2.4 Continuity

Continuity: graph is connected with no breaks or holes.

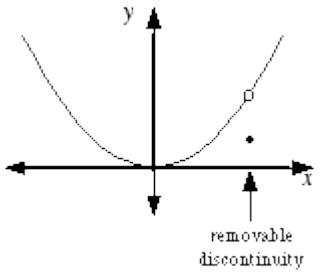
Let $f(x)$ be a graph. Continuity at x = c occurs if all three of the following are true:

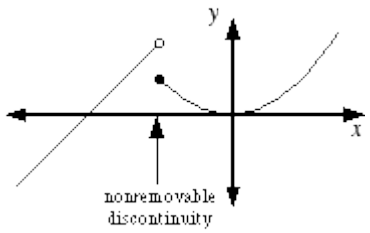
1.) $f(c)$ is defined. (There are no vertical asymptotes or holes at $x = c$)
2.) $\lim_{x \to c} f(x)$ exists. (If it didn't exist, then there must be a break in the graph or a vertical asymptote.)
3.) $\lim_{x \to c} f(x) = f(c)$ (Since these y-values are equal, there can't be a hole.)

If even one of the above is false then the function is discontinuous. There are two types of discontinuities:

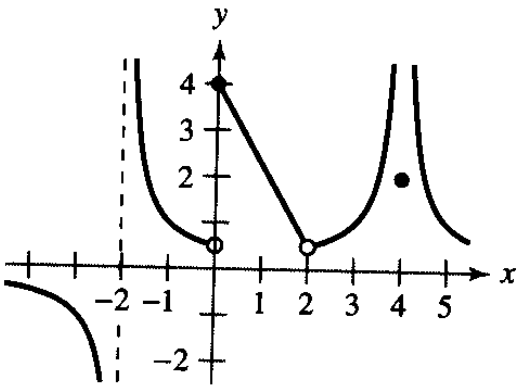
1.) Removable discontinuity: A point can be assigned to "plug up" the hole.

2.) Nonremovable discontinuity: No point can cause the graph to be connected.





Let's look at the below graph from the previous section. Can you tell which places the graph is discontinuous? Which ones are removable and nonremovable?



EXAMPLE: Describe the set of x-values where the function is continuous, using interval notation.

$f(x) = (3-x)^{\frac{1}{5}}$

EXAMPLE: Describe the set of x-values where the function is continuous, using interval notation.

$f(x) = \sqrt{6x-35}$

EXAMPLE: Describe the set of x-values where the function is continuous, using interval notation.

$f(x) = \dfrac{1}{x^2+1}$

EXAMPLE: Determine the x-values where $f(x)=\dfrac{\tan x}{x^2+1}$ is continuous. Write an exact answer, using π as needed. Write the expression using *n*, where *n* is any **odd** integer.

The function $f(x)=\dfrac{\tan x}{x^2+1}$ is continuous on $(-\infty,\infty)$ except at $x=$ _____

EXAMPLE: Determine the x-values where $f(x)=4\csc(4x)$ is continuous. Write an exact answer, using π as needed. Write the expression using *n*, where *n* is any integer.

The function $f(x)=4\csc(4x)$ is continuous on $(-\infty,\infty)$ except at $x=$_____

EXAMPLE: Indicate the x-values (if any) at which f is not continuous. Then indicate the x-values where f has removable discontinuities and nonremovable discontinuities.

$$f(x) = \frac{x-3}{x^2-9}$$

Discontinuities:_____

Removable:_____

Nonremovable:_____

EXAMPLE: Indicate the x-values (if any) at which f is not continuous. Then indicate the x-values where f has removable discontinuities and nonremovable discontinuities.

$$f(x) = \frac{x+2}{x^2-2x-15}$$

Discontinuities:_____

Removable:_____

Nonremovable:_____