2.1 The Limit of a Function and Limit Laws

Let's look at the graph $y = \frac{x-2}{x^2-4}$. What is y(2)? That's right, its undefined, but what if we wanted to find the y value the graph is approaching as we get close to an x value of 2? This y-value that it is approaching is called a **limit**. Here's some notation for the problem we were just describing.

$\lim_{x \to 2} \frac{x-2}{x^2-4}$ What this means is that we want to find what **y-value** the graph is approaching as x gets close to 2:

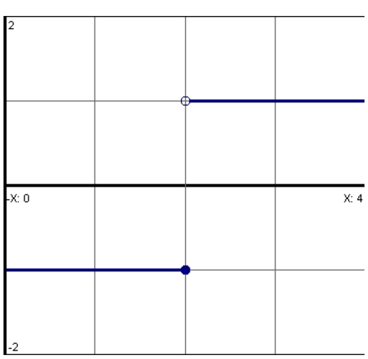
x	1.9	1.99	1.999	2.001	2.01
y(x)	.2564	.2506	.25001	.2499	.2493

By looking at this table, it appears the y-value is approaching 0.25, or 1/4. So you would write your answer as:

$$\lim_{x \to 2} \frac{x-2}{x^2-4} = \frac{1}{4}$$

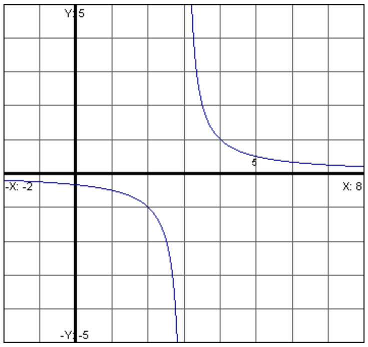
By algebra:

Limits must approach the same number. For example, let's look at the following graph of f(x):



Let's look at $\lim_{x \to 2} f(x)$.

What about $\lim_{x \to 3} f(x)$ if the graph below is of f(x)?



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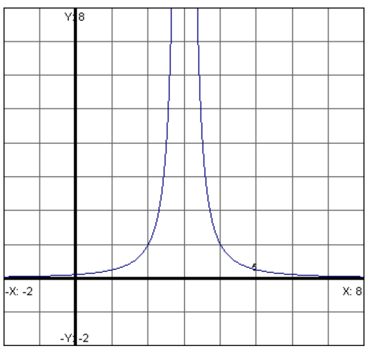
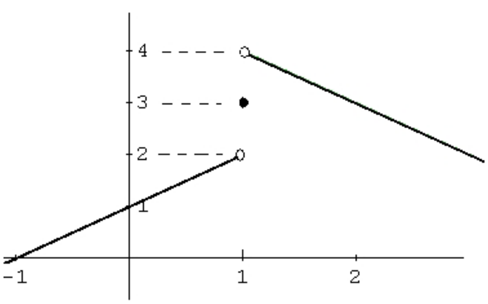


Let's look at the graph below:



First, does $f(1)$ exist?

What is $\lim_{x \to 1} f(x)$?

What is $\lim_{x \to -1} f(x)$?

Limit Laws

1.) *Sum/Difference Rule*: $\lim_{x \to c}\left(f(x) \pm g(x)\right) = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x)$

2.) *Constant Multiple Rule*: $\lim_{x \to c}\left(k \cdot f(x)\right) = k \cdot \lim_{x \to c} f(x)$

3.) *Product Rule*: $\lim_{x \to c}\left(f(x) \cdot g(x)\right) = \lim_{x \to x} f(x) \cdot \lim_{x \to c} g(x)$

4.) *Quotient Rule*: $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$

5.) *Power Rule*: $\lim_{x \to c} [f(x)]^n = \left[\lim_{x \to c} f(x)\right]^n$

6.) *Root Rule*: $\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to c} f(x)}$

EXAMPLE: Find: $\lim_{x \to 5} 2x^2 + 7$ by using limit properties to break it down.

EXAMPLE: Find: $\lim_{x \to 2} \frac{3x^2 - x + 2}{x + 2}$.

EXAMPLE: Find: $\lim_{x \to -3} \frac{x+3}{x^2-9}$.

EXAMPLE: Find: $\lim_{x \to 4} \frac{x^2-5x+4}{x^2-2x-8}$.

EXAMPLE: Find: $\lim_{x \to 3} \frac{\sqrt{x+1}}{x-4}$.

EXAMPLE: Find $\lim_{x \to -31} \sqrt[3]{x+4}$

EXAMPLE: Find: $\lim_{x \to 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$.

EXAMPLE: Find: $\lim_{x \to 3} \frac{x-3}{\sqrt{x+1}-2}$.

Limits with Trigonometric Functions

EXAMPLE: Find $\lim_{x \to \pi} \tan x$

EXAMPLE: Find $\lim_{x \to \frac{\pi}{6}} \cos 3x$

EXAMPLE: Find $\lim_{x \to \frac{\pi}{3}} 3\sin x$

EXAMPLE: Find $\lim_{\theta \to \frac{\pi}{2}} \frac{\tan\theta}{\sec\theta}$

Sandwich (Squeeze) Theorem

Suppose that $g(x) \le f(x) \le h(x)$ for all x in some open interval containing c, except possibly at $x = c$ itself. Suppose also that $\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L$. Then $\lim_{x \to c} f(x) = L$.

EXAMPLE: Find $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x^2}\right)$.

EXAMPLE: If $3x-1 \le f(x) \le x^2+1$, find $\lim_{x\to 2} f(x)$ using the Squeeze Theorem.