

1.1 Review of Algebra and Factoring

In this section we will review various algebra tools necessary for Calculus.

EXAMPLE: Expand: $(x - 10y)^2$.

EXAMPLE: Factor $63x^3 + 21x^2$. Factor out -1 if the leading coefficient is negative.

Difference of Squares Factoring Formula: $a^2 - b^2 = (a + b)(a - b)$

EXAMPLE: Factor using difference of squares: $4x^2 - 9$

EXAMPLE: Factor $x^2 - 10x - 24$ completely. If the polynomial cannot be factored, say it is prime.

EXAMPLE: Factor $7x^3 - 28x^2 - 315x$ completely. If the polynomial cannot be factored, say it is prime.

EXAMPLE: Factor $6x^2 + 23x + 20$ using the grouping method, if possible.

EXAMPLE: Factor $2x^2 + 9x - 35$ using the grouping method, if possible.

Factoring out common factors with fractional exponents

EXAMPLE: Factor completely: $x^{\frac{7}{2}} + 8x^{\frac{5}{2}} + 12x^{\frac{3}{2}}$

EXAMPLE: Solve: $6x^{\frac{7}{3}} - x^{\frac{4}{3}} - 15x^{\frac{1}{3}} = 0$

Radical expressions can be written without the radical symbol. We can use rational (fractional) exponents. The index must be a positive integer. If the index n is even, then a cannot be negative. Below are formulas for converting a rational exponent to a radical.

$$\sqrt[n]{a} = a^{\frac{1}{n}} \qquad \sqrt[n]{a^m} = a^{\frac{m}{n}}$$

EXAMPLE: Convert the radical expression to an equivalent expression involving rational exponents: $\sqrt[9]{x^8}$.

Laws of Exponents

$$a^s \cdot a^t = a^{s+t} \qquad \text{Example:}$$

$$\frac{a^s}{a^t} = a^{s-t} \qquad \text{Example:}$$

$$(a^s)^t = a^{s \cdot t} \qquad \text{Example:}$$

$$(a \cdot b)^s = a^s \cdot b^s \qquad \text{Example:}$$

$$1^s = 1 \qquad \text{Example:}$$

$$a^0 = 1 \qquad \text{Example:}$$

$$a^{-s} = \frac{1}{a^s} \qquad \text{Example:}$$

$$\left(\frac{a}{b}\right)^{-s} = \left(\frac{b}{a}\right)^s \qquad \text{Example:}$$

EXAMPLE: Convert the radical expression to an equivalent expression involving negative rational exponents:

$$\frac{1}{\sqrt[9]{x^{11}}}.$$

EXAMPLE: Simplify $\frac{8x^3 - x^2 + 3\sqrt{x}}{x^3}$ and write with positive exponents.