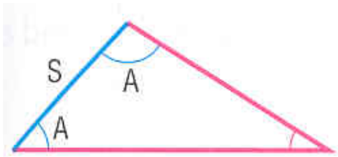
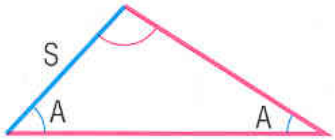
8.2 The Law of Sines

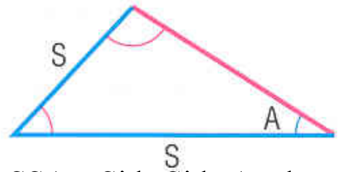
The law of sines is used to solve for missing sides or angles of triangles when we have the following three cases:



ASA – Angle Side Angle



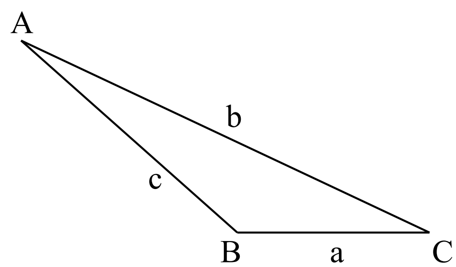
SAA – Side Angle Angle



SSA – Side Side Angle

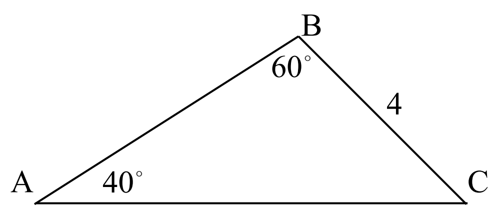
Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Usually you will only use two parts of the above formula, but all three ratios are equal.

EXAMPLE: Solve the triangle:



The first thing we can find is the measurement of angle C by subtracting the given angles from 180 degrees: $m\angle C = 180 - 40 - 60 = 80^\circ$.

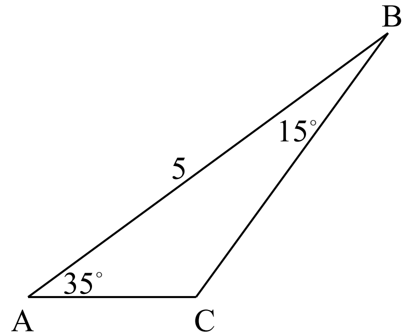
Next we can find side AC, which I will label as b. We will use the law of sines for this. You always need to start with a known side and a side opposite the known side. I will use the 40 degrees and the 4. This will be one side of the equation. The other side is for the side you want to find. Since we want to find side AC we need to use the angle opposite of this side, so we will use the 60 degrees. The equation is: $\frac{\sin 40^\circ}{4} = \frac{\sin 60^\circ}{b}$.

Cross multiplying will give us $b \sin 40^\circ = 4 \sin 60^\circ$. Solving for b we get: $b = \frac{4 \sin 60^\circ}{\sin 40^\circ} \approx 5.39$.

Now we want to find side AB, which I will call c. Once again we will start with a known angle and side opposite this angle. The equation is: $\frac{\sin 40^\circ}{4} = \frac{\sin 80^\circ}{c}$. Cross multiplying will give us $c \sin 40^\circ = 4 \sin 80^\circ$.

Solving for c we get $c = \frac{4 \sin 80^\circ}{\sin 40^\circ} \approx 6.13$. Now the triangle is solved.

EXAMPLE: Solve the triangle:

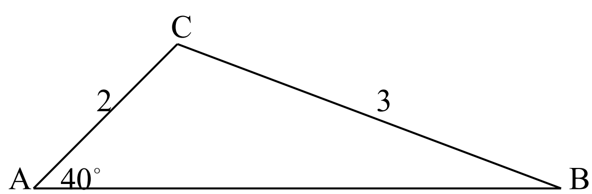


The first thing we can find is the measurement of angle C by subtracting the given angles from 180 degrees: $m\angle C = 180 - 35 - 15 = 130^\circ$.

Next we can find side AC, which I will label as b. We will use the law of sines for this. I will use the 130 degrees and the 5 as my known angle and side. The equation is: $\frac{\sin 130^\circ}{5} = \frac{\sin 15^\circ}{b}$. Cross multiplying will give us $b \sin 130^\circ = 5 \sin 15^\circ$. Solving for b we get: $b = \frac{5 \sin 15^\circ}{\sin 130^\circ} \approx 1.69$.

Now we want to find side BC, which I will call a. Once again we will start with a known angle and side opposite this angle. The equation is: $\frac{\sin 130^\circ}{5} = \frac{\sin 35^\circ}{a}$. Cross multiplying will give us $a \sin 130^\circ = 5 \sin 35^\circ$. Solving for c we get $a = \frac{5 \sin 35^\circ}{\sin 130^\circ} \approx 3.74$. Now the triangle is solved.

EXAMPLE: Solve the triangle:

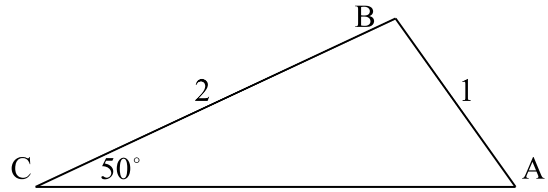


We first want to find one of the missing angles. The only one I can solve for is angle B since I have a side opposite that is given: $\frac{\sin 40^\circ}{3} = \frac{\sin B}{2}$. Now cross multiply: $2 \sin 40^\circ = 3 \sin B$. Solving for $\sin B$ we get: $\sin B = \frac{2 \sin 40^\circ}{3}$. So $\sin B = 0.4285$. Now we will take the inverse sine of both sides to get $m\angle B = 25.37^\circ$.

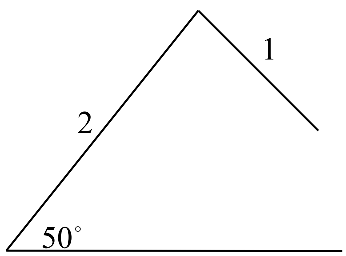
Now we can find angle C: $m\angle C = 180 - 40 - 25.37 = 114.63^\circ$.

Now that we know angle C we can find side AB. I will call this c: $\frac{\sin 40^\circ}{3} = \frac{\sin 114.63^\circ}{c}$. Cross multiplying gives us $c \sin 40^\circ = 3 \sin 114.63^\circ$. Solving for c we get $c = \frac{3 \sin 114.63^\circ}{\sin 40^\circ} \approx 4.24$.

EXAMPLE: Solve the triangle:

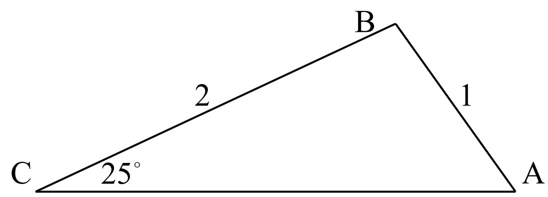


I want to solve for angle A first since there is a side opposite this angle given. We can use the equation: $\frac{\sin 50^\circ}{1} = \frac{\sin A}{2}$. Cross multiplying will give us: $2\sin 50^\circ = \sin A$. So $\sin A = 1.53$. If we try and take the inverse of this in our calculator we will get an error. This is because the domain of the inverse sine function must be between -1 and 1. There is no solution for this problem. That means it is impossible to draw this triangle. The drawing above is not to scale. If we did draw it to scale it would look something like this:

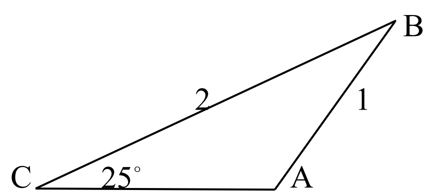


Notice that the side with a 1 is not long enough, so we can't complete the triangle. Again the answer is no solution.

EXAMPLE: Solve the triangle:



I showed on the board that the triangle above could be drawn two different ways. The first way is the above drawing. If we take side AB and swing it to the left, we will get a second triangle:



You will actually get two solutions algebraically. Let's try and solve for angle A again: $\frac{\sin 25^\circ}{1} = \frac{\sin A}{2}$.

Cross multiplying gives us: $2\sin 25^\circ = \sin A$. This will give us $\sin A = 0.8452$. The inverse sine will give us: $A = 57.7^\circ$. Now the calculator just gives us this one angle, which would correspond to our first drawing. Remember that sine is positive in the first and second quadrant, so if 57.7° is a reference angle, then we find another angle in the second quadrant by subtracting this from 180 degrees: $m\angle A = 180^\circ - 57.7^\circ = 122.3^\circ$. So we are going to get two solutions. We will find two different angle B's and two different side AC's .

Go to next page to see both solutions worked out.

Triangle 1:

If $m\angle A = 57.7^\circ$ then we can find angle B: $m\angle B = 180 - 25 - 57.7 = 97.3^\circ$

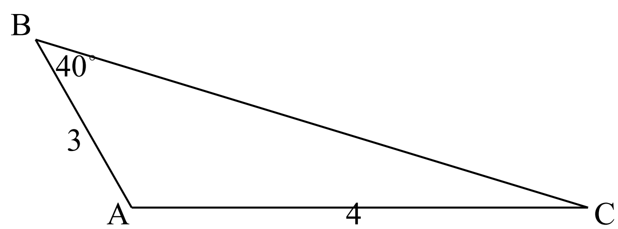
We can use this to find side AC, which I will call b: $\frac{\sin 25^\circ}{1} = \frac{\sin 97.3^\circ}{b}$. Cross multiplying will give us: $b\sin 25^\circ = \sin 97.3^\circ$. Solving for b we get: $b = \frac{\sin 97.3^\circ}{\sin 25^\circ} \approx 2.35$.

Triangle 2:

If $m\angle A = 122.3^\circ$ then we can find angle B: $m\angle B = 180 - 25 - 122.3 = 32.7^\circ$

We can use this to find side AC, which I will call b: $\frac{\sin 25^\circ}{1} = \frac{\sin 32.7^\circ}{b}$. Cross multiplying will give us: $b\sin 25^\circ = \sin 32.7^\circ$. Solving for b we get: $b = \frac{\sin 32.7^\circ}{\sin 25^\circ} \approx 1.28$.

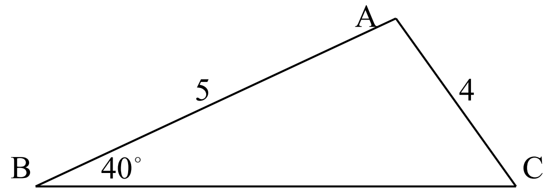
EXAMPLE: Solve the triangle:



The only angle we can solve for here is angle C since we have a side opposite angle C that is given: $\frac{\sin 40^\circ}{4} = \frac{\sin C}{3}$. Cross multiplying will give us $3\sin 40^\circ = 4\sin C$. Then we have: $\sin C = \frac{3\sin 40^\circ}{4}$. This will give us: $\sin C = 0.4821$. When we find the inverse sine our calculator gives us $m\angle C = 28.82^\circ$. Now we need to see if there will be another solution. Once again sine is positive in the second quadrant, so we can find a second solution by subtracting our answer from 180 degrees: $m\angle C = 180 - 28.82 = 151.18^\circ$. Notice that we already have an angle in the triangle that is 40 degrees, so it is impossible to also have an angle of 151.18 degrees because then the sum of the angles would be more than 180 degrees. Therefore we can ignore this second solution. We know for sure that $m\angle C = 28.82^\circ$. Now we can find angle A by subtracting from 180 degrees: $m\angle A = 180 - 28.82 - 40 = 111.18^\circ$. Finally we can find side BC, which I will label as a: $\frac{\sin 40^\circ}{4} = \frac{\sin 111.18^\circ}{a}$. Cross multiplying gives us: $a\sin 40^\circ = 4\sin 111.18^\circ$. Solving for a will give us: $a = \frac{4\sin 111.18^\circ}{\sin 40^\circ} \approx 5.8$.

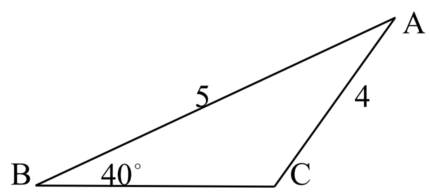
More on next page…

EXAMPLE: Solve the triangle:



We want to find angle C since we have a side opposite angle C that is given to us: $\frac{\sin 40^\circ}{4} = \frac{\sin C}{5}$. Cross multiplying will give us: $5\sin 40^\circ = 4\sin C$. Then we have: $\sin C = \frac{5\sin 40^\circ}{4}$. So $\sin C = 0.8035$. The inverse sine gives us $m\angle C = 53.46^\circ$. Our other solution for angle C is: $m\angle C = 180^\circ - 53.46^\circ = 126.54^\circ$. If we add this to the 40 degree angle already in the triangle we will get an angle less than 180 degrees, so this tells us there will definitely by two solutions to this triangle.

The first solution is the drawing shown above. If we take side AC and swing it to the left, we will get a second triangle:



Now we will solve both triangle separately:

Triangle 1:

If $m\angle C = 53.46^\circ$ then we can find angle A: $m\angle A = 180 - 40 - 53.46 = 86.54^\circ$

We can use this to find side BC, which I will label a: $\frac{\sin 40^\circ}{4} = \frac{\sin 86.54^\circ}{a}$. Cross multiplying will give us:

$a\sin 40^\circ = 4\sin 86.54^\circ$. Solving for b we get: $a = \frac{4\sin 86.54^\circ}{\sin 40^\circ} \approx 6.21$.

Triangle 2:

If $m\angle C = 126.54^\circ$ then we can find angle A: $m\angle A = 180 - 40 - 126.54 = 13.46^\circ$

We can use this to find side BC, which I will label a: $\frac{\sin 40^\circ}{4} = \frac{\sin 13.46^\circ}{a}$. Cross multiplying will give us:

$a\sin 40^\circ = 4\sin 13.46^\circ$. Solving for a we get: $a = \frac{4\sin 13.46^\circ}{\sin 40^\circ} \approx 1.45$.