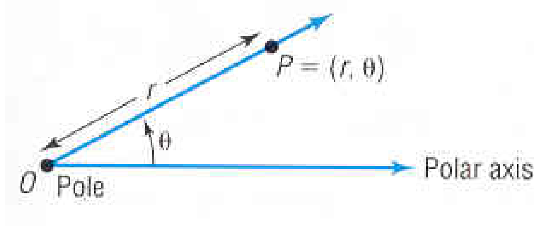
9.1 Polar Coordinates

In this section we will learn a new coordinate system. In this system we plot a point in the form $(r,\ \theta)$. As shown in the picture below you first draw angle θ in standard form. Then you label how long r is:



EXAMPLE: Plot $\left(4,\ \frac{3\pi}{4}\right)$ in the polar coordinate system.

EXAMPLE: Plot $\left(5,\ \frac{5\pi}{3}\right)$ in the polar coordinate system.

EXAMPLE: Plot $\left(-3,\ 120^\circ\right)$ in the polar coordinate system.

EXAMPLE: Plot $\left(-3,\ -\frac{\pi}{2}\right)$ in the polar coordinate system.

EXAMPLE: Plot $\left(-3,\ -\frac{3\pi}{4}\right)$ in the polar coordinate system.

Equivalent Polar Coordinates

There are more than one way to arrive at the same angle. For example in the previous problem, -135 degrees is the same as $360^\circ + \left(-135^\circ\right) = 225^\circ$. If we have 120 degrees then this is the same as $120^\circ + \left(-360^\circ\right) = -240^\circ$. So for negative angles, just add 360 degrees. For positive angles add negative 360 degrees to find the equivalent angle. So basically we can either move clockwise or counterclockwise to arrive at the same angle.

$(r,\ \theta) = (r,\ \theta \pm 2\pi)$ or $(r,\ \theta) = (r,\ \theta \pm 360^\circ)$
$(r,\ \theta) = (-r,\ \theta \pm \pi)$ or $(r,\ \theta) = (-r,\ \theta \pm 180^\circ)$

EXAMPLE: Given the polar coordinate $(5,\ 300^\circ)$, find an equivalent polar coordinate that has the following characteristics: a.) $-360^\circ \le \theta \le 0$, $r > 0$ b.) $0 \le \theta \le 360^\circ$, $r < 0$, c.) $360^\circ \le \theta \le 720^\circ$, $r > 0$

a.) $-360^\circ \le \theta \le 0$, $r > 0$

b.) $0 \le \theta \le 360^\circ$, $r < 0$

c.) $360^\circ \le \theta \le 720^\circ$, $r > 0$

EXAMPLE: Given the polar coordinate $\left(4, \dfrac{3\pi}{4}\right)$, find an equivalent polar coordinate that has the following characteristics: a.) $-2\pi \le \theta \le 0$, $r > 0$ b.) $0 \le \theta \le 2\pi$, $r < 0$, c.) $2\pi \le \theta \le 4\pi$, $r > 0$

a.) $-2\pi \le \theta \le 0$, $r > 0$

b.) $0 \le \theta \le 2\pi$, $r < 0$

c.) $2\pi \le \theta \le 4\pi$, $r > 0$

EXAMPLE: Given the polar coordinate $(-2,\ -120^\circ)$, find an equivalent polar coordinate that has the following characteristics: a.) $-360^\circ \le \theta \le 0,\ \mathrm{r} > 0$ b.) $0 \le \theta \le 360^\circ,\ \mathrm{r} < 0$, c.) $360^\circ \le \theta \le 720^\circ,\ \mathrm{r} > 0$

a.) $-360^\circ \le \theta \le 0,\ \mathrm{r} > 0$

b.) $0 \le \theta \le 360^\circ,\ \mathrm{r} < 0$

c.) $360^\circ \le \theta \le 720^\circ,\ \mathrm{r} > 0$

Conversion formulas from polar to rectangular coordinates.

$x = r\cos\theta$
$y = r\sin\theta$
$x^2 + y^2 = r^2$

EXAMPLE: Convert $\left(5,\ \frac{\pi}{3}\right)$ into a rectangular point.

EXAMPLE: Convert $\left(-3,\ -\frac{\pi}{4}\right)$ into a rectangular point.

EXAMPLE: Convert $\left(-2,\ \frac{2\pi}{3}\right)$ into a rectangular point.

EXAMPLE: Convert the equation $r = 5\sec\theta$ into a rectangular equation.

EXAMPLE: Convert the equation $r = 4$ into a rectangular equation.

EXAMPLE: Convert the equation $r = 2\sin\theta - 4\cos\theta$ into a rectangular equation.

Conversion formulas from rectangular to polar coordinates

$x^2 + y^2 = r^2$

If (x, y) is in the first or fourth quadrant, then $\theta = \tan^{-1}\frac{y}{x}$.

If (x, y) is in the second or third quadrant, then $\theta = \tan^{-1}\frac{y}{x} + \pi$.

EXAMPLE: Convert $(-3,\ 3)$ into a polar coordinate. Express your angle in radians.

EXAMPLE: Convert $\left(-2,\ -2\sqrt{3}\right)$ into a polar coordinate. Express your angle in radians.

EXAMPLE: Convert $\left(\frac{1}{2},\ -\frac{\sqrt{3}}{2}\right)$ into a polar coordinate. Express your angle in radians.

EXAMPLE: Convert the equation $x^2 + y^2 = x$ into a polar equation and solve for r.

EXAMPLE: Convert the equation $4x^2y = 1$ into a polar equation and solve for r.

EXAMPLE: Convert the equation $y^2 = 2x$ into a polar equation and solve for r.