

## 7.7 Product-to-Sum and Sum-to-Product Formulas

### Product-to-Sum Formulas

$$\sin x \sin y = \frac{1}{2} [\cos(x - y) - \cos(x + y)]$$

$$\cos x \cos y = \frac{1}{2} [\cos(x - y) + \cos(x + y)]$$

$$\sin x \cos y = \frac{1}{2} [\sin(x - y) + \sin(x + y)]$$

$$\cos x \sin y = \frac{1}{2} [\sin(x + y) - \sin(x - y)]$$

EXAMPLE: Simplify:  $\sin(6\theta)\sin(4\theta)$  using a product-to-sum formula.

EXAMPLE: Simplify:  $\cos(3\theta)\cos(\theta)$  using a product-to-sum formula.

EXAMPLE: Simplify:  $\sin(3\theta)\cos(5\theta)$  using a product-to-sum formula.

EXAMPLE: Find the exact value of  $\cos\frac{5\pi}{12}\sin\frac{\pi}{12}$  using a product-to-sum formula.

**Sum-to-Product Formulas**

$$\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2 \sin\left(\frac{x-y}{2}\right) \cos\left(\frac{x+y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$

EXAMPLE: Simplify:  $\sin(5\theta) - \sin(3\theta)$  using a sum-to-product formula.

EXAMPLE: Simplify:  $\cos(3\theta) + \cos(2\theta)$  using a sum-to-product formula.

EXAMPLE: Simplify:  $\cos(4\theta) - \cos(7\theta)$  using a sum-to-product formula.

EXAMPLE: Find the exact value of  $\sin 15^\circ + \sin 75^\circ$  using a sum-to-product formula.