPLE: Factor  $\cos^2 x - 13 \cos x + 42$  completely. If the polynomial cannot be factored, say it is prime.

**Steps for factoring  $ax^2 + bx + c$  by grouping (AC Method)**

Multiply  $a$  times  $c$ . Then find two numbers that multiply to  $ac$  but add up to the middle term. From here you will factor by grouping as shown below.

EXAMPLE: Factor  $6x^2 + 23x + 20$  using the grouping method, if possible.

EXAMPLE: Factor  $2 \tan^2 x + 9 \tan x - 35$  using the grouping method, if possible.

The rest of this section will help you practice your trigonometric identities. We are going to establish an identity. What this means is to work out the problem and show that both sides of the identity are the same. First let's look at a list of identities. Most of these are derived directly from the unit circle.

### List of Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta} \quad \csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1 \quad \sin^2 \theta = 1 - \cos^2 \theta \quad \cos^2 \theta = 1 - \sin^2 \theta$$

$$\sec^2 \theta = 1 + \tan^2 \theta \quad \tan^2 \theta = \sec^2 \theta - 1 \quad \csc^2 \theta = 1 + \cot^2 \theta \quad \cot^2 \theta = \csc^2 \theta - 1$$

When working out these identities, you can try one or more of the following techniques (strategies). I will explain each technique with examples:

- 1.) Change everything into sines and cosines.
- 2.) Use factoring to simplify the expression if possible.
- 3.) Get common denominators if there are fractions.
- 4.) Multiply one side by a conjugate.

Of course, as we use the above techniques, be sure to refer back to the list of identities I gave you above. You might need to use some of them to simplify. One you will see come up often is  $\sin^2 \theta + \cos^2 \theta = 1$ .

EXAMPLE: Establish the identity:  $\csc \theta \cdot \tan \theta = \sec \theta$ .

EXAMPLE: Establish the identity:  $\frac{\sin^4 \theta - \cos^4 \theta}{\cos \theta - \sin \theta} = -(\cos \theta + \sin \theta)$ .

EXAMPLE: Establish the identity:  $\frac{\cos x - 2 \sin x \cos x}{\cos^2 x - \sin^2 x + \sin x - 1} = \cot x$ .

EXAMPLE: Establish the identity:  $\cot \theta + \frac{1 - 2 \cos^2 \theta}{\sin \theta \cos \theta} = \tan \theta$ .

EXAMPLE: Establish the identity:  $\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} = 2 \sec x$ .

EXAMPLE: Establish the identity:  $\frac{\tan x + \cot x}{\sec x \csc x} = 1$ .

EXAMPLE: Establish the identity:  $\frac{1 + \sin \theta}{\cos \theta} = \frac{\cos \theta}{1 - \sin \theta}$ .