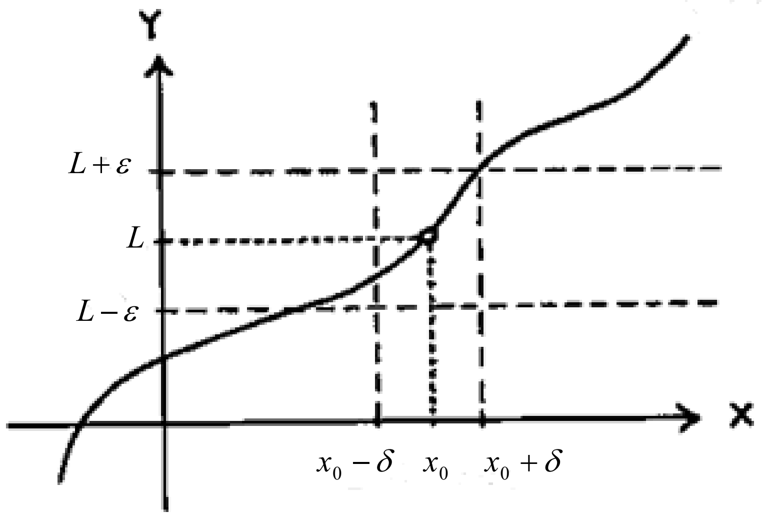
2.2 The Precise Definition of a Limit

Precise definition of a limit



Let *f* be defined on an open interval containing c and let L be a real number. Then:

$\lim_{x \to x_0} f(x) = L$ means that for each $\varepsilon > 0$ there exists a $\delta > 0$ such that if $0 < |x - x_0| < \delta$, then $|f(x) - L| < \varepsilon$.

EXAMPLE: Use the ε-δ definition of a limit to prove that $\lim_{x \to 3} 5x - 4 = 11$.

<u>Proof:</u>
For each $\varepsilon > 0$ there exists a $\delta > 0$ such that if $0 < |x - 3| < \delta$, then $|5x - 4 - 11| < \varepsilon$.

EXAMPLE: Use the ε - δ definition of a limit to prove that $\lim_{x \to 4} \frac{x}{2} + 6 = 8$.

Proof:

For each $\varepsilon > 0$ there exists a $\delta > 0$ such that if $0 < |x - 4| < \delta$, then $\left|\frac{x}{2} + 6 - 8\right| < \varepsilon$.

EXAMPLE: In the following exercises, find an open interval about x_0 on which the inequality $|f(x) - L| < \varepsilon$ holds. Then give the largest value for $\delta > 0$ such that for all x satisfying $0 < |x - x_0| < \delta$ the inequality $|f(x) - L| < \varepsilon$ holds:

a.) $f(x) = 2x - 2$, L = -6, $x_0 = -2$, $\varepsilon = 0.02$

b.) $f(x)=\sqrt{x-7}$, L = 4, $x_0=23$, $\varepsilon=1$

c.) $f(x)=1/x$, L = -1, $x_0=-1$, $\varepsilon=0.1$

d.) $f(x) = x^2$, L = 3, $x_0 = \sqrt{3}$, $\varepsilon = 0.1$ (Round answers to four decimal places)