

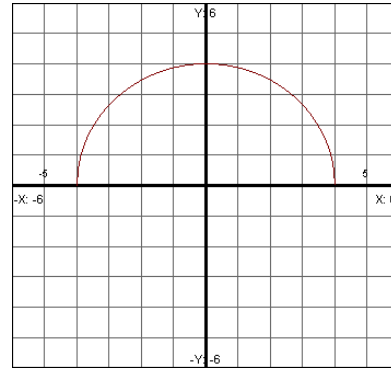
## 2.3 One-Sided Limits

### One-Sided Limits

$\lim_{x \rightarrow c^+} f(x) = L$  This means we are finding the limit of  $f$  as we approach  $c$  from the right (positive side)

$\lim_{x \rightarrow c^-} f(x) = L$  This means we are finding the limit of  $f$  as we approach  $c$  from the left (negative side)

EXAMPLE: Find the limit:  $\lim_{x \rightarrow 4^-} \sqrt{16 - x^2}$

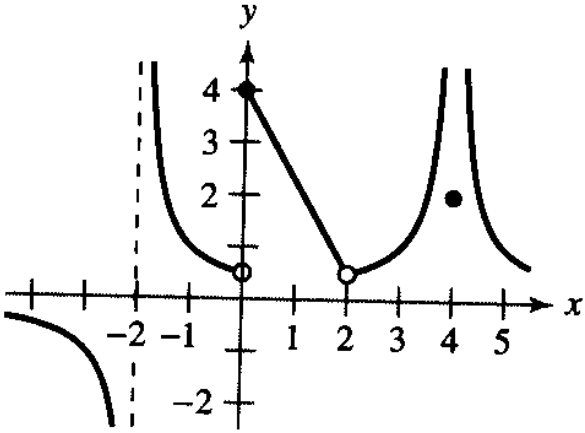


EXAMPLE: Find the limit:  $\lim_{x \rightarrow 2^+} \frac{2 - x}{x^2 - 4}$

### Finding One-Sided Limits Algebraically

EXAMPLE: Find the limit:  $\lim_{x \rightarrow 1^-} \left( \frac{1}{x+1} \right) \left( \frac{x+6}{x} \right) \left( \frac{3-x}{7} \right)$ .

EXAMPLE: Use the graph of  $f(x)$  below to find the following:



a.)  $f(0)$

b.)  $f(2)$

c.)  $f(-2)$

d.)  $f(4)$

e.)  $\lim_{x \rightarrow 2^+} f(x)$

f.)  $\lim_{x \rightarrow 2^-} f(x)$

g.)  $\lim_{x \rightarrow 2} f(x)$

h.)  $\lim_{x \rightarrow 0^+} f(x)$

i.)  $\lim_{x \rightarrow 0^-} f(x)$

j.)  $\lim_{x \rightarrow 0} f(x)$

k.)  $\lim_{x \rightarrow -2} f(x)$ .

**Limits involving  $\frac{\sin \theta}{\theta}$ .**

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin \theta^2}{\theta^2} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin k\theta}{k\theta} = 1 \text{ where } k \text{ is any real number.}$$

EXAMPLE: Find  $\lim_{\theta \rightarrow 0} \frac{\sin 8\theta}{5\theta}$ .

EXAMPLE: Find  $\lim_{\theta \rightarrow 0} \frac{\theta^2 - \theta + \sin \theta}{3\theta}$ .

**Limits involving**  $\frac{1 - \cos \theta}{\theta}$ .

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta} = 0 \quad \text{where } \theta \text{ is in radians.}$$

EXAMPLE: Find  $\lim_{\theta \rightarrow 0} \frac{1 - \cos 3\theta}{6\theta}$

EXAMPLE: Find  $\lim_{x \rightarrow 0} \frac{\sin x - \sin x \cos 3x}{(3x^2 \sin^2 x + 3x^2 \cos^2 x)}$

EXAMPLE: Find  $\lim_{x \rightarrow 0} 9x^2 \cot x \csc(3x)$