1.2 Basic Classes of Functions

Slope Formula

The slope formula is used to find the slope between two points (x_1, y_1) and (x_2, y_2):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- Positive slopes will increase as you move from left to right.
- Negative slopes will decrease as you move from left to right.
- A slope of zero is a horizontal line.
- An undefined or infinity slope is a vertical line.

EXAMPLE: Find the slope of a line passing through the following points. Indicate whether the line increases, decreases, is horizontal or vertical.

a.) (-1, 3) and (2, 4)

b.) (4, -1) and (3, -1)

c.) (3, -2) and (3, -5)

Slope-Intercept Formula– this is the standard from of a line which allows you to easily identify the slope and y-intercept.

$y = mx + b$ Here the slope is m and the y-intercept is (0, b).

Linear Function– this is the same as the slope-intercept form, except with function notation. In general, a linear function begins with $f(x)$ and contains an x with a power of 0 or 1.

$f(x) = mx + b$

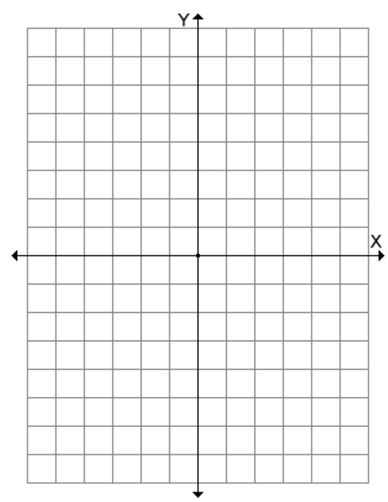
Point-Slope Formula – this is used when you want to find the equation of a line when you are given a slope and another point on the line. This other point does not need to be the y-intercept.

$y - y_1 = m(x - x_1)$

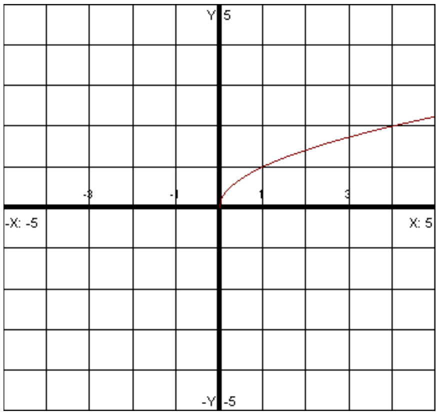
EXAMPLE: Use the information and given conditions to write an equation for each line in slope-intercept form as well as the point-slope form.

Passing through (-3, 6) and (3, -2)

EXAMPLE: Write the following $4x + 6y = -12$ in slope-intercept form and identify the slope and y-intercept. Use this information to graph the equation.



The following is a library of functions that you should know since sketches are sometimes necessary in Calculus.



$y = \sqrt{x}$

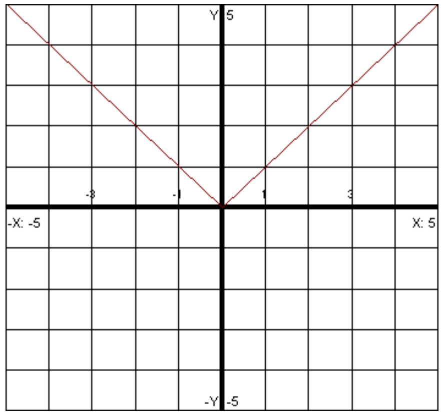
Square Root Function

Domain: $[0, \infty)$

Range: $[0, \infty)$

Increasing: $(0, \infty)$

Decreasing: None



$y = |x|$

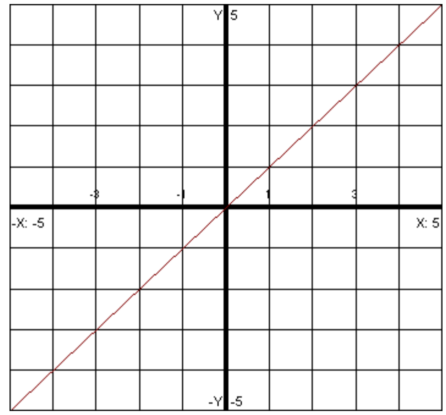
Absolute Value Function

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Increasing: $(0, \infty)$

Decreasing: $(-\infty, 0)$



$y = x$

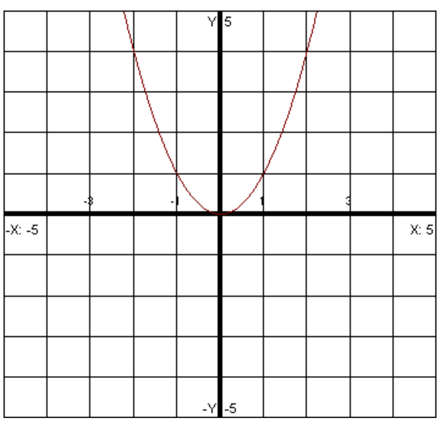
Identity Function

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Increasing: $(-\infty, \infty)$

Decreasing: None



$y = x^2$

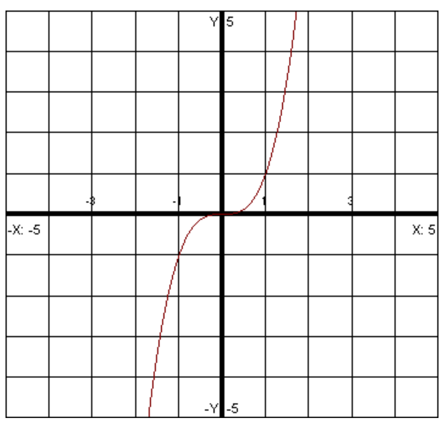
Standard Quadratic Function

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Increasing: $(0, \infty)$

Decreasing: $(-\infty, 0)$



$y = x^3$

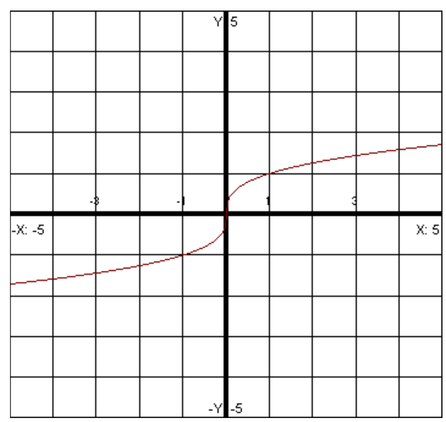
Standard Cube Function

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Increasing: $(-\infty, \infty)$

Decreasing: None



$y = \sqrt[3]{x}$

Cube Root Function

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Increasing: $(-\infty, \infty)$

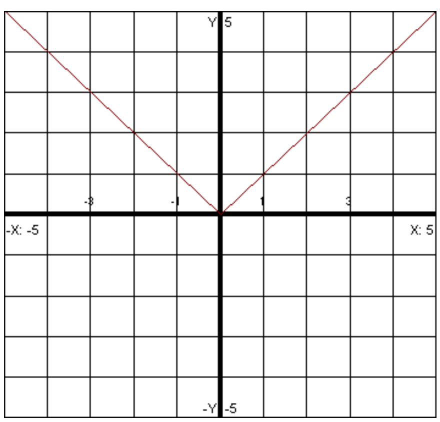
Decreasing: None

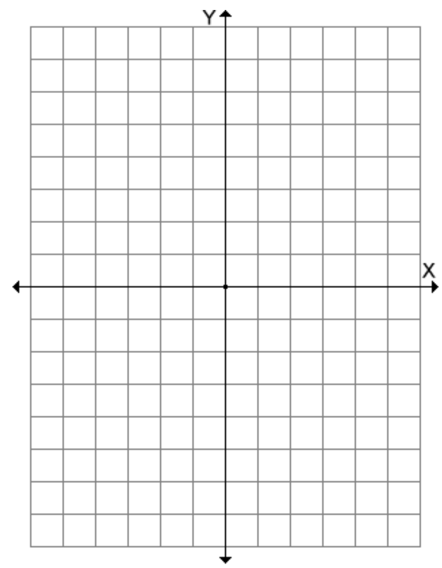
Transformations and Graph Sketches

Suppose $y = f(x)$ is the original function (one we looked at in a previous section)

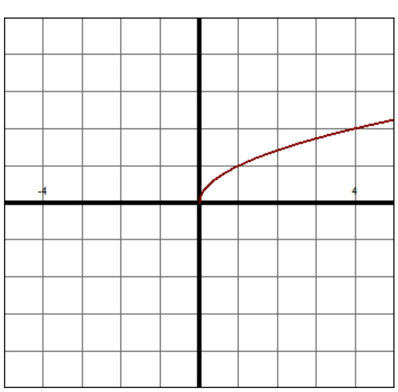
$y = f(x)+k$ moves $f(x)$ k units up
$y = f(x)-k$ moves $f(x)$ k units down
$y = f(x-h)$ moves $f(x)$ h units to the right
$y = f(x+h)$ moves $f(x)$ h units to the left
$y = -f(x)$ flips the graph over the horizontal axis
$y = f(-x)$ flips the graph over the vertical axis
$y = a \cdot f(x)$ If $|a|>1$ then there is a vertical stretch. If $0<|a|<1$, then there is a vertical compression.

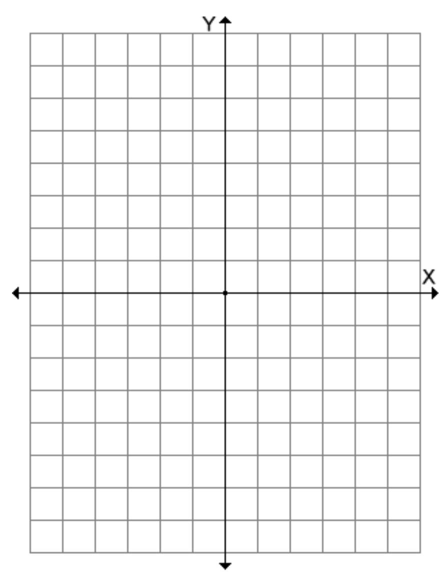
EXAMPLE: Sketch $y = -|x+1|+2$ by using transformations.





EXAMPLE: Sketch $y = \sqrt{4-x}+2$ by using transformations.





Piecewise Functions

These functions are made up of different pieces. Each piece is defined for certain values of x.

EXAMPLE: Use the function $f(x)=\begin{cases} x+2 & if \quad x<-3 \\ x-2 & if \quad x\geq -3 \end{cases}$ to find $f(-4)$, $f(-3)$ and $f\left(-\frac{3}{2}\right)$. Then graph. and use this to determine the graph's range.

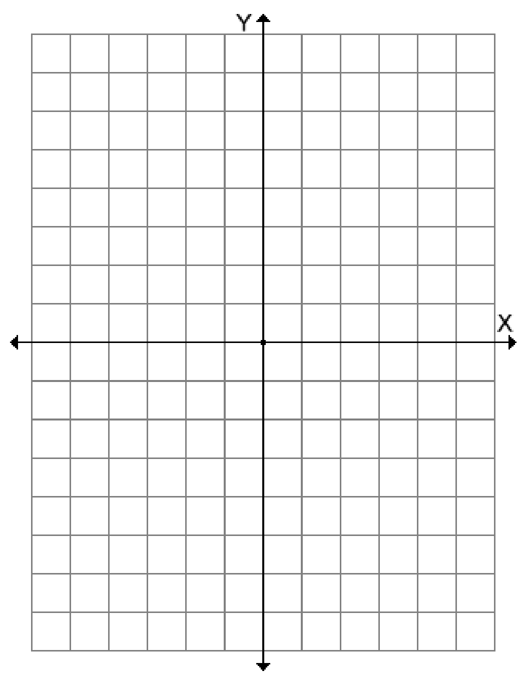
a.) $f(-4)$

b.) $f(-3)$

c.) $f\left(-\frac{3}{2}\right)$ If you don't know if this fraction is less or more than -3 then turn it into a decimal. This is -1.5.

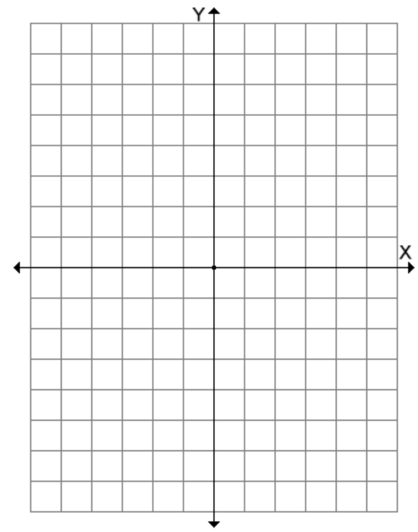
x	$y=x+2$	(x, y)
-5		
-4		
-3		

x	$y=x-2$	(x, y)
-3		
-2		
-1		



EXAMPLE: Find the intercepts, vertex, axis of symmetry, domain, range, and the graph of $y = x^2 - 6x + 5$.

x-int: 0 for y



y-int: 0 for x

Vertex:

Axis of Symmetry:

Domain:

EXAMPLE: Suppose the height of an object shot straight up is given by $h = 512t - 16t^2$ where h is measured in feet and t is in seconds. Find the maximum height and the time at which the object hits the ground.