

## 2.2 The Precise Definition of a Limit

### Precise definition of a limit

EXAMPLE: Use the  $\varepsilon$ - $\delta$  definition of a limit to prove that  $\lim_{x \rightarrow 3} 5x - 4 = 11$ .

Proof:

Scratchwork:

EXAMPLE: Use the  $\varepsilon$ - $\delta$  definition of a limit to prove that  $\lim_{x \rightarrow -4} \frac{x}{2} + 6 = 4$ .

Proof:

Scratchwork:

EXAMPLE: In the following exercises, find an open interval about  $x_0$  on which the inequality  $|f(x) - L| < \varepsilon$  holds. Then give the largest value for  $\delta > 0$  such that for all  $x$  satisfying  $0 < |x - x_0| < \delta$  the inequality  $|f(x) - L| < \varepsilon$  holds:

a.)  $f(x) = 2x - 2$ ,  $L = -6$ ,  $x_0 = -2$ ,  $\varepsilon = 0.02$

b.)  $f(x) = \sqrt{x-7}$ ,  $L = 4$ ,  $x_0 = 23$ ,  $\varepsilon = 1$

c.)  $f(x) = 1/x$ ,  $L = -1$ ,  $x_0 = -1$ ,  $\varepsilon = 0.1$

d.)  $f(x) = x^2$ ,  $L = 3$ ,  $x_0 = \sqrt{3}$ ,  $\varepsilon = 0.1$  (Round answers to four decimal places)