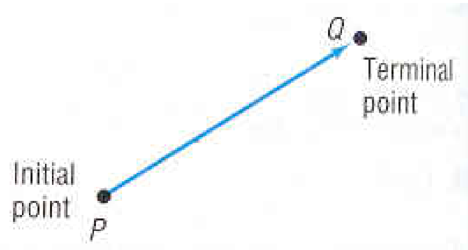
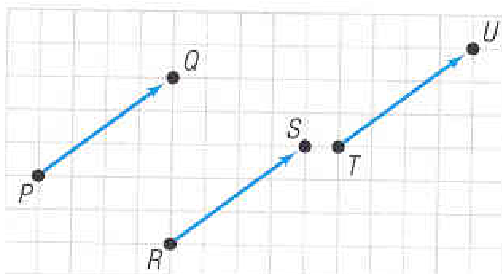
9.4 Vectors

Vectors are needed in physics and engineering courses. A vector is a quantity that has magnitude (size) in a certain direction. You indicate a vector by a ray. The length of the arrow represents the magnitude and the arrowhead indicates the direction of the vector as shown below:



In the picture above the vector starts at point P and ends at Q. It is possible for two vectors to be the same if their magnitude and direction are the same, as shown below.



Notice that vector RS and TU are the same as vector PQ.

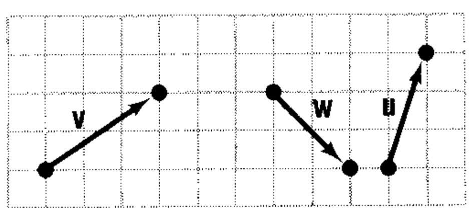
Adding Vectors – when you add two vectors, add them tip to tail as show below:

There is more than one way to go from one point to another as shown below. Notice we are still adding the vectors tip to tail:

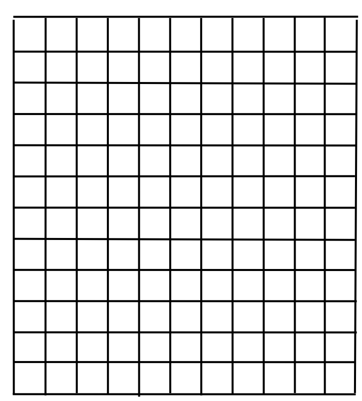
It is possible to have a negative vector. It is drawn the same as the original vector except we change the arrowhead so it goes in the opposite direction:

In order to subtract vectors, it is the same thing as adding the opposite: $v - w = v + (-w)$

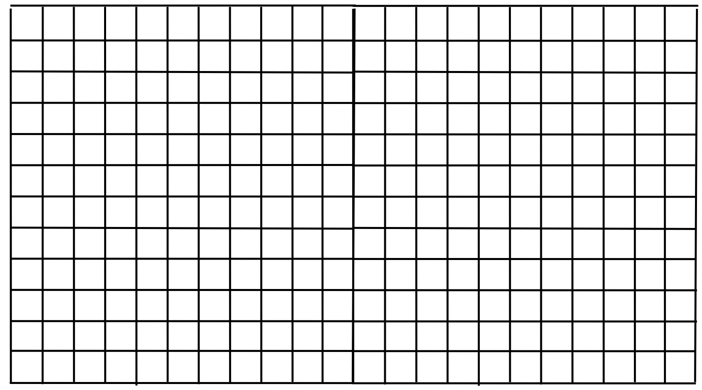
EXAMPLE: Use the following given vectors and sketch the following: a.) $\boldsymbol{v} - \boldsymbol{w}$ b.) $2\boldsymbol{v} + 3\boldsymbol{w}$ c.) $2\boldsymbol{v} - \boldsymbol{w} + \boldsymbol{u}$



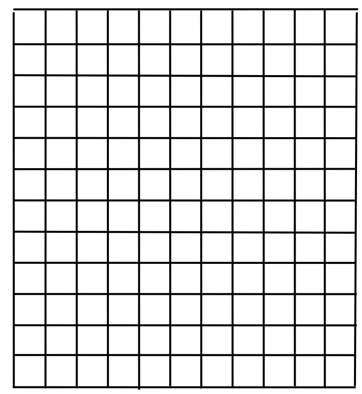
a.) $\boldsymbol{v} - \boldsymbol{w}$



b.) $2\boldsymbol{v} + 3\boldsymbol{w}$



c.) $2\boldsymbol{v} - \boldsymbol{w} + \boldsymbol{u}$



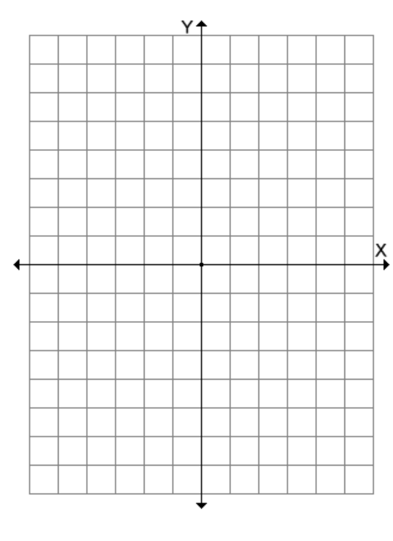
Algebraic Vector

This is expressed as $v = \langle a,\ b \rangle$. The a is the horizontal component and b is the vertical component.

Position Vector

This is an algebraic vector with the starting point at the origin. Suppose that v is a vector with a starting point of $P_1 = (x_1,\ y_1)$ and ending point of $P_2 = (x_2,\ y_2)$. Then $v = \langle x_2 - x_1,\ y_2 - y_1 \rangle$.

EXAMPLE: Find the position vector of the vector $v = P_1P_2$ if $P_1 = (-1,\ 2)$ and $P_2 = (4,\ 6)$. Graph your answer in standard position (graph your position vector).



EXAMPLE: Find the position vector of the vector $v = P_1P_2$ if $P_1 = (-1,\ 4)$ and $P_2 = (6,\ -2)$.

EXAMPLE: Given $r = \langle 2,\ 5 \rangle$, $s = \langle -2,\ -3 \rangle$, $w = \langle -2,\ 5 \rangle$, find 3**r** – (**w** + **s**).

Alternate form of writing $v = \langle a,\ b \rangle$

We can also write this as v = a**i** + b**j**

How to find a vector's magnitude $\|v\|$

To find the magnitude, use the formula $\|v\| = \sqrt{a^2 + b^2}$

EXAMPLE: Given v = 6**i** + 3**j**, find $\|v\|$.

EXAMPLE: Given v = **i** + 2**j** and w = –3**i** + 4**j**, find the following:

a.) -2v + 3w

b.) v – 4w

c.) $\|v + w\|$

Unit Vector – a vector with a magnitude of 1.

Finding a unit vector u in the same direction as v: $u = \frac{v}{\|v\|}$

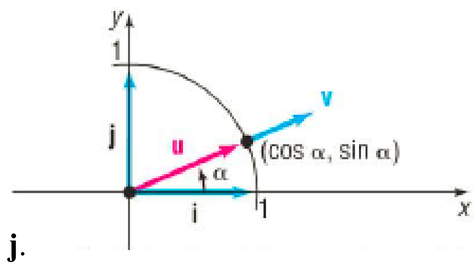
This finds a vector u that is going in the same direction as a given vector v but has a magnitude of 1.

EXAMPLE: Find a unit vector u that has the same direction as the vector v = 12**i** + 5**j**.

EXAMPLE: Find a unit vector u that has the same direction as the vector v = –2**i** + 3**j**.

Vector components

A vector is made up of a horizontal and vertical part. These parts are called components. The picture to the right shows how to break up a vector into components if an angle is given. The formula we will use to break up into components is: $v = \|v\|\cos\alpha\ \mathbf{i} + \|v\|\sin\alpha\ \mathbf{j}$



EXAMPLE: Given $\|v\| = 8$ and $\alpha = 30^\circ$, write the vector **v** in terms of **i** and **j**.

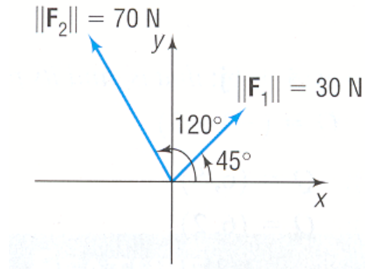
Resultant vector: the result when two vectors are added together.

Direction of Resultant: If the resultant vector is in the form $F = a\hat{i} + b\hat{j}$, then you can find the angle between this vector and the x axis by using the formula: $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ if the resultant vector is quadrant 1 or 4. Also

$\theta = \tan^{-1}\left(\frac{b}{a}\right) + 180^\circ$ if the resultant vector is in quadrant 2 or 3.

Static Equilibrium: The sum of all the forces is zero, which means nothing is moving. The magnitude and direction will both be zero in this case.

EXAMPLE: Two forces of magnitude 30 newtons (N) and 70 newtons act on an object at angle of 45 degrees and 120 degrees with the positive x-axis as shown in the figure. Find the direction and magnitude of the resultant force, that is, find $F_1 + F_2$. Then find what additional force is needed for the object to be in static equilibrium.



EXAMPLE: The magnitude and direction exerted by two tugboats towing a ship are 4200 pounds at $N65^\circ E$ and 3000 pounds at $S58^\circ E$ respectively as shown. Find the magnitude and the bearing of the resultant force.

EXAMPLE: A 9.73 pound picture is hung from 2 wires as shown below. The tension on A is 3.4 pounds at 161 degrees. The tension on B is 9.2 pounds at 69.55 degrees. The tension on C is 9.73 pounds at -90 degrees. Find the magnitude and direction of the resultant force.