

1.4 Inverse Functions

Inverse Function:

Notation to write “the inverse of $f(x)$ ” is $f^{-1}(x)$. If two functions f and f^{-1} are inverses then the following must be true: $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$. Note that these functions must be one-to-one.

EXAMPLE: Given $f(x) = 2x - 1$ and $f^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$ verify that they are inverses.

How to find an inverse algebraically:

Step 1: Replace $f(x)$ with y .

Step 2: Switch x and y .

Step 3: Solve for y .

Step 4: Replace y with $f^{-1}(x)$.

EXAMPLE: Given $f(x) = \sqrt{x+7}$ find $f^{-1}(x)$.

Step 1:

Step 2:

Step 3:

Step 4:

Inverse Trigonometric Functions

From trigonometry we know that $\sin 30^\circ = \frac{1}{2}$. We put in an angle and get a value as a result. In inverse trig functions we put in the value and get an angle: $\sin^{-1} \frac{1}{2} = 30^\circ$. So here we put in the value of one half and got 30 degrees as a result. We are not allowed to put any number into our inverse trig functions. There are restrictions on the domain that are given in the following table:

| | Domain | Range |
|-------------------|------------------------|--|
| $y = \sin^{-1} x$ | $-1 \leq x \leq 1$ | $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ |
| $y = \cos^{-1} x$ | $-1 \leq x \leq 1$ | $0 \leq y \leq \pi$ |
| $y = \tan^{-1} x$ | $-\infty < x < \infty$ | $-\frac{\pi}{2} < y < \frac{\pi}{2}$ |

EXAMPLE: Find $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

EXAMPLE: Find $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$.

EXAMPLE: Find $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$.

EXAMPLE: Find $\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$.

EXAMPLE: Find the exact value: $\cos^{-1}\left(\cos\frac{\pi}{3}\right)$.