2.4 Continuity

Continuity: graph is connected with no breaks or holes.

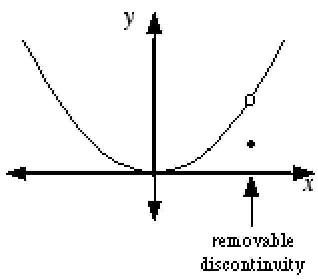
Let $f(x)$ be a graph. Continuity at x = c occurs if all three of the following are true:

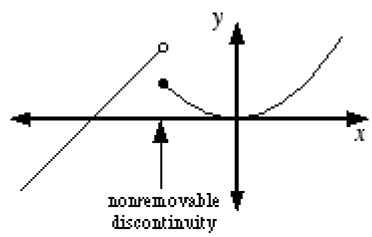
1.) $f(c)$ is defined. (There are no vertical asymptotes or holes at $x = c$)
2.) $\lim_{x \to c} f(x)$ exists. (If it didn't exist, then there must be a break in the graph or a vertical asymptote.)
3.) $\lim_{x \to c} f(x) = f(c)$ (I can plug c in for x since there is not a hole)

If even one of the above is false then the function is discontinuous. There are two types of discontinuities:

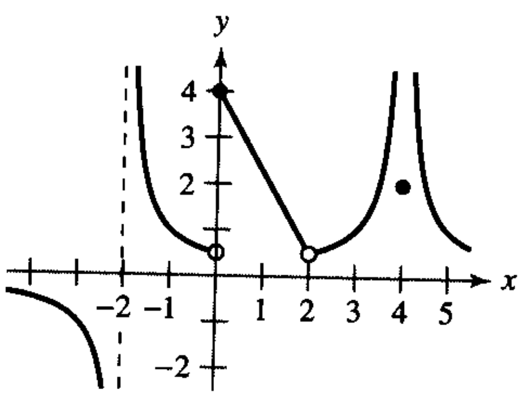
1.) Removable discontinuity: A point can be assigned to "plug up" the hole.

2.) Non-removable discontinuity: No point can cause the graph to be connected.





Let's look at the below graph from the previous section. Can you tell which places the graph is discontinuous? Which ones are removable and non-removable?



EXAMPLE: Describe the set of x-values where the function is continuous, using interval notation.

$$f(x) = (3 - x)^{\frac{1}{5}}$$

EXAMPLE: Describe the set of x-values where the function is continuous, using interval notation.

$$f(x) = \sqrt{6x - 35}$$

EXAMPLE: Describe the set of x-values where the function is continuous, using interval notation.

$$f(x) = \frac{1}{x^2 + 1}$$

EXAMPLE: Describe the set of x-values where the function is continuous, using interval notation. Use π as needed. Write an expression using n, where n is any integer.

$$f(x) = \frac{\tan x}{x^2 + 1}$$

EXAMPLE: Describe the set of x-values where the function is continuous, using interval notation. Use π as needed. Write an expression using n, where n is any integer.

$$f(x) = 4\csc(4x)$$

EXAMPLE: Describe the set of x-values where the function is continuous, using interval notation.

$$f(x) = \frac{x-3}{x^2-9}$$

EXAMPLE: Describe the set of x-values where the function is continuous, using interval notation.

$$f(x) = \frac{x+2}{x^2-2x-15}$$

EXAMPLE: Indicate points of discontinuity, if any, and classify them as removable or non-removable:

$$f(x) = \begin{cases} \dfrac{x^3 - 8}{x^2 - 4}, & x \neq 2 \quad x \neq -2 \\ 3, & x = 2 \\ 4, & x = -2 \end{cases}$$