

## 9.5 The Dot Product

### The Dot Product

If  $u = a_1\mathbf{i} + b_1\mathbf{j}$  and  $v = a_2\mathbf{i} + b_2\mathbf{j}$  then the dot product  $u \cdot v = a_1 \cdot a_2 + b_1 \cdot b_2$ .

EXAMPLE: Given  $u = -3\mathbf{i} + 4\mathbf{j}$  and  $v = 6\mathbf{i} + 5\mathbf{j}$ , find the following:

a.)  $u \cdot v$

b.)  $v \cdot u$

c.)  $u \cdot u$

d.)  $v \cdot v$

e.)  $\|u\|$

f.)  $\|v\|$

**Angle between two vectors**

Given two vectors  $u$  and  $v$ , the angle between the two vectors is given as:

$$\cos \theta = \frac{u \cdot v}{\|u\| \cdot \|v\|}$$

EXAMPLE:  $u = -3\mathbf{i} + 4\mathbf{j}$  and  $v = 6\mathbf{i} + 5\mathbf{j}$ , find the angle between  $u$  and  $v$  and round your answer to two decimal places.

If the angle between the vectors is 0 or 180 degrees, then the vectors are **parallel**.

If the angle between the vectors is 90 degrees then the vectors are **orthogonal**.

EXAMPLE: Given  $u = -3\mathbf{i} + 2\mathbf{j}$  and  $v = 4\mathbf{i} + 6\mathbf{j}$ , find the following:

a.)  $u \cdot v$

b.)  $\|u\|$

c.)  $\|v\|$

d.) The angle between  $u$  and  $v$ .

EXAMPLE: Are the vectors  $u = 2\mathbf{i} + 5\mathbf{j}$  and  $v = 3\mathbf{i} - 7\mathbf{j}$  orthogonal, parallel, or neither?